A NOTE ON THE CALCULATION OF THE AFTER-TAX COST OF DEBT

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When one examines standard financial management texts' treatments of how to calculate the after-tax yield and cost from the issuing firm's perspective of a particular debt issue, one finds apparent inconsistencies and, sometimes, outright errors. A general model to find the after-tax cost of debt is developed and applied to several cases. Any method that includes the conversion $k_d = k_d^*(1 - T)$ will be accurate if there are no flotation costs regardless of the net proceeds to the firm; however, when both flotation cost and a discount (or premium) over face value at issue exist, the conversion is inaccurate. Whether the magnitude of the error is significant is a question of the analyst's subjective judgement.

I. INTRODUCTION

When one examines standard financial management texts' treatments of how to calculate the after-tax yield and cost from the issuing firm's perspective of a particular debt issue,[1] one finds apparent inconsistencies and, sometimes, outright errors. Three specific conditions seem to be at the root of the problem: (1) the case where debt, at issue, is sold at a premium or a discount; (2) the case where a flotation cost makes the proceeds to issuer less than the face value of the debt; and (3) the case where both flotation costs and premia or discounts change the proceeds to the issuer. This note discusses each of these cases, comparing the correct technique to methods found in financial management texts. This note is in contrast to the recent paper by Bey and Collins [1], which examines before- and after-tax yields from the investor's viewpoint.

To accomplish this objective, Section II develops the basic model used for the analysis and presents the problem. In Section III, the basic

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model is used to prove that a two-equation approach to estimate the after-tax cost of debt provides the correct result, even though it may be less appealing than a single-equation method, regardless of the amount of the net proceeds to the firm. Section IV examines the relationship between the two- and single-equation methods when flotation costs are present. The final section summarizes the research effort.

II. THE BASIC MODEL AND PROBLEM

Two methods for estimating a firm's after-tax cost of debt are commonly used. The first is given in Eq(1a) and Eq(1b). The second method is given in Eq(2). For example, Gitman [4], and Gitman, Joehnk and Pinches [5] show that the cost of debt is found by first solving for k_d^* in the debt valuation equation:

$$B_0 = \sum_{t=1}^{n} (I_t + P_t)/(1 + k_d^*)^t \qquad (1a)$$

where,
B_0 = Proceeds from investors at issue (may be at par, premium, or discount);
I_t = interest in period t;
P_t = periodic principal repayment;
n = bond maturity; and
k_d^* = before-tax cost of debt.

In the most general application, one may think of the sum of each period's interest and principal repayment as the periodic cash flow servicing the creditors. The after-tax cost, k_d, is found by:

$$k_d = k_d^*(1 - T), \qquad (1b)$$

where T = firm's marginal tax rate.

Nevertheless, one might question whether this method is equivalent to a straightforward application of the valuation notion. Generally, a valuation is a value calculation based on a stream of cash flows discounted at the appropriate rate; one can approach the problem with either the value or the discount rate as the unknown. Specifically, the after-tax cost of debt application is:

$$B_0 = \sum^{n} (I_t + P_t - TI_t')/(1 + k_d)^t \qquad (2)$$

$$t=1$$

The quantity, $I_t^{'}$, is the portion of the annual cash flow that is tax deductible by IRS regulation.[2]

The calculation of k_d through the two-equation method (Eq(1a) and Eq(1b)) is not the intuitively obvious equivalent to k_d found through the single equation method (Eq(2)). Bierman and Smidt [2: 359] demonstrate that both methods yield the same results in the limited case where neither discounts (or premia) nor flotation costs exist. The broader question here is whether the results differ in the presence of discounts (or premia).

III. A GENERAL PROOF

To answer the question, consider a special case (for reference purposes, this is called Case Q) of Equation 1a, where the interest payments are in each period equal to k_d^{*} times the remaining balance. When the interest payments are subtracted from their respective periodic debt service cash flows, the result is the true stream of payments applied to the principal. In some applications, the true principal payment will be non-positive. With the assumption that the entire principal will have been repaid at period n, the sum of the principal payments will equal the amount B_0. This is true whether the cash flow stream is equal payments, equal principal payments, negative amortization, a sinking fund with optional double payments, or any other imaginable redemption scheme.

The IRS Code [12] states that the deductible interest on discount (premium) at issue debt is found by first solving Equation 1a for k_d^{*} --this rate is the true interest rate. This rate is then used in an amortization table to determine the schedule of deductible interest payments over the debt's life. In other words, the deductible interest is k_d^{*} times the true remaining balance. This is the "effective interest rate" method, as demonstrated in Danos and Imhoff [3].

Now consider the periodic cash flow to service the debt, $CF_t = P_t + I_t$, i.e., the sum of the nominal interest and principal repayments. In each period, one may restate CF to be the true interest payment, $I_t^{'}$, plus the true principal repayment, $P_j = CF_t - I_t^{'}$. Again, the sum of $P_t^{'} = B_0$. Equation 2, by substitution of $I_t + P_t = I_t^{'} + P_t^{'}$ becomes:

$$B_0 = \sum_{t=1}^{n} (I_t^{'} + P_t^{'} - TI_t^{'})/(1 + k_d)^t . \qquad (3)$$

Rearrangement of terms yields:

$$B_0 = \sum_{t=1}^{n} (I_t^{'}(1-T) + P_t^{'})/(1 + k_d)^t .\qquad(4)$$

Notice that Equation 4 can be thought of as modelling a debt issue whose rate, as applied to the unpaid balance, is $k_d^{*}(1 - T)$. Also, this debt issue's B_0 equals the sum of its principal payments. Thus, the factors fit precisely the definitions for special Case Q of Equation 1a, if and only if the discount rate is $k_d^{*}(1 - T)$. Therefore, regardless of whether debt is issued at par, a discount, or a premium (in consideration of its face value and coupon), its after-tax cost is equal to the before-tax cost times $(1 - T)$; i.e., $k_d = k_d^{*}(1 - T)$.

As demonstrated by Equation 4, as long as the interest expense is the effective interest as required by the IRS, the equality between the two methods exists. To illustrate the equivalence of the two methods in the presence of taxes and discounts, assume a bond with a par value of $1,000, a 5 percent coupon, a maturity of 10 years, a marginal tax rate of 34 percent, and a discount of $307.23. The yield to maturity on this bond is 10 percent. The numerical calculations are shown in Table 1.

Table 1: Illustration of the Basic Model

Year	Starting Balance	Coupon	Interest	Principal Reduction	Ending Balance	Interest Tax Shield	Cash Flow
1	$692.77	$50	$69.28	$-19.28	$712.05	$23.56	$26.44
2	712.05	50	71.20	-21.20	733.25	24.21	25.79
3	733.25	50	73.32	-23.32	756.57	24.93	25.07
4	756.57	50	75.66	-25.66	782.23	25.72	24.28
5	782.23	50	78.22	-28.22	810.45	26.59	23.41
6	810.45	50	81.04	-31.04	841.49	27.55	22.45
7	841.49	50	84.15	-34.15	875.64	28.61	21.39
8	875.64	50	87.56	-37.56	913.20	29.77	20.23
9	913.20	50	91.32	-41.32	954.52	31.05	18.95
10	954.52	50	95.45	954.55	0.00	32.45	1,017.55

Solving for the after-tax cost of debt, using the after-tax cash flows, yields an after-tax cost of debt of 6.6 percent. The same result is found by multiplying the before-tax cost of debt by $(1-T)$, $.10(1-.34)=.066$ or 6.6 percent.

IV. THE ANALYSIS WITH FLOTATION COSTS

The chief cause of questionable discussions of the cost of debt with regard to flotation cost is the omission of tax consequences. The IRS Revenue Ruling [11] states that direct flotation costs for debt may be written off in straight-line fashion over the life of the debt. One plausible explanation for the omission of tax consequences may be that the typical flotation cost of debt is quite small, so that its tax benefits are negligible. Van Horne [13] gives a more sophisticated reason, stating that attaching the flotation cost to each project to be financed is a superior method because attaching it to the cost of capital causes an "excessive penalty." (See Hubbard [7] for a complete discussion and bibliography on this notion.) This argument has merit but runs into some practical difficulties: (1) when several projects are funded at the same time by several different sources, how does one determine which source's flotation cost goes with which project?; (2) the method deals with flotation costs, excluding their tax effect; and (3) as evidenced by many cites in the reference list [2, 4, 5, 6, 8, 9, and 10] and many other financial management texts, academicians and practitioners seem to use the method attaching the flotation costs to the cost of capital.

Addition of the write-off to the cash flow stream to the valuation notion yields a modified form of Eq(2), where F is the direct flotation cost in dollars:

$$B_0 - F = \sum_{t=1}^{n} (I_t' + P_t' - TI_t' - TF/n)/(1 + k_d)^t. \qquad (5)$$

Note that the simple conversion, $k_d = k_d^*(1 - T)$, is inaccurate for this case (Unfortunately, several authors incorrectly advocate this method, e.g., Moyer, McGuigan and Kretlow [9]; Petty, Scott, Keown, and Martin [8]; Gup [6]; and perhaps Pinches [10].) This "popular" method appears as Equation 6:

$$B_0 - F = \sum_{t=1}^{n} (I_t + P_t)/(1 + k_d)^t \qquad (6)$$

For example, consider a bond with the following traits:

$B_0 = \$692.77$; $n = 10$ years; $I_{1 \text{ to} 10} = \$50$; $P_{10} = \$1,000$; $F = 34.64$ or 5 percent of B_0; $T = .34$

If one applies the method initiated by Equation 1a, k_d^* is 10 percent, ignoring flotation costs, and Equation 1b will produce $k_d = 7.09$ percent. The accurate method found through Equation 5 finds k_d to be 7.08 percent from the cash flows in Table 2. Thus, a small error comes from ignoring the

tax effect of the flotation costs. Flotation cost always increases the cost of new debt. The popular method overstates the amount of the increase in the after-tax cost of debt. Examination of Equation 5 shows that the best estimate of the after-tax cost of debt lies between the rates yielded by the $k_d = k_d^*(1 - T)$ method and the popular method. Table 3 shows the magnitude of the difference for various debt characteristics and flotation costs.

Table 2: Illustration of Equation 5

Year	After-Tax Cash Flow*	Flotation Cost Tax Shield	Cash Flow
1	$ 26.44	$-1.18	$ 25.26
2	25.79	-1.18	24.62
3	25.07	-1.18	23.90
4	24.28	-1.18	23.90
5	23.41	-1.18	23.11
6	22.45	-1.18	21.28
7	21.39	-1.18	20.22
8	20.23	-1.18	19.06
9	18.95	-1.18	17.78
10	1,017.55	-1.18	1,016.38

V. CONCLUSION

Eq(5) applies generally to all cases whether B_0 is at a premium or a discount. Any method that includes the conversion $k_d = k_d^*(1 - T)$ will be accurate if there are no flotation costs regardless of the net proceeds to the firm; however, when both flotation cost and a discount (or premium) over face value at issue exist, the conversion is inaccurate.Whether the magnitude of the error is significant is a question of the analyst's subjective judgment.

Table 3: After-Tax Cost of Debt Computations Using Equation 5

Market rate=10.0 %; Face value=$1,000; Maturity=10 years; Tax rate=34%

After-Tax Cost of Debt with Flotation Cost at:

Rate	Issue	0%	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%

10%	$1,000.00	6.60	6.71	6.81	6.92	7.03	7.15	7.26	7.38	7.49	7.61	7.73
9	938.55	6.60	6.70	6.81	6.92	7.03	7.14	7.26	7.36	7.47	7.59	7.71
8	877.11	6.60	6.70	6.80	6.91	7.02	7.12	7.23	7.34	7.45	7.57	7.68
7	815.66	6.60	6.70	6.80	6.90	7.00	7.11	7.21	7.32	7.43	7.54	7.65
6	754.21	6.60	6.70	6.79	6.89	6.99	7.09	7.20	7.30	7.40	7.51	7.62
5	692.77	6.60	6.69	6.79	6.88	6.99	7.08	7.18	7.28	7.38	7.48	7.58
4	631.33	6.60	6.69	6.78	6.87	6.96	7.06	7.15	7.25	7.35	7.44	7.54
3	569.88	6.60	6.69	6.77	6.86	6.95	7.04	7.13	7.22	7.31	7.41	7.50
2	508.43	6.60	6.68	6.76	6.85	6.93	7.01	7.14	7.17	7.27	7.36	7.45
1	446.99	6.60	6.68	6.75	6.83	6.91	6.99	7.07	7.15	7.23	7.31	7.40
0	385.54	6.60	6.67	6.74	6.81	6.88	6.96	7.03	7.11	7.18	7.26	7.23

NOTES

1.   One should not confuse the notion of the cost of debt for a particular issue with the marginal cost of debt capital. The latter is a function of many factors, including the firm's operational and financial risk, interest rates in the market, and portfolio effects with other securities. The former may differ because of market imperfections, callability, sinking fund provisions, and other indenture items.

2.   Yawitz and Maloney [14] in their study of original issue discount bonds discuss the tax effects to both issuer and investor, when the effective interest rate method is used.

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