THE EFFECT OF ALTERNATIVE COMMON COST ALLOCATIONS ON MANAGERAL UTILITY AND PRODUCTION DECISIONS UNDER UNCERTAINTY

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The effect of alternative cost allocations on managerial utility and production decisions under price uncertainty is examined in a decentralized organizational setting where the allocation of common costs affects local manager's opportunity set for expected reported profit and risk. Four alternative allocation schemes are considered: (1) the lump-sum allocation, (2) the output-based allocation, (3) the sales-based allocation, and (4) the profit-based allocation. Comparative statistic analyses show that decreasing absolute risk-averse managers produce less output under the first two allocation schemes, and more under the last two allocation schemes. Furthermore, the level of the risk-averse manager's utility decreases at the new optimum under the first two schemes while it increases under the last two schemes. These results suggest a testable implication that the last two schemes would be more frequently observed than the first two schemes in decentralized firms whose profit centers are faced with price uncertainty.

I. INTRODUCTION

Numerous studies in the accounting literature have examined the issue of cost allocations in various contexts and have suggested a variety of criteria for evaluating alternative allocation proposals.[1] However, the past literature on cost allocations was predominantly *normative* in that its major concern was to prescribe how costs *should or should not* be allocated rather than to explain and predict why firms allocate costs. In this regard, Biddle and Steinberg [6:34-35] observed:

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A striking aspect of the cost allocation literature to date is its normative tone. Equally striking is the limited impact it has had on cost allocation practices. Foremost among the research areas suggested by this study is a more thorough understanding of the motives for allocating costs.

In contrast, Zimmerman [16:504] attempted to provide *positive* explanations for reasons "why firms continue to allocate costs for internal reporting purposes in spite of educators' continual admonition to the contrary."   He examined the question in the context of internal agency relationships in a decentralized firm where headquarter's expenses are allocated to profit centers. Following Williamson's [15] model of expense preference, Zimmerman showed that: (1) when headquarter's expenses are allocated to profit centers on the lump-sum basis, the center's consumption of perquisites decreases at the new optimum, and (2) it would increase if the amount of allocation is in direct proportion to reported profits. Based on this result, he suggested a testable implication that the lump-sum allocation scheme would be more frequently observed than the profit-based allocation scheme.

Zimmerman's analysis relies upon the differential effect of the two alternative schemes on the perquisite consumption. As illustrated in his Figure 1, his analysis showed, however, that the allocations of common costs (or headquarter's expenses) under both the lump-sum and profit-based schemes lead to a decrease in the opportunity set available to the manager of a profit center, which in turn causes the level of utility to decrease at the new optima. As a result his analysis leaves still unanswered the question of why the rational, maximizing manager would be willing to accept utility-decreasing allocation schemes. Furthermore, his analysis did not take into account the manager's attitude towards risk (by assuming that reported profits of a profit center are known with certainty). It is well documented in the theory of the firm that economic agents behave differently under uncertainty than under certainty (Baron [5], Sandmo [12], Leland [9], and Hawawini [7]). In the accounting literature, Adar, Barnea, and Lev (ABL) [1:147] applied the theory of the firm under uncertainty to their examination of the effects of fixed-cost changes on the firm's short-run output decision when the market for firm's output is perfectly competitive, and stated the following in their concluding remarks:

> Particularly, the effect of fixed-cost changes under uncertainty on the short-run output decision has obvious implications to a wide range of cost accounting problems, such as *common cost allocation*, joint product pricing and transfer pricing systems for divisionalized firms. While under certainty, fixed costs in those cases are irrelevant for decision making (e.g., transfer prices do not include fixed costs); under uncertainty a different attitude toward

fixed costs (and *particularly, fixed cost allocation*) is called for.
(emphasis added)

Following ABL's lead, this paper attempts to explain and predict how the allocations of common costs under alternative schemes differentially affect managerial behavior under uncertainty. Our analysis differs from Zimmerman's in the following ways. First, we are primarily concerned with the differential effect of alternative cost allocations on the *risk-averse* manager's production decision and the level of managerial utility associated therewith, while Zimmerman focuses on their differential effect on the (risk-neutral) manager's consumption of perquisites under certainty. Second, unlike Zimmerman, we explicitly incorporate manager's risk preference into the analysis of the cost allocation effects. In his model, the utility of a center manager is a positive function of reported profit and perquisite both of which are known with certainty. In this paper, however, managerial utility is a positive function of *expected* reported profit but a negative function of risk (as measured by the standard deviation of uncertain reported profit). While Zimmerman's analysis focuses on how cost allocations lead to a change in the trade-off relation between reported profit and perquisite, our analysis focuses on how alternative cost allocations differentially affect the utility-maximizing, risk-averse manager's optimization through their effect on the trade-off relation between expected reported profit and risk. In doing so, we consider four alternative allocation schemes: (1) the lump-sum allocation, (2) the output-based allocation, (3) the sales-based allocation, and (4) the profit-based allocation.

The rest of the paper is organized as follows: In Section II, we develop a model of the local manager's optimization under uncertainty assuming that no common cost is allocated to decentralized units. In Section III, we derive the new optima under the four alternative allocation schemes and compare them with the initial optimum under no allocation. In doing so, we derive the optima for both risk averse and risk neutral managers. The final section summarizes and concludes the paper.   The Appendix demonstrates that the results of this study remain unchanged even when the market for center's output is not perfectly competitive.

II. BASIC MODEL UNDER NO ALLOCATION

Consider a firm that is decentralized into local entities on the profit-center basis. The headquarter delegates production and sales decisions to profit

centers, while it retains the prerogative of investment decision, and provides them with common resources or services. Similar to ABL, each profit center produces and sells a single product; and a center manager does not know the selling price at the time of production decision but does know at least its subjective probability distribution.

While ABL employ a linear cost function as it is commonly used in most cost-volume-profit (CVP) models, we introduce a quadratic cost function. In particular, it is assumed that the cost function for a profit center is known with certainty and is given by:

$$C(x) = F + V(x) = F + vx^2 \tag{1}$$

where C denotes the total costs, x the production quantity, F the total fixed costs (which incur at and/or are directly traceable to a profit center), and V the total variable costs that are assumed to show increasing marginal cost and to be equal to vx^2 $(v>o)$.[2]

In their analysis of the fixed cost effect in a context of CVP analysis, ABL assume that the output market is perfectly competitive. In contrast, we assume that the market for a center's output is either perfectly competitive or imperfectly competitive. For the expositional simplicity, however, our analysis to follow is based upon the assumption of perfect competition. It is shown in the Appendix that the result of the paper does not alter even in the imperfectly competitive market where the demand function for a center's output is downward sloping.

The manager of a profit center is evaluated and rewarded based upon reported profit, that is operational profit minus allocated common costs, perhaps because reported profit is only the cost-effective, observable measure of performance. In this simplified setting, rational manager will take into account how alternative cost allocations differentially affect reported profit. The *ex-ante* reported profit for a center under no allocation (π), which is equivalent to the *ex-ante* operational profit, is given by:

$$\pi = px - C(x) = px - F - vx^2 \tag{2}$$

where p is the *ex-ante* selling price, assumed to be a random variable with mean p and standard deviation σ_p. It follows from (2) that:

$$E(\pi) = px - F - vx^2 \tag{3}$$

$$\sigma(\pi) = x\sigma_p \tag{4}$$

where E is the expectation operator and $s(\pi)$ is the standard deviation of profits. The decision problem faced by the manager of a profit center is to produce output which maximizes the expected utility of reported profit, that is:

$$\text{Max } E[U(\pi)], \text{ subject to (2)} \qquad\qquad (5)$$

where $U(\pi)$ is a von Neuman-Morgernstern utility-of-profit function with $U'(\pi) > 0$ (nonsatiation) and $U''(\pi) < 0$ (risk aversion) or $U''(\pi) = 0$ (risk neutrality).

To illustrate graphically the process of determining the manager's optimum, this study modifies the formulation in (5) by using the mean-standard deviation framework. Following Tobin [14], Sharp [13], Lintner [10] and Hawawini [7,8], it can be shown that when the price distribution can be specified solely by the first two moments, the decision problem in (5) is equivalent to:[3]

$$\text{Max } G(E(\pi), s(\pi)), \text{ subject to (3) and (4)} \qquad\qquad (6)$$

where G is a utility function with $\partial G/\partial \sigma(\pi) > 0$ and $\partial G/\partial \sigma(\pi) < 0$ (risk aversion) or $\partial G/\partial \sigma(\pi) = 0$ (risk neutrality). Although the primary concern of this study is to incorporate risk aversion into the analysis of cost allocation effects, risk neutrality will also be considered so that the cost allocation effect under both cases can be compared.

The constraints (3) and (4) indicate that both the expected profit and the risk (measured by $\sigma(\pi)$) are a function of the level of output, x. In other words, the manager of a profit center can trade off between the expected profit and the risk by controlling the output level. The (feasible) opportunity set for this trade-off can be easily derived by substituting (4) into (3), which leads to the following strictly concave function (with respect to $\sigma(\pi)$):

$$E(\pi) = -(v/\sigma_p^2)\sigma^2(\pi) + (p/\sigma_p)\,\sigma(\pi) - F \qquad\qquad (7)$$

Thus, the decision problem in (6) is equivalent to maximizing G(•) subject to (7). Note that the strictly concave, quadratic opportunity set in (7) allows us to locate the optimum not only for risk aversion but also for risk neutrality.

Figure 1 illustrates graphically the optimal solution for the manager of a profit center under both risk aversion ($\partial G/\partial \sigma(\pi) < 0$) and risk neutrality ($\partial G/\partial \sigma(\pi) = 0$) when common costs (e.g., home office expenses) are not allocated to profits centers.

The utility function, G($\bullet$), is represented by the indifference curve in the mean-standard deviation-of-profit plane. The curve I and the line I′ represent the indifference curves for risk-averse and risk-neutral managers, respectively. The curve H represents the opportunity set in (7).

The optimum is obtained at the point of tangency between the indifference curve (I or I′) and the opportunity curve H. The points A and B represent the optima for risk-averse and risk-neutral managers, respectively. Once the optimal levels of expected profit and risk are determined in the first quadrant, the optimal level of output can be found in the fourth quadrant by using the line OQ representing equation (4). The optimal level of output for risk-averse managers is represented by X_0 which corresponds to the optimal level of risk, Oa, and the expected profit, OE_0, while that for risk neutral managers by $X_0′$. Note that the optimal level of output under risk neutrality can be regarded as that under certainty since the risk does not affect the production decision of a risk-neutral manager. Thus, as depicted in Figure 1, risk-averse managers produce less output at the optimum than risk-neutral managers ($X_0 < X_0′$).

The next section will focus on how the manager's attitude toward risk differently affects production decisions and how cost allocations under alternative schemes change the opportunity set, which, in turn, leads to new optima. The effect of cost allocations on the local manager's optimizations

will be determined by comparing the initial optimum derived in this section and the new optima under alternative allocation schemes.

III. COST ALLOCATION EFFECTS UNDER ALTERNATIVE SCHEMES

Common costs are the costs of providing common resources or services shared by multiple users (e.g., salaries for the chief executive officer and company-wide advertising). In this study, it is assumed that common costs are not directly traceable to profit centers and are to be allocated by using a predetermined allocation rate. The predetermined allocation rate is also assumed to be known to local managers prior to their production decisions.[4] Four alternative allocation schemes (or predetermined rates) will be considered:

The Lump-Sum Allocation Scheme

In this paper, the lump-sum allocation is said to occur if there is nothing the manager of a profit center can do to alter the amount of costs allocated to him. Put differently, the amount of allocated costs under this scheme depends totally on the headquarter's decision. In this sense, the allocated costs may be regarded as a lump-sum tax imposed on profit centers. Suppose, for example, that the headquarter selects as an activity base the size of center's fixed assets for which the manager of a profit center has neither responsibility nor authority to control. In such a case, the amount of common costs allocated to a profit center (A_l) is neither controllable nor affected by the center manager's production and sales decisions. Accordingly, the *ex-ante* reported profits under this allocation scheme (π_1) can be expressed as follows:

$$\pi_1 = px - F - vx^2 - A_l \tag{8}$$

Thus,

$$E(\pi_1) = px - F - vx^2 - A_l \tag{9}$$

$$\sigma(\pi_1) = \sigma_p x \tag{10}$$

Equations (9) and (10) indicate that the lump-sum allocation causes the expected profit to decrease by the amount of A_l with the level of risk unaffected. Substituting (10) into (9) yields the new opportunity set:

$$E(\pi_1) = - (v / \sigma_p^2) \sigma^2(\pi_1) + (p /\sigma_p) \sigma(\pi_1) - (F + A_l) \qquad (11)$$

Figure 2 illustrates the new optima for both risk-averse and risk-neutral managers given the new opportunity set in (11) and compares them with the initial optima.

The new opportunity set in (11) is represented by the curve H_1 while the initial opportunity set in (7) by the curve H. Since the indifference map for risk-neutral managers looks like the set of lines denoted by I′ and II′, the new optimum for *risk-neutral* managers will be obtained at the tangent point D. As a result, the optimal level of output remains unchanged ($X_0′$), and the level of utility at the new optimum decreases relative to that at the initial optimum, a finding consistent with Zimmerman who considers only the certainty case. Comparison of the two opportunity sets in (7) and (11) reveals that the lump-sum allocation is equivalent to an increase in center's (direct) fixed costs by the amount of A_l. No change in the optimal level of output for risk-neutral managers is not surprising given that: (1) the optimal output under risk neutrality is equal to that under certainty; and (2) under certainty, an increase in fixed costs (resulting from the lump-sum allocation) does not affect production decisions unless the contribution margin changes.

In contrast, the new optimum for *risk-averse* managers depends on the Pratt-Arrow absolute risk aversion function of reported profit, which in turn determines the structure of the indifference map in the mean-standard deviation-of-profit plane. Decreasing (increasing) absolute risk aversion means that, for a given level of risk, the magnitude of risk aversion, which can be measured by the slope of the tangent line for the indifference curve in the mean-standard deviation-of-profit plane (i.e., the marginal rate of substitution between expected profit and risk), decreases (increases) as the expected profit increases.[5] Consequently, the indifference map for decreasing (increasing) absolute risk-averse managers should look like the set of curves I and II_1 (I and II_3) in Figure 2. Constant absolute risk aversion means that the magnitude of risk aversion remains unchanged for a given level of risk as the expected profit increases. Thus the indifference map for constant absolute risk-averse managers should be of a type depicted by the set of curves I and II_2.[6]

In the most likely case of decreasing absolute risk aversion, the cost allocation under this scheme causes the optimum to shift from A to C. As a consequence, the optimal level of output decreases from X_0 to X_1 ($X_0 > X_1$), and the level of utility at the new optimum decreases relative to that at the initial optimum. In the unlikely case of increasing and constant absolute risk aversion, the new optimum is obtained at the tangent points E and J, respectively, and the corresponding level of output at X_3 ($> X_0$) and X_2 ($= X_0$), respectively.

In sum, for managers displaying risk neutrality and constant absolute risk aversion, the lump-sum allocation of common costs does not affect the local manager's optimal production decision. However, it causes decreasing (increasing) absolute risk-averse managers to produce less (more) output at the new optimum than at the initial optimum. Furthermore, the lump-sum allocation causes both risk-neutral and risk-averse managers' levels of utility to decrease at the new optimum. This decrease in managerial utility arises because the lump-sum allocation results in a decrease in expected profit with the level of risk unchanged as manifested in equations (9) and (10).

Arrow [4], among others, provides an explanation of why decreasing absolute risk aversion is a reasonable approximation of risk-averse behavior. Consequently, only decreasing absolute risk aversion will be considered in the remaining sections.

The Output-Based Allocation Scheme

When the output level is used as an activity base, the amount of common costs allocated to a profit center (A_x) is in direct proportion to the center's output level:

$$A_x = r_x x \tag{12}$$

where r_x is the output-based predetermined rate. It is assumed that the amount of allocated common costs is less than the expected sales but is greater than zero (i.e., $0 < r_x < p$).[7] Since the center's output level, x, is controllable by the manager of a profit center, so is the allocated amount, A_x. The *ex-ante* reported profit for a center under this allocation (π_2) can thus be written as:

$$\pi_2 = px - F - vx^2 - r_x x \tag{13}$$

Accordingly

$$E(\pi_2) = (p - r_x)\, x - F - vx^2 \tag{14}$$

$$\sigma(\pi_2) = \sigma_p x \tag{15}$$

Substitution of (15) into (14) leads to the new opportunity set as follows:

$$E(\pi_2) = -\left(v / \sigma_p^2 \right) \sigma^2(\pi_2) + \left((p - r_x) / \sigma_p \right) \sigma(\pi_2) - F \tag{16}$$

As shown in equations (14) and (15), the output-based allocation causes the expected reported profit to decrease at the optimum with the level of risk unaffected. Furthermore, comparisons of equations (3) and (14) and of the opportunity sets (7) and (16) indicate that the output-based allocation has the same effect on the production decision as does a decrease in the expected price by the amount of r_x, which, other things being equal, leads to decrease in production. Thus the output-based allocation induces both risk-averse and risk-neutral managers of a profit center to reduce production under price uncertainty.
    Figure 3 illustrates the new optima given the opportunity set in (16) for the risk-neutral and decreasing absolute risk-averse managers.

The output-based allocation shifts down the opportunity curve in a nonparallel fashion from the curve H to the curve H_2. This is so because under the assumption of $0 < r_x < p$, the level of risk satisfying the first order condition for (16) and the corresponding level of expected profit are lower than those derived from (7) (i.e., Od < 0b and Dd < Bb).[8] The set of curves I and II represents the indifference map for decreasing absolute risk aversion while the set of curves I¢ and II¢ for risk neutrality. For decreasing absolute risk-averse managers, the new optimum will be obtained at the point C where the new opportunity curve H_2 and the indifference curve II are tangent. The level of output corresponding to the new optimum is obtained at X_2 by using the line OQ. The output-based allocation causes decreasing absolute risk-averse managers to produce less output at the new optimum than at the initial optimum under no allocations ($X_0 > X_2$). For risk-neutral managers, the new optimum will be obtained at the point D, and the corresponding level of output at X. In sum, the cost allocation under this scheme leads to a decrease in the optimal level of output for both decreasing absolute risk aversion and risk neutrality, while it causes the levels of utility of both risk-averse and risk-neutral managers to decrease at the new optimum.

The Sales-Based Allocation Scheme

When common costs are allocated by using the level of sales dollars as an activity base, the *ex-post* amount of common costs allocated to a profit center (A_s) is determined as follows:

$$A_s = r_s \cdot s, \tag{17}$$

where
r_s = the predetermined allocation rate $(0 < r_s < 1)$; and
s = sales dollars realized at a profit center.

Since the manager of a profit center does not know the level of sales dollars (s) until sales are realized, the *ex-ante* amount of common costs allocated to a profit center (A_s) is a random variable given by:

$$A_s = r_s (x) \tag{18}$$

Accordingly, the *ex-ante* reported profit for a center under this allocation scheme (π_3) can be expressed as:

$$\pi_3 = px - F - vx^2 - r_s (px) \tag{19}$$

Equation (19) indicates that the cost allocation under this scheme, from the *ex-ante* point of view, is equivalent to imposing a sales tax on the center's sales. It follows from (19) that:

$$E(\pi_3) = (1 - r_s) px - F - vx^2 \tag{20}$$

$$\sigma(\pi_3) = (1 - r_s) \sigma_p x \tag{21}$$

Equations (20) and (21) show that the cost allocation under this scheme causes both the levels of expected profit and risk to decrease. Substituting (21) into (20) leads to the following opportunity set:

$$E(_3) = -(v / (1 - r_s)^2 \sigma_p^2) \sigma^2(\pi_3) + (p / \sigma_p) \sigma(\pi_3) - F \tag{22}$$

In Figure 4, the new optimum given the new opportunity set in (22) is illustrated and compared to the initial optimum.

With respect to the new opportunity curve, H_3, the following points deserve mentioning. First, the curve H_3 should have a shape such that Od < Ob and Dd < Bb since the level of risk satisfying the first-order condition for (22) and the corresponding level of expected profit are less than those derived from (7). Second, the slope of the tangent line for the curve H_3 is greater than that for the curve H for a given level of risk because the value of the first derivative of (7) is less than that of (22) for the Od domain of risk. This implies that cost allocations give rise to an increase in the magnitude of expected profits required to compensate for additional risk within the Od domain.

For decreasing absolute risk-averse managers, the indifference map should be of a type depicted by the curves I and II, and thus the point of tangency between the indifference curve II and the new opportunity set H_3 can only occur within the domain of Od. In Figure 4, the new optimum for risk-averse managers is represented by the tangent point, C. Note that, unlike the lump-sum and output-based allocations, the sales-based allocation leads to an increase in the level of the risk-averse manager's utility at the new optimum relative to that at the initial optimum. This is not surprising given that the sales-based allocation reduces not only the expected profit (the marginal utility of which is positive) but also the level of risk (the marginal utility of which is negative) as manifested in equations (20) and (21). In other words, the negative effect of the decrease in expected profit on managerial utility is dominated by the positive effect of risk reduction. Note further that the cost allocation under this scheme shifts down the line representing the relation between the expected profit and the risk from OQ to OQ_3. The optimal level of output will thus be obtained at X_3, which is determined by using OQ_3, not OQ. Unlike the previous two allocation

schemes, the optimal level of output increases from X_0 to X_3 ($X_0 < X_3$) as a result of cost allocations.

For risk-neutral managers, the sales-based allocation will cause the optimum to shift from B to D. The direction of the change in the optimal level of output depends on the magnitude of the predetermined rate, which in turn determines the slope of the line OQ_3. In Figure 4, the predetermined rate, r_s, was assumed to be equal to r so that the optimal level of output at the new optimum could be equal to that at the initial optimum for risk-neutral managers. However, if r_s is greater (less) than r (but less than unity), then the optimal level of output at the new optimum would decrease (increase) relative to that at the initial optimum.

The Profit-Based Allocation Scheme

Suppose that common costs are allocated by using the level of profits as an activity base. Then the *ex-post* amount of common costs allocated to a profit center (A_π) is in direct proportion to center's reported profit:

$$A_\pi = r_\pi \times p, \tag{23}$$

where
r_π = the predetermined rate ($0 < r_p < 1$); and
p = actual level of center's reported profits under no allocation.

Since the selling price is a random variable, the manager of a profit center would expect the (*ex-ante*) allocated amount of common costs to be $r_p \times \pi$ where π is as defined in equation (2). Accordingly, the *ex-ante* reported profit for a center under this allocation scheme (π_4) can be expressed as:

$$\pi_4 = \pi - r_\pi . \pi = (1 - r_\pi)\pi \tag{24}$$

Equation (24) implies that the profit-based allocation, from the *ex-ante* point of view, can be regarded as a proportional tax on the center's profits. The expected profit and the risk under this scheme are:

$$E(\pi_4) = (1 - r_\pi) (px - vx^2 - F) \tag{25}$$

$$\sigma (\pi_4) = (1 - r_\pi) \sigma_p x \tag{26}$$

The profit-based allocation thus decreases both the levels of the expected profit and the risk by $100r_p$ percent which in turn changes the trade-off

relation between the two. The new opportunity set can be obtained by substituting (26) into (25) as follows:

$$E_{(4)} = - (v / (\sigma^2_p(1 - r_\pi)) \sigma^2(\pi_4) + (p / \sigma_p) \sigma(\pi_4) - (1 - r_\pi) - F \quad (27)$$

Figure 5 illustrates the new optimum given the new opportunity set in (27).

For the same reasons described in the preceding subsection, the shape of the new opportunity set should look like H_4 (Od < Ob and Dd < Bb), and the indifference map for decreasing absolute risk-averse managers should be of the type depicted by the set of curves I and II. Given the new opportunity curve H_4 and the indifference curve II, the new optimum for decreasing absolute risk-averse managers will be obtained at the tangent point C, and the corresponding level of output at X_4, which is determined by using the line OQ_4 instead of OQ. Thus, the profit-based allocation causes decreasing absolute risk-averse managers to increase the level of output from X_0 to X_4 ($X_0 < X_4$). Note further that, similar to the sales-based allocation, the profit-based allocation leads to an increase in the level of the risk-averse manager's utility at the new optimum relative to that at the initial optimum, a finding contrary to Zimmerman's. For risk-neutral managers, the new optimum is obtained at the point D, but the corresponding level of output depends on the magnitude of the predetermined rate for the same reasons explained in the preceding subsection.

IV. SUMMARY AND CONCLUDING REMARKS

The effects of alternative common cost allocations on managerial utility and production decisions under price uncertainty are examined in a decentralized organizational setting. Four alternative allocation schemes are considered: (1) the lump-sum allocation, (2) the output-based allocation, (3) the sales-based allocation, and (4) the profit-based allocation. Under the first two schemes, the allocation of common costs leads to a decrease in expected profit with the level of risk unchanged, while under the last two schemes, it leads to a decrease in both expected profit and risk. Through their effects on expected reported profit and risk, cost allocations give rise to a change in the opportunity set, which in turn leads to new optima.

Comparison of the initial optimum (under no allocation) with the new optima under alternative schemes reveals the following: First, *risk-neutral* managers do not change the level of output under the lump-sum allocation scheme, and produce less output under the output-based allocation scheme. However, the optimal level of output under the sales-based and profit-based allocation schemes depends on the magnitude of the predetermined allocation rate. Second, decreasing absolute *risk-averse* managers produce less output at the new optimum under the first two schemes, but more output under the last two schemes, which supports ABL's intuition cited earlier. Finally, under the first two schemes, the level of the risk-averse manager's utility decreases at the new optimum while it increases under the last two schemes, a finding inconsistent with Zimmerman.

The adoption of the first two (utility-decreasing) schemes may not be cost-effective from the standpoint of headquarter because: (1) it not only makes local managers less happy but also would create job searching or other incongruent behavior if managerial utility decreases below the level of the opportunity utility determined in the market for managerial services; and (2) the decrease in center's output (resulting from cost allocations) could mean a decrease in the center's contribution to firm's overall profits especially when profit centers are operated in the perfectly competitive market. Thus, managers of the headquarter would prefer the last two schemes to the first two schemes to the extent that they are concerned with management strategies which increase the utilities of local managers with the least cost. This suggests a testable implication that the sales-based and/or profit-based allocation schemes would be preferred to the lump-sum and/or output-based allocation schemes in decentralized firms whose profit centers are faced with price uncertainty. This implication is contradictory with what was suggested in Zimmerman [16].

However, the result of this study should be compared with Zimmerman's cautiously. While his model ignores the differential effect of

alternative cost allocations on a change in the risk borne by the manager, our model fails to explicitly model their effect on the perquisite consumption. Thus the challenge in future research is to reconcile the difference between the two models by developing a more complete model which takes into account both the perquisite consumption and the risk. Further research on this direction may provide more useful insight for explaining and predicting why different firms adopt different allocation practices.

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APPENDIX

This Appendix demonstrates that the relaxation of the perfect competition assumption does not change the major results of this study. Suppose that under imperfect competition, the demand function for a profit center facing uncertain demands is given by:

$$p = f(\tilde{x}, e) = f(x)\tilde{} + e \qquad \text{(al)}$$

where f denotes a function and e is a random element which is assumed to be additively separable and normally distributed with zero mean and constant variance, s. Assume, for simplicity, that

$$f(x) = a - bx \qquad a, b > 0 \qquad \text{(a2)}$$

It follows from (al) and (a2) that the expected price (p) and standard deviation of price (σ_p) is:

$$\bar{p} = f(x) = a - bx \qquad \text{(a3)}$$

$$\sigma_p = \sigma_e \qquad \text{(a4)}$$

Note that in the absence of cost allocations, the *ex-ante* reported profit is:

$$\pi = f(x)x - F - vx^2$$

$$\tilde{}$$

$$= (a - bx + e)x - F - vx^2 \tag{a5}$$

Thus,

$$E(\pi) = - (b + v)x^2 + ax - F \tag{a6}$$

$$\sigma(\pi) = x \times \sigma_e = x \times \sigma_p \tag{a7}$$

Substitution of (a7) into (a6) leads to the opportunity set:

$$E(\pi) = - ((b + v) / \sigma^2_p) \sigma^2(p) + (a / \sigma_p) \sigma(\pi) - F \tag{a8}$$

which corresponds to that under perfect competition in (7).

Under imperfect competition, the *ex-ante* reported profits under the lump-sum, output-based, sales-based, and profit-based allocation schemes can be expressed as in equations (a9), (a10), (a11), and (a12), respectively:

$$\pi_1 = (a - bx + e)x - F - vx^2 - A_l \tag{a9}$$
$$\pi_2 = (a - bx + e)x - F - vx^2 - r_x x \tag{a10}$$
$$\pi_3 = (1 - r_s)(a - bx + e)x - F - vx^2 \tag{a11}$$
$$\pi_4 = (1 - r_\pi)[(a + bx + e)x - F - vx^2] \tag{a12}$$

Following the same procedures used to obtain (a8), we can derive the opportunity sets under four alternative schemes as follows:

(1)   The lump-sum allocation:

$$E(\pi_1) = - ((b + v) / \sigma^2_p)\sigma^2(\pi_1) + (a /\sigma_p)\sigma(\pi_1) - F - A_l \tag{a13}$$

(2)   The   output-based   allocation:

$$E(\pi_2) = - ((b + v) /\sigma^2_p)\sigma^2(\pi_2) + (a - r_x) /\sigma_p)\sigma(\pi_2) - F \tag{a14}$$

(3)   The sales-based allocation:

$$E(\pi_3) = -((1 - r_s)(b + v) /((1-r_\pi)^2\sigma^2_p)\sigma^2(\pi_3) + (a/\sigma_p) \sigma(\pi_3) - F \tag{a15}$$

(4)   The profit-based allocation:

$$E(\pi_4) = - ((b + v)/(1 - r_p)\sigma^2_p)\sigma^2(\pi_4) + (a/\sigma_p)\sigma(\pi_4) - (1 - r_\pi) - F \tag{a16}$$

The opportunity sets under imperfect competition in (a13), (a14), (a15), and (a16) correspond to those under perfect competition in (11), (16), (22), and (27), respectively. Note that if a = p and b = 0 in (23), the case of imperfect competition then reduces to that of perfect competition.

Comparison of the initial opportunity set in (a8) with the new ones in (a13), (a14), (a15), and (a16) reveals that although cost allocations cause the (initial) opportunity set to shift in various ways, the cost allocation effect of each scheme on the opportunity set under imperfect competition is the same with that under perfect competition. Therefore, Figures 2, 3, 4, and 5 can be used to illustrate the cost allocation effect of alternative schemes under imperfect competition. In sum, the relaxation of the perfect competition assumption (A4) does not alter the results of this study.

NOTE

1. Biddle and Steinberg [6] provide an excellent survey of the cost allocation literature.
2. While most CVP analyses, including ABL's, employ a linear cost function, we assume a form of quadratic cost function as in equation (1) because it allows us to obtain more general results. First, as will be explained later, it gives rise to a strictly concave opportunity set which guarantees the existence of the optima for both risk aversion and risk neutrality. Second, in the case of risk aversion, the results of this study are robust to whether the cost function is quadratic or linear. Finally, if the cost function is linear, the optimal level of output under certainty becomes infinite while it can be uniquely determined if the cost function is quadratic.
3. The formulation in (5) is equivalent to that in (6) when the utility function, U(), is quadratic. This study, however, excludes the assumption of quadratic utility function because it is appropriate only for the unlikely case of increasing absolute risk aversion. For more details, see Alexander and Francis [2], chapters 2 and 3.
4. Consistent with its definition used in most cost accounting textbooks, the predetermined allocation rate is defined as the ratio of budgeted costs (to be allocated) to the budgeted level of activity (or cost driver). In a decentralized firm, local managers may influence the determination of the allocation rate through their participation in budgeting process. In this paper, it is, however, assumed to be exogenously given to the manager of a profit center.
5. See Amihud [3] for an excellent exposition on the relationship between the Arrow-Pratt absolute risk aversion function and the

marginal rate of substitution between expected profit and risk in the mean-standard deviation plane.

6. For more detailed discussion on this issue, see Amihud [3], Miller [11], and Hawawini [7,8].

7. As will be later explained in more detail, the assumption of $0 < r_x < p$ is necessary to locate the optimum under the output-based allocations. See also endnote (8).

8. It can be shown that: $\sigma(\pi_2)^* = (p - r_x) \sigma_p / 2v$ and $\sigma(\pi)^* = p \sigma_p / 2v$, where $\sigma(\pi_2)^*$ and $\sigma(\pi)^*$ represent the levels of risk satisfying the first order conditions for (16) and (7), respectively. It thus follows that $\sigma(\pi_2)^* < \sigma(\pi)^*$ with the assumption of $0 < r_x < p$. By substituting $\sigma(\pi_2)^*$ and $\sigma(\pi)^*$ into (16) and (7), respectively, it can be also shown that $E(\pi_2)^* < E(\pi)^*$ where $E(_2)^*$ and $E(\pi)^*$ are the levels of expected risk corresponding to $\sigma(_2)^*$ and $\sigma(\pi)^*$, respectively.   Note that if $r_x > p$, then $\sigma(\pi_2)^* < 0$ which is contradictory to the definition of standard deviation, and $E(\pi_2)^*$ in (14) could not be positive (i.e., always negative) regardless of the level of costs. The assumption of $0 < r_x < p$, however, eliminates the above possibility.

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