

Construction of Directional Volatility Index

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ABSTRACT

Is it possible to change the common idea that volatility is not directional? This paper provides a new volatility index - Conditional VIX has directionality. The change of Conditional VIX has a very high directional relationship with the changes of futures price at the same time, and can transform multiple market sentiments into market consensus. This paper further explores the famous double-slit experiment in the field of quantum mechanics, which shows that whether measuring photons or not has a significant difference in the change of bright and dark bands. Does this phenomenon also exist in the macro world? This paper studies whether the Futures Exchange measuring the loss of trader or not, which will have an impact on the trading behavior. The VIX index is an important sentiment index in the financial market, but the past literature mostly discussed the extreme reactions of fear and greed. This paper is the first to discuss the differences of eight kinds of sentiment changes according to the theory of the eight trigrams (I Ching), and draw candlestick charts with eight colors, so that futures price changes can be reflected the combination of sentiment and options volatility at the same time. Finally, we study the relationship between whether the Futures Exchange measures the loss of trader or not and future price changes (movement patterns). The results show that when the market volatility is high, traders are more willing to express their inner thoughts when the trader's loss is not measured (that is, no maintenance margin call is required), so the predictive power of futures price changes (movement patterns) is more obvious.

JEL Classification: G4, G13, G17, C83

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I. INTRODUCTION

After the New York Stock Exchange (NYSE) adopted the circuit breaker system to deal with the problem of excessive abnormal fluctuations in the market, the Chicago Board of Options Exchange (CBOE) immediately began to try to compile the VIX index to observe the volatility of the market. The VIX began on January 19, 1993 when CBOE announced that it would release the real-time Chicago Options Exchange Market Volatility Index. It was changed from CBOE's S&P100 Index (OEX) to CBOE's S&P500 Index (SPX) on September 22, 2003. The concept of the VIX index is to reflect the out-of-the-money options volatility of the S&P500 index over the next 30 days. When the index is higher, it means that investors are uneasy about the prospects of the S&P500 index, and there may be a large amount of buying or selling due to high uncertainty, on the contrary, if investors believe that the future performance of the S&P500 index is stable and the volatility is small, the VIX index will fall or remain at a relatively low level. Hancock (2012) found that the main benefit to investors of using VIX index-related commodities is to join asset portfolios that are negatively correlated with market changes, which can significantly reduce the overall portfolio risk. However, the emotional reactions of investors are complex. When the VIX index suddenly rises, do they feel anxious? or self-confidence? or fear? Is it even possible to be manipulated? The practical debates have always existed. How to transform multiple emotional reactions into rational and operable methods is an urgent need for market traders. Because the VIX index uses the data of out-of-the-money options in the near month and next month, it should not only include expectations for the future, but also be able to separate the impact of changes in call options and put options on VIX, and then distinguish the difference between fear, greed, optimism and pessimism, even be transformed into a rational operation of market consensus.

Firstly, this paper will review the past literature on the research ideas and results of decomposing the VIX index, and then explain the method of this paper and propose an innovative directional Conditional VIX and sentiment candlestick charts to integrate various emotional responses and options volatility combinations, and create a measure method of market consensus to turn emotional responses into rational operations. Finally, this paper uses Taiwan's weekly options data in the predictive empirical research to investigate the effectiveness of our Conditional VIX index.

The main contributions of this paper are the following three aspects. First, this paper constructs a directional volatility index, Conditional VIX, which is closely related to the direction changes of futures price. Second, it is not only closely related to the options volatility combinations, but also successfully transform the market sentiment into the market consensus. Finally, this paper finds that the traders are more willing to express their inner thoughts in the options trading behavior where there is no observed trader's loss (that is, no maintenance margin call is required) during the after-hours trading session, and the volatility increases sharply, so it will be immediately reflected in the changes of Conditional VIX. This paper uses these features to predict the direction of the futures close price and movement pattern, and the empirical results have shown good predictive power of the proposed measurements.

II. PREVIOUS RESEARCH

According to the White Paper of CBOE's VIX index, the data comes from S&P500 Index out-of-the-money options, so will the sentiment of the stock market affect the price of S&P500 options? Han (2008) found that when the market sentiment becomes more bearish (bullish), the S&P500 options volatility smile curve will be steeper (flatter), and the skewness of monthly index returns will be more (less) negative. Therefore, the VIX index is an important indicator for the current market to measure fear. Carr (2017) also used investors' risk aversion, VIX^2T , the correlation between VIX and the S&P500 index, and the Black Scholes model to prove that VIX is indeed a reflection of an indicator of fear. However, the final conclusion derived from the Black Scholes model is that the VIX index is actually a sentiment indicator that implies both fear and greed in the market. For the VIX index, it should be viewed from the perspective of fear and greed. Moreover, Kownatzki (2016) research found that in the middle of the financial crisis in 2008, the VIX index underestimated the real volatility by as much as 180 basis points, and overestimated the volatility in normal times. The traders have questioned whether the VIX index may be artificially manipulated? Osterrieder et al. (2019) found that the possible way is to actively post orders, on the one hand, it can change the quotation of options, and on the other hand, by subtly placing orders, it can affect the width of the tail. Saha et al. (2019) constructed a regression model and found that the daily level changes of the VIX index are explained by market fundamentals rather than by manipulation. Therefore, this paper believes that it is necessary to clarify how to decompose VIX index and quickly reflect the meaning of current options volatility.

The most common method of decomposing volatility in many existent articles is to subtract the implied volatility of at-the-money call options from the implied volatility of at-the-money put options to obtain the difference in implied volatility value, also known as Implied Volatility Spread (IVS). Atilgan et al. (2015) conducted research on the intertemporal relationship between IVS and the expected return of the overall stock market. Han and Li (2021) took the average of all stock IVS to predict market returns and overall economic trends, and this method showed better predictive power. Cao et al. (2019) confirmed that IVS can reflect market sentiment and market advantage information trading behavior. Gao et al. (2020) used SEC's EDGAR log files to observe investors' attention to US companies' financial reports, and found that if the attention of investors increases, it will also increase the predictive power of IVS on the trading behavior of superior information. From the above literature, we can find that the decomposition of volatility helps to understand the degree of market fear and greed.

However, IVS represents only the volatility of at-the-money options to reflect the relative sentiment of fear and greed. To capture the fear and greed of all options traders as much as possible, the VIX index must be further decomposed into the VIX of fear and the VIX of greed. Serur et al. (2021) believed that the increase of VIX represents the increase of market fear, which is a wrong perception and will mislead investors. By individually calculating the VIX of out-of-the-money call options and the VIX of out-of-the-money put options, they verified the characteristics of the two VIX indices and confirmed that VIX is a measure of investor sentiments, and called them the greed and fear index respectively. Bevilacqua et al. (2019) defined the positive VIX as the VIX of at-the-money and out-of-the-money calls options; the negative VIX as VIX of at-the-money and out-of-the -money put options, and analyzed the risk premium of positive and

negative historical VIX, and found that the positive risk premium is more related to the overall economic factors, while the negative risk premium is more related to market uncertainties and geopolitical risks. Bevilacqua et al. (2020) further compared the convergence speed of the positive VIX and negative VIX defined above with the terrorist attacks that occurred around the world in history, and found that terrorist attacks have a greater impact on negative VIX.

In addition, there are also decomposition of the VIX index for different time periods. For example, Chen et al. (2021) decomposed VIX into overnight period and intraday period, and found that VIX tended to increase in overnight period and decrease in intraday period, and found that there were the non-trading day effects, meaning that the overnight increase of VIX on weekends or holidays is significantly higher than that of two consecutive trading days. Therefore, they suggested that this feature should also be taken into consideration when conducting research on trading strategies related to the VIX index.

To sum up, for decomposing the VIX index, much literature has concluded that the VIX index is more sensitive to the emotion of fear, it also represents the emotion of greed behind it, and the interactive relationship between positive VIX and negative VIX has important implications in the volatility of financial markets.

III. RESEARCH METHOD

A. Conditional VIX

According to the 2007 annual report of the Taiwan Futures Exchange, TAIFEX discloses information about the volatility of Taiwan Index Option Index (TAIWAN VIX), for promoting market transparency and providing traders with more data to create their trading strategies. TAIWAN VIX is calculated by interpolation based on the contract volatility of the near month and the next near month of TAIEX Options (TXO). The contract volatility of a single month is determined by at-the-money (ATM) strike price based on the price of the corresponding Taiwan stock futures (TX) of the same month, and selects out-of-the-money (OTM) sequence of call and put quotes, and then use the following formula to calculate TAIWAN VIX:

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2 \quad (1)$$

where $i = 1, \dots, n$ marks the options strike price available on that specific date, T is the TXO duration, F is the price of the Taiwan stock futures (TX), K_i is the strike price of the i th OTM sequence of the contracts in this month, ΔK_i is the strike price interval defined as a half of the strike price gap above and below K_i , K_0 is the strike price of the price level sequence and is equal to or lower than the first strike price corresponding to the TX price (F) of the same month, R is the risk-free interest rate applicable during the contract duration of the month, and $Q(K_i)$ is the middle price of the bid and the best ask price of the strike price of sequence K_i .

According to the composition of VIX in the above Equation (1), we can know that TAIWAN VIX uses monthly options contract data, because Taiwan's monthly options market transactions are relatively small, it is not conducive to investigate the relationship

between VIX and futures price changes. The most active transaction is the weekly options market and is more suitable for the analysis of this paper, in addition, according to much literature (Bevilacqua et al., 2019; Chen et al., 2021; Serur et al., 2021) all suggested to decompose VIX index into call VIX and put VIX as Equations (2) and (3):

$$\sigma_{call}^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} C(K_i) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2 \quad (2)$$

$$\sigma_{put}^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} P(K_i) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2 \quad (3)$$

The above Equations (2) and (3) show that the futures price is included in both VIX indices of call options and put options, but if we want to know the futures price changes through put-call parity, we must compare the VIX of different trading days. We can use the changes of the call options, the changes of the put options, and the changes of the strike price of the ATM, and then analyze the relationship with the change of the futures price. Therefore, this paper intends to assume the strict condition that the change of the futures price and the change of the ATM strike price are not known. The duration (T) is also assumed to be unchanged because T will not affect the correlation between the price changes of the options and the futures price although T will inevitably decrease. Then, the Conditional VIX proposed in this paper is a combination of *Conditional* σ_{call}^2 and *Conditional* σ_{put}^2 as the Equation (4) below, that is, under these assumptions that the futures price, ATM strike price and duration T are all unchanged, the same strike price range as in the previous trading session is used, called Conditional VIX. In addition, it should be reminded that when the weekly options are finally settled every Wednesday, there will be a very large difference in the duration of the new and old contracts on the previous and subsequent trading days. To reduce the huge influence of duration in different contracts, we calculate *Conditional* σ_{call}^2 and *Conditional* σ_{put}^2 in the last after-hours trading session before the weekly contract renewal and VIX must use the settlement date of the new contract.

$$\text{Conditional VIX} = \text{Conditional } \sigma_{call}^2 + \text{Conditional } \sigma_{put}^2 \quad (4)$$

Since futures price changes can be explained by put-call parity, this paper proposes Conditional VIX to analyze that we only use the same range of strike prices as the previous trading session under the condition that the futures price change and the ATM strike price change are unknown. To clarify the relationship between Conditional VIX changes and futures price changes, we define the *Conditional* $\sigma_{call,t}^2$ and *Conditional* $\sigma_{put,t}^2$ changes in Equations (5) and (6):

$$\Delta \text{Conditional } \sigma_{call,t}^2 = \text{Conditional } \sigma_{call,t}^2 - \sigma_{call,t-1}^2 \quad (5)$$

$$\Delta \text{Conditional } \sigma_{put,t}^2 = \text{Conditional } \sigma_{put,t}^2 - \sigma_{put,t-1}^2 \quad (6)$$

According to Equations (2) to (6), the $\Delta \text{Conditional VIX}_t$ changes can be known as in Equation (7):

$$\Delta \text{Conditional VIX}_t = \text{Conditional VIX}_t - \text{VIX}_{t-1} \quad (7)$$

According to the relationship of $\Delta\text{Conditional } \sigma_{\text{call}}^2$ and $\Delta\text{Conditional } \sigma_{\text{put}}^2$ changes, there are eight combinations, which are described in the following relationships:

Buy Call > Sell Put (BC > SP), if $\Delta\text{Conditional } \sigma_{\text{call}}^2 > \text{ABS}(-\Delta\text{Conditional } \sigma_{\text{put}}^2)$

Buy Call > Buy Put (BC > BP), if $\Delta\text{Conditional } \sigma_{\text{call}}^2 > \Delta\text{Conditional } \sigma_{\text{put}}^2$

Sell Call > Buy Put (SC > BP), if $\text{ABS}(-\Delta\text{Conditional } \sigma_{\text{call}}^2) > \Delta\text{Conditional } \sigma_{\text{put}}^2$

Sell Call > Sell Put (SC > SP), if $\text{ABS}(-\Delta\text{Conditional } \sigma_{\text{call}}^2) > \text{ABS}(-\Delta\text{Conditional } \sigma_{\text{put}}^2)$

Buy Call < Buy Put (BC < BP), if $\Delta\text{Conditional } \sigma_{\text{call}}^2 < \Delta\text{Conditional } \sigma_{\text{put}}^2$

Buy Call < Sell Put (BC < SP), if $\Delta\text{Conditional } \sigma_{\text{call}}^2 < \text{ABS}(-\Delta\text{Conditional } \sigma_{\text{put}}^2)$

Sell Call < Sell Put (SC < SP), if $\text{ABS}(-\Delta\text{Conditional } \sigma_{\text{call}}^2) < \text{ABS}(-\Delta\text{Conditional } \sigma_{\text{put}}^2)$

Sell Call < Buy Put (SC < BP), if $\text{ABS}(-\Delta\text{Conditional } \sigma_{\text{call}}^2) < \Delta\text{Conditional } \sigma_{\text{put}}^2$

This paper will use the above eight kinds of options volatility combinations to study the relationship with the directional changes of futures price.

B. The Relationship between Options Volatility Combination and Eight Trigrams (I Ching)

How to explain the eight options volatility combinations through the theory of the eight trigrams (I Ching)? Each trigram of the eight trigrams is composed of three lines, and each line may be broken or unbroken, representing yin and yang respectively. Baynes (1950) mentioned for changes of the eight trigrams (I Ching) that at the same time, they were held to be in a state of continual transition, one changing state into another, just as transition from one phenomenon to another is continually taking place in the physical world. The change of volatility in the financial market can be explained by the eight trigrams (I Ching), since this paper thinks that the yin and yang represent the change of power and volatility is also a kind of power. The first line is to determine the priority order of call and put in options, then the second line is to determine whether the power of the buyer or the seller is strong, and finally the third line is to determine whether there is a conflict between long and short forces. Consequently, these three lines can produce the relationship between options volatility combination and eight trigrams (I Ching). The primordial eight trigrams include Ch'ien, Tui, Li, Chên, Sun, K'an, Kên, K'un. For example, for the trigram Li, the first line is yang line, recorded as Call > Put (C > P). The second line is yin line. We record it as SC >, because the seller is more powerful. The third line is yang line, forming a border trigram, meaning that there is no conflicting force. It suggests Buy Put (BP). Thus, the options volatility combination corresponding to the trigram Li is Sell Call > Buy Put (SC > BP). In the trigram Sun, the first line is yin line, so it is Call < Put (C < P). The second line is yang line, so the buyer has greater power, noted as BC <. The third line is yang line, forming a corner trigram, which means that there is conflicting force, with Buy Put (BP). Thus, the options volatility combination is BC < BP. Table 1 shows the relationship between options volatility combinations and the eight trigrams (I Ching).

Table 1

The relationship between options volatility combinations and the eight trigrams (I Ching)

Eight trigrams and name	The first line	The second line	The third line	Options volatility combinations
☰ Ch'ien	Call > Put	BC >	Border	BC > SP
☷ Tui	Call > Put	BC >	Corner	BC > BP
☲ Li	Call > Put	SC >	Border	SC > BP
☵ Chên	Call > Put	SC >	Corner	SC > SP
☱ Sun	Call < Put	BC <	Corner	BC < BP
☴ K'an	Call < Put	BC <	Border	BC < SP
☶ Kên	Call < Put	SC <	Corner	SC < SP
☳ K'un	Call < Put	SC <	Border	SC < BP

In Figure 1, we can see that the options volatility combinations are different between the border and corner trigrams. For example, the trigram Chên is between the trigram K'un and the trigram Li, so it has the nature of SC. Because the trigram K'un and the trigram Li are border trigrams, they will use BP without conflict, and the trigram Chên only has SP available, forming SC > SP, meaning that there is conflicting. When the third line appears, we can see the complete image of each trigram.

Why do we have to introduce the concept of the eight trigrams (I Ching) in such a complicated manner? Why not simply distinguish their size relationship between BC, SC, BP and SP? Because the financial transaction is a transaction between a trader and his counterparty, not a transaction between his left hand and his right hand. In the options market, the call options and the put options cannot be viewed separately, which are independent components. The financial market cannot exogenously exist independently but maintains a certain degree of endogenous correlation. For example, when a trader enters the options market, he has three interrelated questions that need to be answered. The first question is whether he prefers to use the call options or the put options, so there are two choices, $C > P$ or $C < P$; the second question is whether you want to stand on the buyer's or the seller's willingness. This idea is combined with the result of the first question to form a preference position, so there will be four kinds of results (BC>, SC>, BC<, SC<). The third question is whether the ambition is without suspense or there are some conflicting ideas, which is the meaning of the border or corner trigrams.

Let's go back to eight combinations relationship. We want to explore what kind of combination of options volatility will cause futures price to move in which direction. Among them, $BC > BP$, because $\Delta Conditional \sigma_{call}^2$ and $\Delta Conditional \sigma_{put}^2$ increase, $\Delta Conditional VIX_t$ should increase is obvious. Why is the corresponding futures price rising? According to Put-Call parity, the $\Delta Conditional \sigma_{call}^2$ greater than $\Delta Conditional \sigma_{put}^2$, so the futures price should rise. We need to pay attention to whether there is the same result in our empirical research. In the same way, it is obvious that $SC > SP$ will cause the $\Delta Conditional VIX_t$ to fall, but why is the futures price falling? Is the volatility change like an options trading strategy? This also requires to be proved empirically. As for comparing an increase in the futures price of the trigram K'an and the trigram Tui, but which one will increase more? If according to the third line of the eight trigrams (I Ching), the trigram K'an has no opposing force, the increase in futures price may be larger. Similarly, the decline in futures price of the trigram Li should be larger than the trigram Sun, and these possible slight differences also need empirical evidence. The possible relationship among all options volatility combinations, the eight trigrams (I

Ching), futures price changes and $\Delta \text{Conditional VIX}_t$ changes are summarized in Table 2.

Figure 1
The Positions of the Eight Trigrams (I Ching), in Which the East-west, North-south Directions Are the Border Trigrams, And the Rest Are the Corner Trigrams

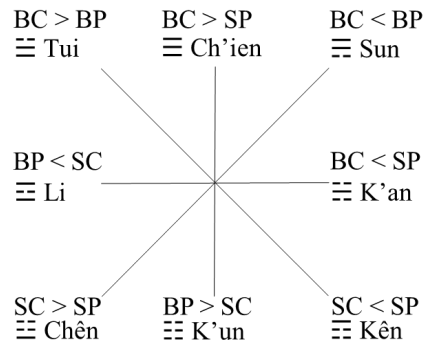


Table 2
The Relationship among the Eight Trigrams (I Ching), Options Volatility Combinations, Possible Changes of Futures Price and Possible Changes of $\Delta \text{Conditional VIX}$

Eight trigrams and name	Options volatility combinations	Possible direction of futures price	Possible changes of $\Delta \text{Conditional VIX}$
☰ Ch'ien	BC > SP	Up	Increase
☱ Tui	BC > BP	Up	Increase
☲ Li	SC > BP	Down	Decrease
☳ Chên	SC > SP	Down	Decrease
☵ Sun	BC < BP	Down	Increase
☶ K'an	BC < SP	Up	Decrease
☷ Kên	SC < SP	Up	Decrease
☱ K'un	SC < BP	Down	Increase

Therefore, we expect the following two properties of Conditional VIX:

1. We can clearly describe the relationship between the changes of futures price and Conditional VIX.
2. The difference in the options volatility combinations is closely related to the degree of futures price changes.

This paper will draw candlestick charts with eight colors, so that futures price changes can be reflected in the combination of sentiment and options volatility at the same time.

Next, in the world of market sentiment, sentiment is complex although the fear and greed in the market can be clearly felt by everyone. Can there be more detailed differences? How high is the current market consensus? It should be what market traders want to know more and can it be quantified? It is necessary for market traders to turn the

emotional feeling into a substantial consensus. Therefore, we want to further analyze whether futures price changes and Conditional VIX changes are integrated into a sentiment indicator, or even a market consensus indicator, so we create the following indicators:

$$Z_{\Delta F} = \frac{\text{mean of } \Delta F}{\text{standard deviation of } \Delta F} \quad (8)$$

$$Z_{\Delta \text{Conditional VIX}} = \frac{\text{mean of } \Delta \text{Conditional VIX}}{\text{standard deviation of } \Delta \text{Conditional VIX}} \quad (9)$$

$$MC = ABS\left(\frac{Z_{\Delta F}}{Z_{\Delta \text{Conditional VIX}}}\right) \quad (10)$$

Equations (8) and (9) are standardized, which can show the relative degree, but we need to integrate futures price changes and Conditional VIX changes together, to create the Equation (10) to measure market consensus (MC). Why does MC represent the degree of market consensus? If the price changes sharply with relatively lower volatility, it means that there is no huge difference in the market's views of long and short traders, that is, the MC is high. On the contrary, if the price changes violently with relatively higher volatility, it means that the long-short forces in the market are in a tug-of-war, and the seemingly losers have not withdrawn from the market but are preparing to wait for subsequent opportunities. Therefore, this paper expects that by calculating the MC, everyone's multiple sentiment reactions can be transformed into substantial tradability. Why is MC helpful for trading? Because according to the theory of the eight trigrams (I Ching), the trigram Ch'ien and the trigram K'un should reflect a higher degree of MC, these two trigrams have the purest power and clear direction in the eight trigrams. As for the trigram K'an and the trigram Li, because they are also the border trigrams, they should have a relatively higher MC compared to the corner trigrams. Why may the trigram Tui and the trigram Sun have the lowest degree of MC? Because the trigram Tui is $BC > BP$ or the trigram Sun is $BC < BP$, there occurs to be a conflict between long and short forces. Then, the degree of MC should be the lowest. Therefore, we show the relationship between MC and the eight trigrams (I Ching) as shown in Table 3. Through empirical evidence, we will know more clearly whether Conditional VIX is a good method to measure volatility changes.

Table 3
The Possible Relationship between the Eight Trigrams (I Ching) and the Degree of Market Consensus

The border/corner trigrams	Eight trigrams and name		Degree of market consensus	
Border	☰ Ch'ien	☷ K'un	Higher	Highest
	☵ K'an	☲ Li		Higher
Corner	☱ Kên	☶ Chên	Lower	Lower
	☴ Tui	☳ Sun		Lowest

Young (1832) double-slit experiment proved that light behaves like waves rather than particles because the interference pattern of light on a screen looks similar to that of water. His experiments came later, allowing photons to fall on a screen through a slit, and various observations were collected. When we look at the slit/screen without measuring/observing equipment, the photons act like waves and an interference pattern

appears on the screen. When we look at the slit/screen with a measuring/observing device, the photons behave like particles and no interference pattern appears on the screen. Since there are only two slits, there are only two bright bands. This paper is not going to study the physical properties of the double-slit experiment, but want to use this phenomenon to analyze whether there is an observer effect in the macro world. Because in Taiwan's options trading market, traders need to be monitored in real time to see if there is a requirement to make up the maintenance margin due to insufficient maintenance margin during the regular trading session. However, there is no such regulation in the after-hours trading session, so this paper wants to analyze the impact on traders. What is the effect of behavioral influence? We want to judge whether there is a difference from the perspective of MC because after-hours trading does not require margin calls for traders' losses and traders should be more willing to express their inner thoughts and trading behaviors. Perhaps there are some differences in the market between the two consensus variables on transaction time.

C. Conditional VIX Predictive Power Analysis

In order to further understand the predictive power of Conditional VIX, this paper will analyze the predictive power for futures price changes and movement patterns. As explained above, Taiwan's options market is divided into regular hours session and after-hours session. For the transactions in after-hours session, the loss of traders will not be measured, so maybe easier to find informed trader's pattern than regular hours session. Therefore, this paper has the following two analysis.

1. What is the difference in predictive power of Conditional VIX in different trading hours session?
2. Do different options volatility combinations have different predictive power?

We can give an example to understand the content of the second research is that the volatility state after the transaction between trader A and counterparty B is $BC > SP$ where the volatility state of Call is BC after the transaction between the buyer and the seller and the volatility state of Put after the transaction is SP. The power of BC is greater than the power of SP where we only see the volatility state of $BC > SP$ although we do not see the volatility state of $SC < BP$, but there must be the opposite volatility force. To express the volatility power of buyers and sellers in the training set at the same time through the design of the difference in sample characteristics, we further show training samples. For example, when the current volatility state is $BC > SP$, we need to include $SC < BP$ volatility. The volatility state is $BC > SP$ corresponding to the samples whose close price of the futures in the subsequent trading session is "rising", and then takes out; we also include the samples whose volatility state is $SC < BP$ corresponding to the close price of the futures in the subsequent trading session is "falling". Thus, the training samples collected together. Therefore, this paper divides the training sample set into four groups, and uses four models generated by four different training sets to analyze whether different options volatility combinations have different predictive power. In the past, there were many similar methods for the construction of strategy combinations, for example, the three-factor construction of Fama and French (1993) also had a similar approach.

Table 4
Training Set of Paired Sampling

Training name	Paired set	Options volatility combinations	Direction of subsequent trading session
Ch'ien_K'un	≡ Ch'ien	BC > SP	Up
	≡ K'un	SC < BP	Down
Tui_Sun	≡ Tui	BC > BP	Up
	≡ Sun	BC < BP	Down
K'an_Li	≡ K'an	BC < SP	Up
	≡ Li	SC > BP	Down
Kên_Chên	≡ Kên	SC < SP	Up
	≡ Chên	SC > SP	Down

In this paper, we want to predict the following two purposes, and to avoid affecting the analysis, this study will exclude the data that the futures close price has not changed.

1. Futures close price up/down in subsequent trading sessions.
2. Types of futures price movement in subsequent trading sessions.

Since the training samples in this paper will only retain about 200 samples after the sampling design, it is impossible to use deep learning, and in the relevant past literature on the combination of machine learning algorithms and VIX index, Rosillo et al. (2014) used the SVM algorithm combined with the VIX index data to predict the weekly return of the S&P500 index, and the results showed that the predictive effect was better than only the VIX index alone or the SVM algorithm alone. Prasad and Bakhshi (2022) combined logistic regression and ensemble learning with VIX index to predict the rise and fall of the Indian index's daily return has a very stable predictive result, and it is recommended that XGBoost be used alone to have a good predictive result. Therefore, this paper intends to use SVM, XGboost and Softmax Regression with multiple classifications as modeling algorithms:

$$y_{t+1} = f(\Delta Conditional VIX_t, R\sigma_{call,put,t}^2, D\sigma_{call,put,t}^2) \quad (11)$$

In Equation (11), where y_{t+1} are the predictive variables of the subsequence trading session, If the futures price is rising, let y_{t+1} be 1; if the futures price is falling, y_{t+1} denotes 0.

To simplify the prediction of price movement, this article divides each sample into ten groups from the lowest price to the highest price during the trading session and creates four categories of forecasting future movement according to the following rules.

1. BC is defined as the sample (y_{t+1}) where the open price is in the lowest group and close price is in the highest group, labeled as category 0.
2. BP is defined as the sample (y_{t+1}) where the open price is in the highest group and close price is in the lowest group, labeled as category 1.
3. SP is defined as the sample (y_{t+1}) where the closing price is rising except with the classification of BC and BP, labeled as category 2.

4. SC is defined as the sample (y_{t+1}) where the closing price is falling except with the classification of BC and BP, labeled as category 3.

In Equation (11), there are three input variables for training set. $\Delta\text{Conditional VIX}_t$ from the Equation (7) is benefit in explaining the changes of futures price in the same trading session. This study examines whether $\Delta\text{Conditional VIX}_t$ can predict the direction changes of futures price in subsequent trading session.

$$R\sigma_{call,put,t}^2 = ABS(\Delta\sigma_{call,t}^2 / \Delta\sigma_{put,t}^2) \quad (12)$$

where $R\sigma_{call,put,t}^2$ is the second input variable and is used to decompose the change of Conditional VIX into call and put force. It is helpful in distinguishing the relative volatility power. The change of Conditional VIX is regarded as a kind of force, and the absolute value of the ratio can be used to measure the relative size of the force, which should be helpful for predicting the direction changes of futures price.

$$D\sigma_{call,put,t}^2 = ABS(\Delta\sigma_{call,t}^2 - \Delta\sigma_{put,t}^2) \quad (13)$$

Where $D\sigma_{call,put,t}^2$ is the third input variable to understand the absolute degree of the difference. Our reason is that, if the volatility changes are related to the direction changes of futures price, the absolute degree of volatility changes may be related to the degree of futures price changes. In addition, due to the small number of training samples in this paper, this study does use input variables that are only directly related to volatility changes.

IV. EMPIRICAL RESULTS

A. Data

Since the trading volume of weekly options is 2-3 times of monthly options in Taiwan, the weekly options are more representative of the market sentiment, this paper uses the weekly options data. Furthermore, the Taiwan Futures Exchange currently only has the VIX index compiled with the monthly options data, and does not have the VIX index with the weekly options data, our method may be used as a reference for the Taiwan Futures Exchange to compile the weekly VIX or launch VIX-related derivative products in the future. Since this paper not only proposes the explanation of various options combinations after decomposing VIX, but also has predictive analysis and the data on the final settlement date of the weekly options is changed to the data of the new contract. In addition, the regular trading session and after-hours trading session will be distinguished. The data of the weekly options trading day include contract name, strike price, best bid price, and best ask price. Furthermore, in order to avoid discontinuous quotations for deep OTM, only the data with the best bid price and the best ask price are obtained for the OTM strike price, and we need the futures open price, the lowest price, highest price and close price. All data are based on the closing time of the Taiwan Futures Exchange website at 13:45 in the regular trading session and 05:00 in the after-hours trading session. Taiwan Futures Exchange launched the after-hours trading session on

May 17, 2017, so the research period of this paper is from May 17, 2017 to December 30, 2022.

B. The Relationship between Changes of Conditional VIX and Futures Price

According to The Five Elements, metal (the trigram Ch'ien and the trigram Tui), wood (the trigram Chên and the trigram Sun), water (the trigram K'an), fire (the trigram Li), and earth (the trigram K'un and the trigram Kên) respectively represent the five colors of white, green, black, red and orange but we need eight colors corresponding the relationship between changes of Conditional VIX and futures price. Hence, we split white into the trigram Ch'ien in white color and the trigram Tui in gray color, split green into the trigram Chên in light green color and the trigram Sun in dark green color, and split orange into the trigram K'un in orange color and the trigram Kên in yellow color. The summary is shown in Table 5 below.

Table 5
Eight Colors Pairing Options Volatility Combinations

Eight trigrams and name	Options volatility combinations	Direction of futures price	Changes of Δ Conditional VIX	Eight colors
☰ Ch'ien	BC > SP	Up	Increase	White
☷ Tui	BC > BP	Up	Increase	Gray
☲ Li	SC > BP	Down	Decrease	Red
☱ Chên	SC > SP	Down	Decrease	Light Green
☳ Sun	BC < BP	Down	Increase	Dark Green
☵ K'an	BC < SP	Up	Decrease	Black
☶ Kên	SC < SP	Up	Decrease	Yellow
☷ K'un	SC < BP	Down	Increase	Orange

We use eight colors to complete the futures price chart as shown in Figure 2, which allows the price of the futures candlestick charts to be displayed simultaneously with the sentiment and options volatility combinations. This paper found that our plots are consistent with the direction of futures price changes in Table 2 and our accuracy rate exceeds 95% during regular trading hours and after-hours trading hours, as shown in Table 6. This result just shows that the futures price change is indeed linked to the change in the options volatility combination through put-call parity. We can see the relationship between the direction of futures price changes and the direction of Conditional VIX changes in Table 7, completely consistent with the relationship in Table 2. Therefore, this paper completes the purpose of Conditional VIX being designed to be directional.

Figure 2
The Futures Candlestick Charts Based on the Eight Trigrams (I Ching)



Table 6
The Accuracy Rate of Futures Movement Direction in Different Trading Session

Trading session	Number of samples	Accuracy rate
Regular	1371	95.48%
After-hours	1370	95.40%

Note: The number of samples has been deducted from the data that the futures price has not changed and the after-hours trading session has not opened.

Table 7
The Statistics in Different Trading Sessions

Eight trigrams and name	Regular trading session				After-hours trading session			
	Change points of futures price		Changes of Δ Conditional VIX		Change points of futures price		Changes of Δ Conditional VIX	
	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
☰ Ch'ien	120.24	89.94	2.0090%	5.5430%	71.88	64.16	1.1195%	2.9896%
☷ Tui	67.50	61.34	3.0075%	2.4442%	-7.00	n/a	0.0420%	n/a
☵ K'an	68.33	55.92	-0.8021%	2.5970%	55.69	46.69	-0.5529%	0.9397%
☶ Kên	17.74	19.89	-1.1414%	1.4347%	10.67	16.65	-1.2150%	1.6791%
☳ Chên	-23.44	22.09	-1.2806%	1.4758%	-18.44	20.29	-1.3203%	1.8672%
☲ Li	-73.82	48.38	-0.9074%	1.1378%	-56.72	43.02	-0.6693%	1.0811%
☱ Sun	-71.27	105.46	3.0195%	3.1287%	-33.00	117.74	4.3518%	5.9472%
☴ K'un	-189.10	132.97	3.3299%	14.6738%	-142.08	93.04	1.8851%	4.4415%

Note: n/a represents there is only one sample of the trigram Tui in the after-hours trading session, so the standard deviation cannot be calculated.

Let's take a closer look at Table 7. The average price increase of the trigram K'an

during the regular trading sessions is only slightly greater than that of the trigram Tui. There is a major difference in the changes of Conditional VIX. The trigram Tui is obviously larger than the trigram K'an and the direction is completely opposite. This just shows that if there is a significant difference in volatility, the changes in the futures price may not necessarily have a significant difference. From the comparison of the trigram K'an and the trigram Tui, we have clarified a problem that has troubled us for a long time, that is, we all know that volatility changes and price changes will not be simply and linearly related, but where is the biggest difference? The answer is that there is a difference between the trigram K'an and the trigram Tui. Similar results are detected for the relationship between the trigram Li and the trigram Sun. Through this major discovery, we know more clearly that there will be a positive correlation between volatility changes and price changes in the trigram Ch'ien, the trigram Tui, the trigram Li and the trigram Chên, and there will be a negative correlation in the trigram K'un, the trigram Sun, the trigram K'an and the trigram Kên. We summarize the results into four quadrants as shown in Table 8:

Table 8
The relationship between the up/down of futures price and the increase/decrease of Δ Conditional VIX in the eight trigrams (I Ching)

Δ Conditional VIX	Direction of futures price	
	Up	Down
Increase	☰ Ch'ien	☷ K'un
	☱ Tui	☲ Sun
Decrease	☱ K'an	☲ Li
	☷ Kên	☰ Chên

This paper further explores the integration of futures price changes and volatility changes into a market consensus based on the theory of the eight trigrams (I Ching). The results are shown in Table 9. The columns of standardized price change ($Z_{\Delta F}$) and standardized volatility change ($Z_{\Delta \text{Conditional VIX}}$) are completely consistent with Table 2. The difference between the properties of the trigram K'an and the trigram Tui are more obvious; through the comparison of the MC, and it can be seen more clearly that the trigram K'an has a higher market consensus than the trigram Tui, and the trigram Li is also higher than the trigram Sun. The overall market consensus is completely consistent with Table 3. We even know why the fear response is stronger than the greedy response, that is, because the market consensus of the trigram K'un is much higher than that of the trigram Ch'ien. The trigram K'an represents a high degree of market consensus on the steady rise of futures price, compared with the trigram Ch'ien and the trigram K'un, suggesting that the market sentiment is calm and rational. So far, we have finally transformed multiple emotional reactions into rational quantitative operations. Through this evidence, we are more convinced that Conditional VIX is a good measure of changes in market volatility.

After we clearly know the relationship between the eight trigrams (I Ching) and market consensus in the results in Table 9, the last thing we want to know is whether the double-slit experiment of quantum mechanics appears in the macro world. The answer is yes.

Table 9
The Relationship between the Eight Trigrams (I Ching) and Market Consensus

Eight trigrams and name	Regular trading session			After-hours trading session		
	Standardized price change ($Z_{\Delta F}$)	Standardized volatility change ($Z_{\Delta Conditional VIX}$)	Market consensus (MC)	Standardized price change ($Z_{\Delta F}$)	Standardized volatility change ($Z_{\Delta Conditional VIX}$)	Market consensus (MC)
☰ Ch'ien	1.34	0.3624	3.6886	1.12	0.3745	2.9917
☷ Tui	1.10	1.2304	0.8942	n/a	n/a	n/a
☱ K'an	1.22	-0.3088	3.9562	1.19	-0.5884	2.0273
☲ Kên	0.89	-0.7955	1.1214	0.64	-0.7236	0.8854
☵ Chên	-1.06	-0.8677	1.2225	-0.91	-0.7071	1.2852
☶ Li	-1.53	-0.7975	1.9132	-1.32	-0.6191	2.1299
☳ Sun	-0.68	0.9651	0.7003	-0.28	0.7317	0.3830
☴ K'un	-1.42	0.2269	6.2668	-1.53	0.4244	3.5978

Note: n/a represents there is only one sample of the trigram Tui in the after-hours trading session, so the $Z_{\Delta F}$, $Z_{\Delta Conditional VIX}$, and MC cannot be calculated.

We standardize the MC in Table 9 for each trading session, as shown in Table 10. This paper finds that in the regular trading session, after measuring the trading loss of the trader, and if the maintenance margin is insufficient, then the trader will receive margin call. The results show that only the trigram Ch'ien, the trigram K'an and the trigram K'un have a standardized MC number greater than 0 during regular trading session, and all other trigrams are less than 0. This phenomenon is like the double-slit experiment when after measuring photons, only two bright stripes will be seen on the screen. In the after-hours trading session, because the Futures Exchange did not measure the trading losses of traders, the standardized MC of the trigram Ch'ien, the trigram K'an and the trigram K'un is greater than 0, and the trigram Li is also greater than 0. These four trigrams are all the border trigrams which there are no conflict between long-short volatility, that is, these are the four situations with the higher MC. Therefore, there is actually one brighter stripe in the after-hours trading session than in the regular trading session, and this result actually appeared in the macro world which bright and dark stripes are more prominent. This paper finds that the extra bright stripe in the after-hours trading session appears in the trigram Li. This phenomenon further shows that traders have a higher degree of MC when the futures price is falling significantly. The greater the MC, the more constructive light waves interference, so the strip will be brighter.

Table 10
The Standardization of Market Consensus in Different Trading Sessions

Eight trigrams and name	Regular trading session	After-hours trading session
☰ Ch'ien	0.6581	1.0307
☷ Tui	-0.8515	n/a
☱ K'an	0.8027	0.1202
☲ Kên	-0.7288	-0.9581
☵ Chên	-0.6742	-0.5805
☶ Li	-0.3010	0.2170
☳ Sun	-0.9563	-1.4323
☴ K'un	2.0510	1.6030

Note: n/a represents there is only one sample of the trigram Tui in the after-hours trading session, so the market consensus cannot be calculated.

The conclusion in Section IV.B first explains the difference between Conditional VIX and VIX, then constructs the option volatility combinations according to the eight trigrams (I Ching) theory, clarifies the relationship between volatility changes and price changes, and finally integrates it into the MC. We can progress from emotional responses to rational operations.

C. Conditional VIX Predictive Power Analysis

1. The Predictive Power Analysis of Different Trading Sessions

This paper will distinguish between the forecast analysis during the regular trading session and the after-hours trading session. In order to make the sampling balance of long and short, all the data should be grouped. If the futures price rise in two consecutive trading sessions, it is a long group. If the futures price fall in two consecutive trading session, it is a short group, and the rest is a sideways group. After such grouping, the ratio is just about one-third each group and the training set samples can be drawn from each group to achieve the purpose of long-short balance, sampling of validation set and test set is also applicable. There are 50% of all samples are training set, 25% are validation set, and 25% are testing set. In addition, bootstrapping was repeatedly sampled 1000 times in verification set and testing set.

Firstly, we find the results of forecasting direction of the futures close price in subsequent after-hours trading session by using the regular trading session samples. In Table 11, the predictive accuracy rate is close to 59% in the SVM model. However, whether it is the regular trading session or the after-hours trading session training set, the other models do not have good predictive power.

Next, we will analyze the forecasting results of the types of futures price movement. In Table 12, we can see whether it is the regular trading session or the after-hours trading session training set, these models do not have good predictive power.

Table 11
Predict Direction of the Futures Close Price

Samples session	Predictive session	Training model	Accuracy rate	
			Validation set	Testing set
Regular	Subsequent after-hours	SVM	58.76%	58.99%
		XGBoost	50.54%	51.38%
		Softmax	45.03%	45.28%
After-hours	Subsequent regular	SVM	52.50%	52.11%
		Softmax	52.57%	52.00%
		XGBoost	49.58%	50.38%

Table 12
Predict the Types of Futures Price Movement

Samples session	Predictive session	Training model	Accuracy rate	
			Validation set	Testing set
Regular	Subsequent after-hours	SVM	53.79%	53.52%
		XGBoost	45.51%	45.93%
		Softmax	45.99%	45.26%
After-hours	Subsequent regular	SVM	48.72%	48.53%
		XGBoost	46.64%	47.01%
		Softmax	49.08%	48.68%

2. Predictive Power Analysis of Different Options Volatility Combinations

We analyze the predictive power of different training sets after pairing the training data with buyer and seller force. Since the training set is a specially selected sample, the validation set and the testing set still have bootstrapping 1000 times although long-short balance processing cannot be performed.

Firstly, we see the results of forecasting direction of the futures close price in subsequent after-hours trading session by using the regular trading session samples. In Table 13, we can clearly find that when the training set is the Ch'ien_K'un group and the Tui_Sun group, the predictive accuracy rates of all models are over 55%. The predictive power of the K'an_Li group and the Kên_Chên group are very poor. The possible reason is that the buyer and seller force difference of the Ch'ien_K'un group and the Tui_Sun group are the larger, so the predictive effect is the better. The training sample characteristic of the K'an_Li group and the Kên_Chên group are the smaller volatility change, so the feature difference is not obvious, which is not helpful for prediction.

Next, we find the results of forecasting direction of the futures close price in subsequent regular trading session by using the after-hours trading session samples. In Table 14, we obviously find that when the training set is the Ch'ien_K'un group, all models have very good predictive power, as high as 64%-65%. We further compared the predictive results with regular trading session in Table 13, it shows that the predictive accuracy using the after-hours trading session samples was higher than that of regular trading session samples in both the Ch'ien_K'un group and the Tui_Sun group. It seems that in the after-hours trading session without measuring the trader's loss (that is, no margin call is required), traders will be more willing to express their inner thoughts when the volatility and price are more significant, and thus has better predictive effect on direction changes of futures close price.

Further, we will analyze the forecasting the types of futures price movement in subsequent after-hours trading session by using the regular trading session samples. In Table 15, we can compare with Table 12 (without paired training samples), the predictive accuracy is improved. This shows that the more the training data has the force of buyers and sellers, the more accuracy to predict the types of futures price movement.

Table 13
Predict Direction of the Futures Close Price of the After-hours Trading Session

Group name of training set	Training model	Accuracy rate	
		Validation set	Testing set
Ch'ien_K'un	Softmax	57.35%	57.38%
	SVM	56.42%	56.45%
	XGBoost	55.64%	55.68%
Tui_Sun	SVM	56.41%	56.46%
	Softmax	55.64%	55.68%
	XGBoost	55.49%	55.51%
K'an_Li	SVM	45.30%	45.24%
	XGBoost	44.36%	44.32%
	Softmax	43.92%	43.83%
Kên_Chên	XGBoost	44.06%	44.01%
	SVM	42.28%	42.06%
	Softmax	42.28%	42.06%

Table 14

Predict the Direction of the Futures Close Price of the Regular Trading Session			
Group name of training set	Training model	Accuracy rate	
		Validation set	Testing set
Ch'ien_K'un	XGBoost	65.06%	65.01%
	SVM	64.91%	64.88%
Tui_Sun	Softmax	64.76%	64.74%
	Softmax	66.65%	66.59%
	XGBoost	57.33%	57.49%
K'an_Li	SVM	54.69%	54.66%
	Softmax	43.83%	43.94%
	SVM	40.70%	40.45%
Kên_Chên	XGBoost	34.94%	34.99%
	Softmax	50.74%	50.85%
	XGBoost	35.24%	35.26%
	SVM	32.95%	32.94%

Table 15

Predict the Types of Futures Price Movement of the After-hours Trading Session			
Group name of training set	Training model	Accuracy rate	
		Validation set	Testing set
Ch'ien_K'un	Softmax	52.53%	52.59%
	SVM	51.90%	51.98%
	XGBoost	41.42%	41.37%
Tui_Sun	SVM	52.19%	52.31%
	Softmax	51.43%	51.51%
	XGBoost	50.23%	50.23%
K'an_Li	SVM	41.73%	41.68%
	XGBoost	40.95%	40.91%
	Softmax	40.51%	40.42%
Kên_Chên	SVM	51.98%	52.20%
	Softmax	40.72%	40.52%
	XGBoost	39.91%	39.78%

Finally, we will analyze the forecasting the types of futures price movement in subsequent regular trading session by using the after-hours trading session samples. In Table 16, we can clearly find that compared with Table 12 (without paired training samples), the SVM and Softmax model of the Ch'ien_K'un group have more than 59% predictive accuracy, and significantly improve. It is even better than in Table 15 (the regular trading session samples).

The conclusion of Section IV.C first analyzes the different results with and without paired training samples and paired training samples, showing that paired training samples significantly improve prediction accuracy. This paper further found that the after-hours trading session has more information than the regular trading session, especially when the volatility is significant, and this very important information is hidden in Conditional VIX changes.

Table 16

Predict the Types of Futures Price Movement of the Regular Trading Session			
Group name of training set	Training model	Accuracy rate	
		Validation set	Testing set
Ch'ien_K'un	SVM	59.29%	59.27%
	Softmax	59.14%	59.13%
	XGBoost	57.86%	57.82%
Tui_Sun	Softmax	61.17%	61.14%
	SVM	50.07%	50.08%
	XGBoost	38.91%	38.79%
K'an_Li	Softmax	42.38%	42.51%
	SVM	36.93%	36.74%
	XGBoost	33.07%	33.12%
Kên_Chên	Softmax	52.02%	52.15%
	SVM	35.79%	35.58%
	XGBoost	20.75%	20.69%

V. CONCLUSION

This paper constructs a directional volatility index, Conditional VIX, which is closely related to the direction changes of futures price. Therefore, it is not only closely related to the options volatility combinations, but also successfully transform the market sentiment into the MC, and has good predictive power. Especially after pairing the training data with buyer and seller force, this paper finds that the traders are more willing to express their inner thoughts in the options trading behavior where there is no observed trader's loss (that is, no maintenance margin call is required) during the after-hours trading session, and the volatility increases sharply, so it will be immediately reflected in the changes of Conditional VIX. This paper uses these features to predict the direction of the futures close price and movement pattern, and the empirical results have shown good predictive power of the proposed measurements.

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