How to Price Efficiently European Options in Some Geometric Lévy Processes Models?

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ABSTRACT

This paper presents the implementation to the class of jump diffusion models of the approach used by Boyarchenko and Levendorskiĭ (2002) in the case of exponential Lévy models. We show that this approach is more computationally efficient than the semi-closed form solutions formerly obtained by Kou (2002). A brand new model is then presented. It extends and generalizes Kou model.

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I.   INTRODUCTION

It is now well recognized that the Gaussian hypothesis for financial assets returns is a convenient assumption but which is clearly rejected when returns are computed with high or medium frequencies. To depart from the traditional hypothesis a general modelling has been put forward for the recent years. It can be expressed in the following way: asset prices are exponentials of Lévy processes, otherwise stated the returns are Lévy processes or asset prices are geometric Lévy processes. The class of Lévy processes is very large and includes arithmetic Brownian motion. Amongst the many candidates to describe financial series and frequently used are: Generalized Hyperbolic, Normal Inverse Gaussian, Meixner, Variance Gamma, CGMY processes and of course jump diffusions. This latter group has been extensively analyzed and constitutes probably a simple and flexible choice. Furthermore any of the previous cited processes can be approximated as jump diffusions.

With geometric Lévy processes the solutions to pricing and hedging problems rarely are given in closed forms. The aim of this research is to see how efficient the Fourier approach is.

This paper is organized as follows: Section II gives the definitions of the considered processes in this article, Section III briefly develops the pricing of options in this context, Section IV presents our numerical analysis and a conclusion ends the paper.

II.   THE CONSIDERED JUMP DIFFUSIONS

In this section firstly we present a simple setting for jump diffusions, then we define Kou processes and in a third part we suggest a new model which extends Kou model to multiple jumps.

A.   Assets Price Dynamics

The three usual ways to define a jump diffusion process are by specifying its dynamics or by expressing its exponential argument, or by giving its characteristic function. Let us denote by S_t the financial asset price at time t. The first definition leads us to say that our jump diffusion process obeys the following stochastic differential equation:

$$\frac{dS_t}{S_{t^-}} = \mu \, dt + \sigma \, dW_t + d\left(\sum_{i=1}^{N_t}(Y_i - 1)\right) \tag{1}$$

The last term models the jumps. In fact a jump is modelled by a random variable Y which transforms the price S_t to YS_t . The difference $Y - 1$ is then the relative change in price when a Poisson jump occurs. The mathematical expectation of this relative change is given by $\kappa = E(Y - 1)$. The intensity or mean arrival rate of the jumps per unit time is λ. W is a standard Brownian motion with $W_0 = 0$. The constants μ and $\sigma > 0$ are respectively the drift and diffusive coefficient of the continuous part of the process. N is a Poisson process with intensity λ.

Using Itō's lemma, we have the expression of the process in an exponential form:

$$S_t = S_0 \exp(X_t)$$

where S_0 is the asset price at time zero and X can be written as

$$X_t = \underbrace{at + \sigma W_t}_{continuous\ part} + \underbrace{\sum_{i=1}^{N_t} J_i}_{jumps} \tag{2}$$

where $a = \mu - \frac{\sigma^2}{2}$. The random variables (J_i) are independent and identically distributed and model jump sizes. They are such that $J_i = \ln(Y_i)$. We denote by $\phi_J(u) = E[e^{iuJ}]$ their characteristic function. The processes W, N and the random variables (J_i) are supposed to be independent. With this modelling, the characteristic function of the random variable X_t writes

$$\phi_{X_t}(u) = e^{t\left(iau - \frac{\sigma^2}{2} + \lambda(\phi_J(u) - 1)\right)} = e^{-t\psi(u)}$$

So the characteristic exponent of X reads

$$\psi(u) = -iau + \frac{\sigma^2}{2} - \lambda(\phi_J(u) - 1) \tag{3}$$

Now we can precisely define the two jump diffusion processes considered in our article: the Kou process and a new process generalizing it.

B.   Kou Model

In this model (Kou 2002), the asset price dynamics is similar to the previous dynamics, what identifies this process is the jumps law. The jumps have a common law J whose asymmetric double exponential density $f_J(y)$ writes

$$f_J(y) = p\,\lambda_1 e^{-\lambda_1 y} 1_{y>0} + q\,p\,\lambda_2 e^{\lambda_2 y} 1_{y\leq 0} \tag{4}$$

with $p \geq 0$, $q \geq 0$, $p + q = 1$, $\lambda_1 > 0$, and $\lambda_2 > 0$.

We call such process a Kou process. The jump part characteristic function writes

$$\phi_J(u) = \frac{p\,\lambda_1}{\lambda_1 - iu} + \frac{q\,\lambda_2}{\lambda_2 + iu}.$$

(5)

This generalized characteristic function ($u \in \mathbb{C}$) is well defined if u belongs to the strip

$$-\lambda_1 < Im(u) < \lambda_2$$

(6)

where $Im(u)$ is the imaginary part of the complex number u.

We can get $E(Y)$ as

$$E(Y) = E(e^J) = \phi_J(-i) = \frac{p\,\lambda_1}{\lambda_1 - 1} + \frac{q\,\lambda_2}{\lambda_2 + 1}.$$

(7)

We must check in the sequel, the condition $\lambda_1 > 1$ so that $E(Y) = E(e^J) < \infty$, which means a jump cannot exceed in mean 100 %. This constraint remains a reasonable condition for this modelling.

C.   A New Jump Diffusion Model

We now suggest a new process which extends Kou process. Let $(\eta_i)_{i \in P \cup N}$ be a finite collection of random exponential variables η_i with parameter λ_i, where P, N are two disjoint sets, and λ_i is strictly positive for all i. Consider the following sequence of positive numbers $(p_i)_{i \in P \cup N}$ such that $\sum_{i \in P \cup N} p_i = 1$. We suggest the following law for the jumps J:

$$J \triangleq \begin{cases} \eta_{k_1}, \text{with probability } p_{k_1}, \\ \qquad \vdots \\ \eta_{k_p}, \text{with probability } p_{k_p}, \\ -\eta_{l_1}, \text{with probability } p_{l_1}, \\ \qquad \vdots \\ -\eta_{l_n}, \text{with probability } p_{l_n}, \end{cases}$$

(8)

where $\{k_1, \dots, k_p\} = P$ and $\{l_1, \dots, l_n\} = N$. The index set P (respectively N) gathers the probabilities of the random variable J taking positive (resp. negative) values.

This jump law is a particular case of the phase-type laws described in great details in Asmussen (2000). In that setting, the negative (respectively positive) jumps follow a hyperexponential law H_n with n (resp. H_p with p) parallel channels. The probability density function of J is

$$f_J(x) = \sum_{i \in P} p_i\,\lambda_i e^{-\lambda_i x}\,1_{\mathbb{R}^+} + \sum_{i \in N} p_i\,\lambda_i e^{\lambda_i x}\,1_{\mathbb{R}^-}$$

and its characteristic function can be written as follows:

$$\phi_J(u) = \sum_{i \epsilon P} \frac{p_i \lambda_i}{\lambda_i - iu} + \sum_{i \epsilon N} \frac{p_i \lambda_i}{\lambda_i + iu},$$

(9)

a well defined function in the following regularity strip:

$$- \min_{i \in P} (\lambda_i) < Im(u) < \min_{i \in P} (\lambda_i).$$ (10)

It is obvious that the Kou jump diffusion model is a particular case of the general model we suggest here by taking $|P| = 1$ and $|Q| = 1$. As in Kou model, we must make sure that the following expectation exists

$$E(Y) = E(e^J) = \phi_J(-i) = \sum_{i \epsilon P} \frac{p_i \lambda_i}{\lambda_i - 1} + \sum_{i \epsilon N} \frac{p_i \lambda_i}{\lambda_i + 1}$$

(11)

by checking that for any $i \epsilon P$, $\lambda_i > 1$, and, consequently, no positive jump can move beyond 100 % in mean.

III.   THE PRICING

As far as the pricing is concerned, we use the fundamental principle of arbitrage in continuous time assuming a constant interest rate, denoted by r. Because of jumps, we are in an incomplete market, the risk-neutral probability is chosen here by using the Esscher measure, see chapter 9 of Cont & Tankov (2004) or Le Courtois & Quittard-Pinon (2007).

Because under the risk-neutral measure Q, discounted prices are martingales, for every $t > 0$, we must have

$$S_0 = E_Q[e^{-rt} S_t]$$
$$= E_Q[e^{-rt} S_0 e^{X_t}]$$
$$S_0 = S_0 e^{-rt} e^{-t\psi_Q(-i)}$$

Otherwise, it can be stated as

$$r + \psi_Q(-i) = 0$$ (12)

It is this condition we call equivalent martingale measure condition. Now we can begin our specific analysis. We first review the Kou solution and then present the Fourier approach.

A.   Closed Form Solutions

Merton (1976) obtained option prices in his jump diffusion model when jumps are Gaussian. His formula expresses option prices as a series of weighted Black and Scholes prices. Similarly, Kou gave a semi-closed form formula recalled below. We assume that under the chosen risk neutral measure X is a Kou process. Let us consider the probability:

$$P(X_t) = \gamma(a, \sigma, \lambda, p, \lambda_1, \lambda_2, a, t) \tag{13}$$

The γ function is defined through a semi-closed form formula and can be found in Kou (2002), p. 1098. Using this function Kou gave the call price in the following compact formula

$$P_{KJ}(S_0, \tau) = S_0 \, \gamma \left(r + \frac{\sigma^2}{2} - \lambda\kappa, \sigma, \lambda', p', \lambda_1', \lambda_2', \ln\left(K/S_0\right), \tau \right)$$

$$-Ke^{-r\tau} \, \gamma \left(r - \frac{\sigma^2}{2} - \lambda\kappa, \sigma, \lambda, p, \lambda_1, \lambda_2, \ln\left(K/S_0\right), \tau \right) \tag{14}$$

where $\kappa = E(Y - 1)$, $\lambda' = \lambda(1 + \kappa)$, $p' = \frac{p}{1+\kappa} \cdot \frac{\lambda_1}{\lambda_1 - 1}$, $\lambda_1' = \lambda_1 - 1$, and $\lambda_2' = \lambda_2 + 1$. The structural parameters λ, λ_1, λ_2, σ, p, and q are Kou parameters in the risk neutral universe. Although this price formula is seemingly very concise, the tail distribution function γ writes as a double series where each term demands the computation of a sequence of the $Hh_n(x)$ special functions (see Abramowitz and Stegun (1974), p. 691) whose implementation and coding alone are rather unwieldy.

B.   Fourier Analysis

Let us consider a European contingent claim whose payoff at maturity T is given through a function payoff g such that the payoff is $F(S_T, T) = g(X_T)$, so F represents the derivative price at maturity T when the underlying price is S_T. The approach used in this paper is based on generalized Fourier transform as expressed by Boyarchenko and Levendorskiĭ (2002). This setting is easier to implement than the Carr and Madan (1998) approach where these authors had to modify option prices to ensure integrability condition, see also Benhamou (2000) for the Fourier approach and convergence conditions.

Suppose g is a measurable function and there exists a real number δ such that $e^{\delta x} g(x)$ is integrable ($\in \mathcal{L}^1$). Let us define by $\hat{g}$ the following generalized Fourier transform of g:

$$\hat{g}(u) = \int_{\mathbb{R}} e^{-iux} g(x) dx$$

$$\tag{15}$$

It can be extended on the line that $Im(u) = \delta$.

1.   General pricing formula

It is possible (see Boyarchenko and Levendorskiĭ (2002)) to get the following formula for every European derivative

$$F(S_0 e^x, t) = \frac{1}{2\pi} \int_{\mathbb{R}+i\delta} e^{iux} e^{-\tau(r+\psi(u))} \hat{g}(u) du$$

(16)

For European options, the general pricing formula writes:

$$F(S_t, t) = K \frac{1}{2\pi} \int_{\mathbb{R}+i\delta} \frac{e^{iux} e^{-\tau(r+\psi(u))}}{(-iu)(-iu+1)} du$$

(17)

where $x = \ln(S_t/K)$. With the constraint $\delta < -1$ on the integration line, we obtain the arbitrage free European call price

$$C(S_t, t) = -K \frac{1}{2\pi} \int_{\mathbb{R}+i\delta} \frac{e^{iux} e^{-\tau(r+\psi(u))}}{(u)(u+1)} du$$

(18)

while we get the pricing of a European put with $\delta > 0$.

2.   Numerical implementation

With a choice for the g function and its Fourier transform $\hat{g}$, with equation (18), using a variable change $u \rightarrow u + i\delta$ gives (up to the factor K)

$$f(x, t) = \frac{1}{2\pi} \int_{\mathbb{R}} e^{iux} e^{-\tau r - \delta x} \frac{e^{-\tau\psi(u+i\delta)}}{(-iu+\delta)(-iu+\delta+1)} du$$

where $f(x, t) = F(S_0 e^x, t)/K$.
Grouping the x and u terms leads to rewrite the general formula for European options (17) in the following synthetic expression

$$f(x, t) = R(x, \delta) \times \frac{1}{2\pi} \int_{\mathbb{R}} e^{iux} \varphi(u, \delta) du$$

(19)

where $R(x, \delta) = e^{-\tau r - \delta x}$ and $\varphi(u, \delta) = \frac{e^{-\tau\psi(u+i\delta)}}{(-iu+\delta)(-iu+\delta+1)}$.
This last expression immediately calls for using the Fast Fourier Transform (FFT), which permits to obtain N simultaneous results in a computation time of order $O(N \log_2(N))$.

IV.   NUMERICAL ANALYSIS

In this section, from a numerical viewpoint we revisit Kou model and its multiple jumps extension. We can only present in this latter case the Fourier method results for we do not have, as far as we know, explicit or quasi explicit formula. We postulate an initial value for the underlying asset of $S_0 = 100$, the constant interest rate is $r = 0.05$, the maturity is one half year $\tau = 0.05$, the diffusive coefficient $\sigma = 0.16$, the intensity $\lambda = 1$ and we take the following values for the double exponential density $p = 0.4, \lambda_1 = 10$, and $\lambda_2 = 5$.

We consider two series: C_1 and C_2. The first one corresponds to contracts C_1 where exercise prices vary from 90 to 110 by a step of 2, while the second series of contracts C_2 varies a little more than half a basis point precisely with a step of 0.615 %. We shall see later what the meaning of this series is.

A.   Kou Semi Closed form Solution

Table 1 gives European call prices. The convergence of the double series which intervenes to compute the probability (13) does not necessitate many terms. Experience shows that seven terms are sufficient to attain an absolute error less than 10^{-6}. With thirteen terms our tests show that the relative error is never greater than 10^{-16}. These prices are reported in Table 1.

Table 1

European call option pricing in Kou's double exponential jump diffusion model. Prices computed from formula (14) and rounded to the nearest to seven decimals.

Kou's double exponential model European call option $S_0 = 100, r = 0.05, \sigma = 0.16, \tau = 0.5, \lambda = 1, p = 0.4, \lambda_1 = 10, \lambda_2 = 5$			
C_1 series		C_2 series	
Strike	Call price	Strike	Call price
90.00	14.8118905	97.00	9.7789477
92.00	13.2764024	97.60	9.3989071
94.00	11.8139684	98.20	9.0253635
96.00	10.4346054	98.80	8.6586420
98.00	9.1473173	99.41	8.2990574
100.00	7.9594292	100.02	7.9469119
102.00	6.8760520	100.64	7.6024934
104.00	5.8997425	101.26	7.2660732
106.00	5.0303905	101.88	6.9379047
108.00	4.2653317	102.51	6.6182210
110.00	3.5996498	103.14	6.3072339

The contract in the C_1 series whose exercise price is $K = 98$ corresponds to an example given by Kou (2002) where the option price is 9.14732, which is exactly what we obtain here, rounding to the nearest to five decimals.

B.   Kou Model Results with FFT

From equations (3) and (5), the Kou process characteristic exponent is

$$\psi_Q(u) = -iau + \frac{\sigma^2}{2} - \lambda\left(\frac{p\,\lambda_1}{\lambda_1 - iu} + \frac{q\,\lambda_2}{\lambda_2 + iu} - 1\right)$$

where $p + q = 1$, $\lambda_1 > 1$, $\lambda_2 > 0$, and convergence condition (6) $-\lambda_1 < Im(u) < \lambda_2$. In this case the equivalent martingale measure restriction writes:
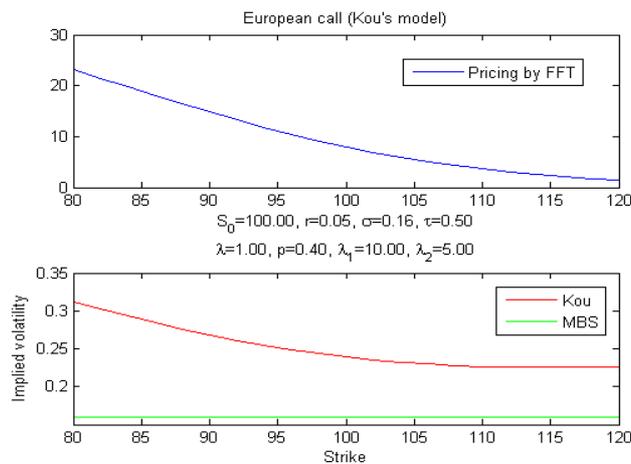
$$0 = r + \psi_Q(-i) = r - a - \frac{\sigma^2}{2} - \lambda\left(\frac{p\,\lambda_1}{\lambda_1 - 1} + \frac{q\,\lambda_2}{\lambda_2 + 1} - 1\right),$$

or:

$$a = r - \frac{\sigma^2}{2} - \lambda\left(\frac{p\,\lambda_1}{\lambda_1 - 1} + \frac{q\,\lambda_2}{\lambda_2 + 1} - 1\right).$$

Figure 1 gives the volatility smile with the Kou model in contrast with the Black and Scholes model where the volatility is constant.

Figure 1
European call option prices and volatility smile – Kou model

In table 2 are the errors with respect to the semi closed formulae results given in Table 1 when using call formula (19). These options refer to the same contracts C_2 and C_2 as before. The chosen integration contour is $\mathbb{R} + i\delta$ with δ the midpoint of $-\lambda_1$ and -1.

Table 2
Error made when pricing via fast Fourier transform against prices obtained from Kou's semi-closed form formula.

Kou's double exponential model
European call option – pricing by FFT
$S_0 = 100, r = 0.05, \sigma = 0.16, \tau = 0.5, \lambda = 1, p = 0.4, \lambda_1 = 10, \lambda_2 = 5$

C_1 series		C_2 series	
Strike	Error	Strike	Error
90.00	-1.51487e-008	97.00	1.95399e-014
92.00	-1.60534e-008	97.60	-7.10543e-015
94.00	-1.16162e-008	98.20	1.77636e-014
96.00	-3.77282e-009	98.80	3.01981e-014
98.00	2.01567e-009	99.41	4.9738e-014
100.00	3.05118e-008	100.02	7.10543e-015
102.00	4.61735e-008	100.64	2.75335e-014
104.00	3.27544e-008	101.26	1.59872e-014
106.00	1.53197e-008	101.88	3.9968e-014
108.00	7.85505e-009	102.51	4.9738e-014
110.00	9.24877e-009	103.14	4.70735e-014

Table 3
Performance comparison of the two approaches in Kou and Merton models. Time measured on a 2.8 GHz Pentium[©] 4 PC with 1 GB of memory.

Model	Time for 101 contrats	Mean time	FFT N=4096
Merton	$\approx$ 0.73 s	≈ 0.007 s	≈ 0.015 s
Kou	$\approx$ 6 mn 40 s	≈ 4 s	≈ 0.016 s

Computing times for these two approaches have been taken on a stock computer equipped with a Pentium[©] 4 processor running at 2.8 GHz and 1 GB of memory. The results are reported in table 3. The first column refers to computing time necessary to price one hundred contracts with Kou semi closed form formula. The second shows the average computing time to price each contract in this way. Then, the third column gives the computing time obtained with FFT algorithm using 4096 points. The code has been carried out with Matlab.

C.   Multiple Jumps Model Results with FFT

We can now consider the equivalent martingale measure condition that the multiple jumps model must verify under Q. According to equations (3), (9) and (12), this restriction can be written as follows:

$$0 = r - a - \frac{\sigma^2}{2} - \lambda \left(\sum_{i \epsilon P} \frac{p_i \lambda_i}{\lambda_i - 1} + \sum_{i \epsilon N} \frac{p_i \lambda_i}{\lambda_i + 1} - 1 \right),$$

which implies:

$$a = r - \frac{\sigma^2}{2} - \lambda \left(\sum_{i \epsilon P} \frac{p_i \lambda_i}{\lambda_i - 1} + \sum_{i \epsilon N} \frac{p_i \lambda_i}{\lambda_i + 1} - 1 \right).$$

We kept for X the same parameters for the continuous part of the model as in the previous case. The parameters for the multiple jumps law are given in Table 4. The first jumps law presents a positive jump with a 0.4 probability. In the second law, there are no more positive jumps: the probability mass has been transferred on the lower parameter. It is well known that exponential variables with low parameters are the ones which have the more impact on option prices. This is because the mean of each exponential random variable, modelling the jumps, is the reciprocal of its parameter.

Table 4
Multiple exponential jumps laws. Each pair (p, λ) gives the probability p of a jump whose law is exponential with parameter λ.

Jumps law	Positive jumps	Negative jumps
L_1	(0.4, 7)	(0.3, 5)-(0.2, 7)-(0.1, 9)
L_2	None	(0.7, 5)-(0.2, 7)-(0.1, 9)

Along C_1 contracts prices, we also give implicit volatilities in Table 5. The underlying asset price is always S_0=100 at initial time, the constant interest rate is r=0.05 and maturity $\tau = 0.5$. Here again, the computing time with FFT is around 0.015 second while the asset price process is more general than the original Kou model, which is costly in computing time with our code.

If prices obtained with law L_2 are above the prices with law L_1 up to the contract with exercise price 106, it is the inverse which prevails after this threshold. We have the same phenomenon with implicit volatilities. Let us note in both cases that the implicit volatility is more important than the a priori volatility of the classical model. It presents a pronounced convexity with respect to the exercise price with L_1 law but this effect is less important for L_2 law. Anyway it is not constant, and we find back the empirical observations already made on market data at the beginning of this study.
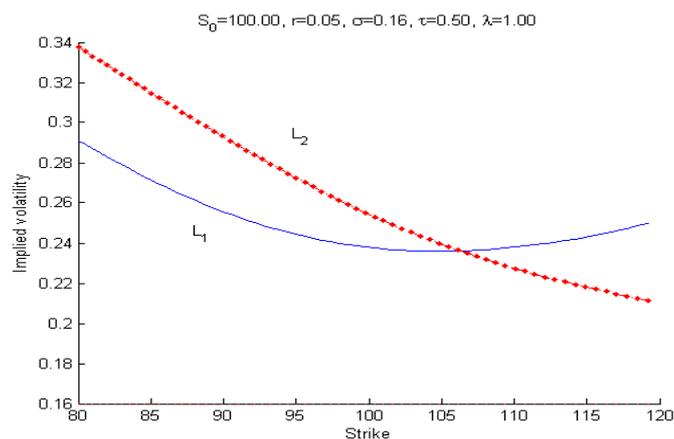
Figure 2
Volatility smiles – Multiple jumps model.

Table 5

Call options on the C_1 contracts series and corresponding implied volatilities in the new multiple exponential jumps model.

Multiple exponential jumps model				
European call option				
$S_0 = 100, r = 0.05, \sigma = 0.16, \tau = 0.5, \lambda = 1$				
Jumps law	L_1		L_2	
K	Price	Volatility	Price	Volatility
90.00	14.5478818	25.549 %	15.3323568	29.291 %
92.00	13.0393977	25.041 %	13.8074665	28.445 %
94.00	11.6145671	24.612 %	12.3414812	27.631 %
96.00	10.2828945	24.262 %	10.9427522	26.856 %
98.00	9.0521296	23.989 %	9.6197015	26.123 %
100.00	7.9276887	23.790 %	8.3803040	25.438 %
102.00	6.9122876	23.662 %	7.2315400	24.801 %
104.00	6.0058246	23.601 %	6.1788850	24.215 %
106.00	5.2055082	23.604 %	5.2259009	23.677 %
108.00	4.5061915	23.667 %	4.3739751	23.186 %
110.00	3.9008541	23.787 %	3.6222355	22.739 %

D.   Discussion

We see in Table 2 that prices obtained via formula (19) using a FFT with N=4096 points are very accurate. Let us also remark that, for the obtained prices for contracts

C_1 where a cubic spline has been used to interpolate FFT prices, the maximum error is never above 10^{-7}. Series C_2 corresponds to contracts which are direct FFT output. We must emphasize that the error, in this case, is always under 10^{-13}.

As far as performance is concerned, we see on the two first columns in table 3 the important computing time cost due to the use of special function intervening in Kou formula. For example if we compute option prices with Merton model, it is nearly 500 times faster than in Kou model.

In the third column, we see that the computing time using FFT with 4096 points is always less than 2/100 second whatever the considered model. In fact, once N is chosen, the computing cost remains stable. With more than ten contracts, in our experiment the FFT method outperforms the semi explicit approach. If you had only one contract to price the FFT method performs at least as well as its challenger.

V.   CONCLUSION

In this article we have shown that generalized Fourier approach associated with FFT gives a very powerful tool to price European options in jump diffusion models. It avoids indirect modifications of option payoffs to ensure integration convergence as is the case in Carr and Madan (1998) and in Benhamou's (2000) papers. We introduce a new model with multiple exponential jumps which extends Kou process this process as well as the original Kou process are used in our study as benchmarks to test the suggested method. The numerical analysis shows that it gives very fast and accurate results, furthermore it is easier to implement than the Kou algorithm. This method is very general and could be, as shown by Boyarchenko and Levendorskiĭ (2002), applied for other underlying processes whenever the Laplace exponent is known. An immediate application for the speed and accuracy of this method is its use in calibration of Lévy processes.

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