

The Time Structure of Investment Behavior and Output Price Expectations: Solution of A Paradox

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This paper presents a model for the analysis of the time structure of investment behavior and output price expectations. We consider the firm's problem of various dynamic expectation models in the face of the adjustment of capital accumulation and its price. We allow for the adjustment expectations not only in different model settings, including adaptative, extrapolative, and partial adjustment expectations, but also in its general model settings in the real and nominal factors of the investment behavior. Herein lies a paradox of different time structure in the derived regressions. A more appropriate specification for the estimated regressions is derived.

I. INTRODUCTION

The relationship between investment behavior and its underlying determinants has always been a focus of intense research within the field of macroeconomics. A central part of this research has been devoted to studying the time structure of this relationship¹. In their restatement of the time structure of investment behavior, Jorgenson and Stephenson [22] argued that an arbitrary lag distribution may be approximated to any desired degree of accuracy by a rational distributed lag model so that no a priori specification of the form of the lag distribution is required. At the same time, many lag distributions are in fact special cases of the general Pascal distribution. Barry [1] analyzed the demand for investment as a function of the distributed lag and changes in desired capital. This also has been the approach of many applied studies. For example, Koyck [28] employed a geometric lag distribution, Solow [51] proposed the Pascal lag distribution for this purpose, and Leeuw [33] used an inverted-V shaped distribution. Recently, Petersen, et al. [46] used only lag one in their investment cyclical study, but Yoon [58] presented lag one of Tobin q in his lag distributions. Each of these classes of lag distributions has the undesirable property that makes it impossible to approximate an arbitrary lag distribution by a member of the class.

Since the seminal work of Jorgenson [16], the direct effects of real balance on business investment have been studied extensively. On the other

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hand, few have studied the effects of price through a framework relating the real and nominal investment behavior decisions of the firm. The gap may be due to the long-held view that the firm's investment decisions are independent of its financing decisions (the Modigliani-Miller separation theorem), especially as it concerns the role of price in investment behavior. Recently, however, the bulk of the argument has clearly shifted against the separation view, and several questions concerning the impact of asset prices on business investment remain unanswered.

Among the few reported attempts to measure empirically dynamic or expectational effects through a framework relating the real and nominal (or financial) decisions, the most ambitious are those of Brimmer and Sinai [3] and of Sinai, et al. [50]. In these studies, the integration of the firm's real and nominal decisions is made by incorporating financial flow-of-funds constraints into the Jorgenson model of investment behavior. Unfortunately, these studies focused only on the static effects to the firm's decision. For example, it is unclear how to specify expectations for investment and price. Another criticism of the models is that they rely heavily on accounting identities, making comparative statical analysis relatively difficult.

There are many competing views explaining why investment should be characterized by a distributed lag. Among the most widely known hypotheses are the accelerator model; the neoclassical investment model, emphasizing the cost of capital and stock adjustments; and the cash flow model under conditions of imperfect capital markets. To date, there is no widespread agreement on which view of investment is most consistent with the facts concerning the expectation of capital adjustment or which view takes care the expected rates of inflation.

In this paper, we explore the adjustment of expectations of output price and capital as it relates to the firm's investment decision. We consider the firm's problem, assuming various dynamic expectations models in the face of the adjustment of capital accumulation and price. The form of expectations adopted will be seen to influence the lag length required to complete a project. Furthermore, in many studies the inflation rate plays a critical role in investment decisions. It is not intuitively clear why that real capital accumulation should be sensitive to moderate expected rates of inflation. Certainly, from theoretical considerations, in real stock adjustment models with dynamic expectations, there is little role for the expected rate of inflation. Herein lies the paradox. While theory and common sense seem to dictate that both expected and actual inflation play small roles in real capital accumulation, many studies continue to use these variables in an explanatory capacity.

This paper reconciles previous works by showing that with a more appropriate specification, the expected or actual rate of inflation is redundant

and should not be significant. Previous models typically used either a partial or static quantity adjustment or employed various expectation formation mechanisms (e.g., adaptive, extrapolative, or general model).

The time structure of investment behavior in the context of various models is presented below in the second section. An analysis of investment behavior and output price is then undertaken in the third section. In the fourth section, a composite of different expectation models is formed to give a generalized adjustment model. The fifth section presents a description of the real and nominal adjustment model and expectation model. The sixth section is the implication of the study and its comparative results. The last section concludes the paper.

II. THE TIME STRUCTURE OF INVESTMENT BEHAVIOR AND MODEL SETTING

Our study is based on a theory of investment behavior that has as its empirical counterpart a distributed lag in net investment and changes in desired capital services. We let I_t denote gross investment, K_t the capital stock, and K_t^e the desired level of capital services. If replacement is proportional to the capital stock, net investment may be denoted $I_t - \delta K_t$, where δ is the rate of replacement. Denoting output by q_t , the price of output by P_t , and the price of capital services by r_t , the long-run desired level of capital services K is generally assumed to be²

$$K_t^e = AP_t q_t^m r_t^n, \quad (1)$$

$$\text{or } k_t^e = Aq_t^m r_t^n, \quad (1a)$$

where $k_t^e = K_t^e / P_t$ is a real term of desired level of capital. In the short run, firms adjust towards K_t^e by assuming log-linear rules as follows³:

A. Static expectation model setting

$$\ln k_t^e = \ln k_{t-1}, \quad (2)$$

B. Adaptive expectation model setting

$$\ln k_t^e - \ln k_{t-1}^e = \lambda (\ln k_t - \ln k_{t-1}^e), \quad (3)$$

where λ is the speed of adjustment in the process of expectation, $0 < \lambda \leq 1$,

C. Extrapolative expectation model setting

$$\ln k_t^e = (1 - \lambda) \ln k_{t-1} + \lambda \ln k_{t-1}^e, \quad (4)$$

D. Partial adjustment expectation setting

$$\ln k_t - \ln k_{t-1} = \lambda (\ln k_t^e - \ln k_{t-1}). \quad (5)$$

By algebraic manipulation, different equations with various price variables in the estimated equations are specified below.

II.A. Real Adjustment Expectation Models

A. Static expectation-estimated equation form

$$\ln K_{t-1} = \ln A + m \ln q_t + n \ln r_t + \ln P_t + \ln P_{t-1}, \quad (6)$$

$$\ln k_{t-1} = \ln A + m \ln q_t + n \ln r_t + \ln P_t, \quad (6a)$$

where m and n are both the coefficient of the estimated regressions and may be the elasticity of the independent variable of q_t and r_t , respectively.

B. Adaptive Expectation Estimated Equation Form

$$\ln K_t = m_0 + m_1 \ln q_t + m_2 \ln q_{t-1} + m_3 \ln r_t + m_4 \ln r_{t-1} + m_5 \ln P_t, \quad (7)$$

$$\ln k_t = m_0 + m_1 \ln q_t + m_2 \ln q_{t-1} + m_3 \ln r_t + m_4 \ln r_{t-1}, \quad (7a)$$

where $m_0 = \ln A$;

$$m_1 = m / \lambda ;$$

$$\begin{aligned}
m_2 &= -(1-\lambda)/\lambda ; \\
m_3 &= n/\lambda ; \\
m_4 &= -(1-\lambda)n/\lambda ; \text{ and} \\
m_5 &= 1 .
\end{aligned}$$

C. Extrapolative Expectation - Estimated Equation Form

$$\ln K_{t-1} = t_0 + t_1 \ln q_t + t_2 \ln r_t + t_3 \ln K_{t-2} + t_e \pi_e, \quad (8)$$

$$\ln k_{t-1} = t_0 + t_1 \ln q_t + t_2 \ln r_t + t_3 \ln K_{t-2}, \quad (8a)$$

where $t_0 = [1/(1-\lambda)] \ln A$;
 $t_1 = m/(1-\lambda)$;
 $t_2 = n/(1-\lambda)$;
 $t_3 = -\lambda(1-\lambda)$; and
 $t_e \pi_e = \ln P_{t-1} - [\lambda(1-\lambda)] \ln P_{t-2}$.

D. Partial Adjustment Expectation - Estimated Equation Form

$$\ln K_t = k_0 + k_1 \ln q_t + k_2 \ln r_t + k_3 K_{t-1} + k_p \pi_p, \quad (9)$$

$$\ln k_t = k_0 + k_1 \ln q_t + k_2 \ln r_t + k_3 \ln k_{t-1}, \quad (9a)$$

where $k_0 = \lambda \ln A$;
 $k_1 = \lambda m$;
 $k_2 = \lambda n$;
 $k_3 = 1-\lambda$; and
 $k_p \pi_p = \ln(P_t / P_{t-1}) + \lambda \ln P_{t-1} = \ln P_t - (1-\lambda) \ln P_{t-1}$.

Note that the above various expectations have different estimated equation forms and that each form has a different time structure by virtue of having various lag periods. Moreover, the behavior of output price has also been derived from an expectations setting.

II.B. Nominal Adjustment Expectation Models

E. Static expectation - estimated equation form

$$\ln K_{t-1} = \ln A + \ln P_t + m \ln q_t + n \ln r_t, \quad (10)$$

$$\ln k_{t-1} = \ln A + m \ln q_t + n \ln r_t + \ln(P_t / P_{t-1}). \quad (10a)$$

F. Adaptive expectation - estimated equation form

$$\ln K_t = e_0 + e_1 \ln q_t + e_2 \ln q_{t-1} + e_3 \ln r_t + e_4 \ln r_{t-1} + e_A \pi_A, \quad (11)$$

$$\ln k_t = e_0 + e_1 \ln q_t + e_2 \ln q_{t-1} + e_3 \ln r_{t-1} + e_A \pi_A, \quad (11a)$$

where $e_0 = \ln A$;

$$e_1 = m / \lambda ;$$

$$e_2 = -m(1 - \lambda) / \lambda ;$$

$$e_3 = n / \lambda ;$$

$$e_4 = -n(1 - \lambda) / \lambda ; \text{ and}$$

$$e_A \pi_A = (1 / \lambda) \ln P_t - [(1 - \lambda) / \lambda] \ln P_{t-1} .$$

G. Extrapolative expectation - estimated equation form

$$\ln K_{t-1} = d_1 + d_2 \ln q_t + d_3 \ln r_t + d_4 \ln K_{t-2} + d_E \pi_E, \quad (12)$$

$$\ln k_{t-1} = d_1 + d_2 \ln q_t + d_3 \ln r_t + d_4 \ln k_{t-2} + d_E \pi_E, \quad (12a)$$

where $d_1 = [1 / (1 - \lambda)] \ln A$;

$$d_2 = m / (1 - \lambda) ;$$

$$d_3 = n / (1 - \lambda) ;$$

$$d_4 = -\lambda(1 - \lambda) ;$$

$$d_E \pi_E = [1 / (1 - \lambda)] \ln P_t ; \text{ and}$$

$$d_E \pi_E = [1 / (1 - \lambda)] \ln P_t + \ln P_{t-1} - [\lambda / (1 - \lambda)] \ln P_{t-2} .$$

H. Partial adjustment expectation - estimated equation form

$$\ln K_t = g_0 + g_1 \ln q_t + g_2 \ln r_t + g_3 \ln K_{t-1} + g_p \pi_p, \quad (13)$$

$$\ln k_t = g_0 + g_1 \ln q_t + g_2 \ln r_t + g_3 \ln k_{t-1} + g_{p'} \pi_{p'}, \quad (13a)$$

where $g_0 = \lambda \ln A$;

$$g_1 = \lambda m ;$$

$$g_2 = \lambda n ;$$

$$g_3 = 1 - \lambda ;$$

$$g_p \pi_p = \lambda \ln P_t ; \text{ and}$$

$$g_{p'} \pi_{p'} = \ln(P_t / P_{t-1}) + \lambda(\ln P_t + \ln P_{t-1}) = (1 + \lambda) \ln P_t - (1 - \lambda) \ln P_{t-1} .$$

III. AN ANALYSIS OF INVESTMENT BEHAVIOR AND OUTPUT PRICE

We now consider an example from partial adjustment expectation setting in equation (5) above to show the behavior of investment⁴. Assume that

$$\ln(K_t / P_t) - \ln(K_{t-1} / P_{t-1}) = \lambda [\ln(K_t^e / P_t) - \ln(K_{t-1} / P_{t-1})] \quad (14)$$

Taking logarithm of (1) and substituting into (14) yields

$$\ln(K_t / P_t) = \alpha_0 + \alpha_1 \ln q_t + \alpha_2 \ln r_t + \alpha_3 \ln(K_{t-1} / P_{t-1}), \quad (15)$$

where $\alpha_0 = \lambda \ln A$;

$$\alpha_1 = \lambda m ; \text{ and}$$

$$\alpha_2 = \lambda n .$$

Denoting real magnitudes with lower case letters, (15) can be expressed as

$$\ln k_t = \alpha_0 + \alpha_1 \ln q_t + \alpha_2 \ln r_t + \alpha_3 \ln k_{t-1} . \quad (16)$$

Here we see the time structure of investment behavior, with capital lagged one period but with output, q_t , and capital service, r_t , influencing k_t only contemporaneously.

Testing whether expected inflation also enters as an explanatory variable

is then accomplished by including expected (or actual) inflation in (1) and thus in (16), which yields an estimating equation

$$\ln k_t = \alpha_0 + \alpha_1 \ln q_t + \alpha_2 \ln r_t + \alpha_3 \ln k_{t-1} + \alpha_4 \ln \pi, \quad (17)$$

where π is the actual rate of inflation⁵ or $\pi = P_t / P_{t-1}$.

We next assume that firms adjust towards K_t^e by a nominal adjustment of the capital stock; i.e.,

$$\ln K_t - \ln K_{t-1} = \lambda (\ln K_t^e - \ln K_{t-1}), \quad (18)$$

which gives, upon substitution from (1)

$$\ln(K_t / P_t) = \beta_0 + \beta_1 \ln q_t + \beta_2 \ln r_t + \beta_3 \ln(K_{t-1} / P_t), \quad (19)$$

where $\beta_0 = \lambda \ln A$;

$$\beta_1 = \lambda m ;$$

$$\beta_2 = \lambda n ; \text{ and}$$

$$\beta_3 = 1 - \lambda .$$

The difference between (15) and (19) is found in the last term. Under the hypothesis that nominal terms adjust with a lag, the last period's capital stock should be deflated by this period's price. An alternative way of writing (19) is

$$\ln k_t = \alpha_0 + \alpha_1 \ln q_t + \alpha_2 \ln r_t + \alpha_3 \ln k_{t-1} + \alpha_4 \ln(P_t / P_{t-1}), \quad (20)$$

$$\text{where } \alpha_3 = -\alpha_4 . \quad (21)$$

Equations (20) and (16) differ by the term $\ln(P_t / P_{t-1})$, which is approximately the rate of inflation itself. Consequently, under a nominal adjustment mechanism, the rate of inflation will automatically enter the equation. It follows that equation (17) is evidence of misspecification of the model, with the role of the price variable, P_t , depending on whether nominal or real capital is being analyzed.

There is a simple test for misspecification. If a nominal adjustment outperforms a real adjustment, then equation (20) should be significantly better

than equation (16), and the implied restriction (21) should hold. This restriction is that the coefficients on $\ln k_{t-1}$ and $\ln(P_t / P_{t-1})$ are equal and opposite in sign.

Notice that equation (20) can be rewritten as

$$\ln k_t = \alpha_0 + \alpha_1 \ln q_t + \alpha_2 \ln r_t + \alpha_3 \ln(K_{t-1} / P_t) + (\alpha_3 + \alpha_4) \ln(P_t / P_{t-1}) \quad (22)$$

The test of whether (21) holds is thus a test that $\alpha_3 + \alpha_4 = 0$, i.e., it is the test that the coefficient of the price variable (in the rate of inflation form) in equation (22) is zero. Consequently, the test that the nominal specification is sustained by the data (i.e., (21) holds) is exactly the test that the price variable is not significantly different from zero in equation (22). In addition to the partial adjustment expectation model being implied by the restriction, other expectation models are also implied by the restriction.

From the above nominal and real adjustments, we conclude that great care must be taken in specifying models for which the price variable is relevant.

IV. GENERALIZED EXPECTATIONS ADJUSTMENT MODEL AND COMPARISON

To characterize the time pattern of response of investment to change in the conditions determining the desired level of capital services and output price, the distributed lag in desired capital and desired output may be written in various forms using Waud's General Model as follows⁶:

A. Real Adjustment / Expectations Models

A1. k_t is the Nerlove setting, while q_t is the Cagan setting

1. $\ln k_t^e = \ln A + m \ln q_t^e + n \ln r_t$,
2. $\ln k_t^e - \ln k_{t-1}^e = \lambda_1 (\ln k_t - \ln k_{t-1}^e)$, $0 < \lambda_1 \leq 1$,
3. $\ln q_t - \ln q_{t-1} = \lambda_2 (\ln q_t^e - \ln q_{t-1})$, $0 < \lambda_2 \leq 1$,

Solving for the expectation yields

$$\ln k_t = a_0 + a_1 \ln q_t + a_2 \ln q_{t-1} + a_3 \ln q_{t-2} + a_4 \ln r_t + a_5 \ln r_{t-1}, \quad (23)$$

where $a_0 = \ln A$;

$$a_1 = m / \lambda_1 \lambda_2 ;$$

$$\begin{aligned}
a_2 &= -m(\lambda_2 - \lambda_1) / \lambda_1 \lambda_2 = -a_1(\lambda_2 - \lambda_1) ; \\
a_3 &= -m(1 - \lambda_1)(1 - \lambda_2) / \lambda_1 \lambda_2 = -a_1(1 - \lambda_1)(1 - \lambda_2) ; \\
a_4 &= n / \lambda_1 ; \text{ and} \\
a_5 &= -n(1 - \lambda_1) / \lambda_1 = -a_4(1 - \lambda_1) .
\end{aligned}$$

If the speed of adjustment is the same, $\lambda_1 = \lambda_2 = \lambda$, then the equation may be rewritten as

$$\ln k_t = b_0 + b_1 \ln q_t + b_2 \ln q_{t-1} + b_3 \ln q_{t-2} + b_4 \ln r_t + b_5 \ln r_{t-1} , (24)$$

where $b_0 = \ln A = a_0$;

$$b_1 = m / \lambda^2 ;$$

$$b_2 = 0 ;$$

$$b_3 = (m / \lambda^2)(1 - \lambda)^2 ;$$

$$b_4 = n / \lambda ; \text{ and}$$

$$b_5 = -n(n / \lambda)(1 - \lambda) = -b_3(1 - \lambda) .$$

Note that when the speed of adjustment is the same, the explanatory variable q_t of the equation no longer appears (i.e. $b_1 = 0$). Furthermore, output price plays no role in the determination of k_t .

A2. k_t is the Cagan setting, while q_t is the Nerlove setting

$$1. \ln k_t^e = \ln A + m \ln q_t^e + n \ln r_t ,$$

$$2. \ln k_t - \ln k_{t-1} = \lambda_1 (\ln k_t^e - \ln k_{t-1}) , \quad 0 < \lambda_1 \leq 1 ,$$

$$3. \ln q_t^e - \ln q_{t-1}^e = \lambda_2 (\ln q_t - \ln q_{t-1}^e) , \quad 0 < \lambda_2 \leq 1 ,$$

Solving for the expectation, we obtain the following equation:

$$\ln k_t = c_0 + c_1 \ln q_t + c_2 \ln r_t + c_3 \ln r_{t-1} + c_4 \ln k_{t-1} + c_5 \ln k_{t-2} , \quad (25)$$

where $c_0 = \lambda_1 \lambda_2 \ln A$;

$$c_1 = -\lambda_1 \lambda_2 m ;$$

$$c_2 = \lambda_1 n ;$$

$$\begin{aligned} c_3 &= -\lambda_1 n(1 - \lambda_2) = -c_2(1 - \lambda_2) ; \\ c_4 &= (1 - \lambda_1) + (1 - \lambda_2) ; \text{ and} \\ c_5 &= -(1 - \lambda_1)(1 - \lambda_2) . \end{aligned}$$

If the speed of adjustment is the same, $\lambda_1 = \lambda_2 = \lambda$, then

$$\ln k_t = d_0 + d_1 \ln q_t + d_2 \ln r_t + d_3 \ln r_{t-1} + d_4 \ln k_{t-1} + d_5 \ln k_{t-2} , (26)$$

where $d_0 = \lambda^2 \ln A$;

$$d_1 = -\lambda^2 m ;$$

$$d_2 = \lambda n ;$$

$$d_3 = -d_2(1 - \lambda) = -\lambda(1 - \lambda)n ;$$

$$d_4 = (1 - \lambda) + (1 - \lambda) = 2(1 - \lambda) ; \text{ and}$$

$$d_5 = -(1 - \lambda)^2 .$$

Comparing case A1 to case A2, where both are real adjustment models, the time pattern is seen to be different. We get a time lag in capital stock for case A2 but not for case A1, and the time lag variable q_{t-1} in case A2 still holds whether or not the speed of adjustment is the same.

B. Nominal adjustment / Expectations models

One point to note is the somewhat asymmetric assumption regarding the adjustment of capital and output, as represented by equations in case A1 and case A2, which have firms adjusting their capital and output instantly with respect to price, but slowly with respect to capital or output. However, since it is nominal terms which are always adjusted, clearly a nominal adjustment mechanism makes more sense.

B1. K_t is the Nerlove setting, while Q_t is the Cagan setting⁷

$$1. \ln K_t^e = \ln A + \ln P_t + m \ln q_t^e + n \ln r_t ,$$

$$2. \ln K_t^e - \ln K_{t-1}^e = \delta_1 (\ln K_t - \ln K_{t-1}^e) , \quad 0 < \delta_1 \leq 1 ,$$

$$3. \ln Q_t - \ln Q_{t-1} = \delta_2 (\ln Q_t^e - \ln Q_{t-1}) , \quad 0 < \delta_2 \leq 1 .$$

Solving for the expectation, we get the following equation:

$$\ln K_t = e_0 + e_1 \ln Q_t + e_2 \ln Q_{t-1} + e_3 \ln Q_{t-2} + e_4 \ln r_t + e_5 \ln P_t, \quad (27)$$

where $e_0 = \ln A$;

$$e_1 = m / \delta_1 \delta_2 ;$$

$$e_2 = (m / \delta_2) [(1 - \lambda_1) + (1 + \lambda_2)] ;$$

$$e_3 = (m / \delta_2) (1 - \delta_1) (1 - \delta_2) ;$$

$$e_4 = n ; \text{ and}$$

$$e_5 = 1 - m .$$

If $\delta_1 = \delta_2 = \delta$, then

$$\ln K_t = d_0 + d_1 \ln Q_t + d_2 \ln Q_{t-1} + d_3 \ln Q_{t-2} + d_4 \ln r_t + d_5 \ln P_t, \quad (28)$$

where $d_0 = \ln A$;

$$d_1 = m / \delta^2 ;$$

$$d_2 = -2m(1 - \delta) / \delta ;$$

$$d_3 = m(1 - \delta)^2 / \delta = -(1 - \delta)d_2 / 2 ;$$

$$d_4 = n ; \text{ and}$$

$$d_5 = 1 - m .$$

Note that for the time pattern of nominal terms adjustment, we have a two-period lag in the explanatory variable output Q_t and the sign alternates. However, output price is not affected by the speed of adjustment (δ) , with the elasticity of output (m) being the unique factor. If the elasticity of output is equal to one, then output price plays no role in the nominal adjustment model and seems to accord with a rational expectation model.

Considering now the model in real terms, the above equation may be rewritten as

$$\ln k_t = f_0 + f_1 \ln q_t + f_2 \ln q_{t-1} + f_3 \ln q_{t-2} + f_4 \ln r_t + f_5 \pi_t, \quad (29)$$

where $f_0 = d_0$;

$$\begin{aligned} f_1 &= d_1 ; \\ f_2 &= d_2 ; \\ f_3 &= d_3 ; \\ f_4 &= d_4 ; \text{and} \end{aligned}$$

$$f_5 \pi_t = [2 - m - (m/\delta)] \ln P_t - [2m(1-\delta)/\delta] \ln P_{t-1} - [m(1-\delta)^2/\delta] \ln P_{t-2} .$$

Note that price will now have two lags and is combined with the elasticity of output (m) and the speed of adjustment (δ). It is obvious that when empirically testing investment behavior or the level of desired capital, we shouldn't treat output price arbitrarily. Rather, price must obey the given models.

B2. K_t is the Cagan setting, while Q_t is the Nerlove setting⁸

$$\begin{aligned} 1. \ln K_t^e &= \ln A + \ln P_t + m \ln q_t^e + n \ln r_t , \\ 2. \ln K_t - \ln K_{t-1} &= \delta_1 (\ln K_t^e - \ln K_{t-1}) , \quad 0 < \delta_1 \leq 1 , \\ 3. \ln Q_t^e - \ln Q_{t-1}^e &= \delta_2 (\ln Q_t - \ln Q_{t-1}) , \quad 0 < \delta_2 \leq 1 . \end{aligned}$$

Solving for the expectation, we get the following equation:

$$\ln K_t = g_0 + g_1 \ln Q_t + g_2 \ln r_t + g_3 \ln r_{t-1} + g_4 \ln K_{t-1} + g_5 \ln K_{t-2} + g_6 \pi_t , (30)$$

where $g_0 = \delta_1 \delta_2 \ln A$;

$$g_1 = \delta_1 \delta_2 m ;$$

$$g_2 = n \delta_1 ;$$

$$g_3 = -n \delta_1 (1 - \delta_2) ;$$

$$g_4 = (1 - \delta_1) + (1 - \delta_2) ;$$

$$g_5 = -(1 - \delta_1)(1 - \delta_2) ; \text{and}$$

$$g_6 \pi_t = \delta_1 (1 - m) [\ln P_t - (1 - \delta_2) \ln P_{t-1}] .$$

If $\delta_1 = \delta_2 = \delta$, then the equation can be rewritten as

$$\ln K_t = h_0 + h_1 \ln Q_t + h_2 \ln r_t + h_3 \ln r_{t-1} + h_4 \ln K_{t-1} + h_5 \ln K_{t-2} + h_6 \pi_t , (31)$$

where $h_0 = \delta^2 \ln A$;

$$\begin{aligned}
h_1 &= \delta^2 m ; \\
h_2 &= n\delta ; \\
h_3 &= -n\delta(1-\delta) ; \\
h_4 &= 2(1-\delta) ; \\
h_5 &= -(1-\delta)^2 ; \text{ and} \\
h_6\pi_t &= \delta(1-m)[\ln P_t - (1-\delta)\ln P_{t-1}] .
\end{aligned}$$

Note that output price is also combined with the speed of adjustment (δ) and the elasticity of output, and the treatment is not arbitrary. It is obvious that the time structure or time pattern will specify different time lags. In the designed equation, capital has a lag of two periods, while output is not lagged.

Comparing case A, the real adjustment model, to case B, the nominal adjustment model, the form of the lag between various expectations is quite different. We can see that output price enters equations with different time lags and structured forms. The time structure of capital and output, and even the other variable have shown different form and lag structures.

V. REAL VERSUS NOMINAL MODELS AND THE RATE OF INFLATION

In the above treatment, we assumed that firms adjust their capital stock instantly with respect to price. This led to some interesting results, including various time patterns and investment behaviors. Now we will relax the assumption, by letting firms adjust slowly with respect to price. We also use the Waud General Model, which combines both Nerlove and Cagan settings, to test the role of price in the different adjustment for real and nominal terms.

A. Real Adjustment for Capital

1. $\ln K_t^e = \ln A + \ln P_t^e + m \ln q_t + n \ln r_t$,
2. $\ln k_t - \ln k_{t-1} = \delta_1 (\ln k_t^e - \ln k_{t-1})$, $0 < \delta_1 \leq 1$,
3. $\ln P_t^e - \ln P_{t-1}^e = \delta_2 (\ln P_t - \ln P_{t-1})$, $0 < \delta_2 \leq 1$.

Solving for the expectation, we get the following equation:

$$\ln k_t = i_0 + i_1 \ln q_t + i_2 \ln r_t + i_3 \ln k_{t-1} + i_4 \ln k_{t-2} + i_5 \ln q_{t-1} + i_6 \ln r_{t-1} + i_7 \pi_t, \quad (32)$$

where $i_0 = \delta_1 \delta_2 \ln A$;
 $i_1 = \delta_1 m$;
 $i_2 = \delta_1 n$;
 $i_3 = (1 - \delta_1) + (1 - \delta_2)$;
 $i_4 = -(1 - \delta_1)(1 - \delta_2)$;
 $i_5 = -\delta_1(1 - \delta_2)m$;
 $i_6 = -\delta_1(1 - \delta_2)n$; and
 $i_7 \pi_r = \delta_1(\delta_2 - 1) \ln(P_t / P_{t-1})$.

If $\delta_1 = \delta_2 = \delta$, then the equation becomes

$$\ln k_t = j_0 + j_1 \ln q_t + j_2 \ln r_t + j_3 \ln k_{t-1} + j_4 \ln k_{t-2} + j_5 \ln q_{t-1} + j_6 \ln r_{t-1} + j_7 \pi_t , \quad (33)$$

where $j_0 = \delta^2 \ln A$;
 $j_1 = \delta m$;
 $j_2 = \delta n$;
 $j_3 = 2(1 - \delta)$;
 $j_4 = -(1 - \delta)^2$;
 $j_5 = -\delta(1 - \delta)m$;
 $j_6 = -\delta(1 - \delta)n$; and
 $j_7 \pi_t = \delta(\delta - 1) \ln(P_t / P_{t-1})$.

For this equation, the real terms and price are not instantly adjusted. We can see that the square of speed of adjustment is equal to the coefficient of the explanatory variable P_t . Furthermore, the capital stock, k_t , shows a two-period lag in the equation.

B. Nominal Adjustment for the Capital

1. $\ln K_t^e = \ln A + \ln P_t^e + m \ln q_t + n \ln r_t$,
2. $\ln K_t - \ln K_{t-1} = \delta_1 (\ln K_t^e - \ln K_{t-1})$, $0 < \delta_1 \leq 1$,

$$3. \ln P_t^e - \ln P_{t-1}^e = \delta_2 (\ln P_t - \ln P_{t-1}^e) \quad , \quad 0 < \delta_2 \leq 1 .$$

Solving for the expectation, we get the following equation:

$$\ln K_t = t_0 + t_1 \ln q_t + t_2 \ln q_{t-1} + t_3 \ln r_t + t_4 \ln r_{t-1} + t_5 \ln K_{t-1} + t_6 \ln K_{t-2} + t_7 \ln P_t \quad , \quad (34)$$

where $t_0 = \delta_1 \delta_2 \ln A$;

$$t_1 = \delta_1 m \quad ;$$

$$t_2 = -\delta_1 (1 - \delta_2) m \quad ;$$

$$t_3 = \delta_1 n \quad ;$$

$$t_4 = -\delta_1 (1 - \delta_2) n \quad ;$$

$$t_5 = (1 - \delta_1) + (1 - \delta_2) \quad ;$$

$$t_6 = -(1 - \delta_2)(1 - \delta_1) \quad ; \text{ and}$$

$$t_7 = \delta_1 \delta_2 \quad .$$

If $\delta_1 = \delta_2 = \delta$, the same speed of adjustment for K_t and P_t , then

$$\ln K_t = u_0 + u_1 \ln q_t + u_2 \ln q_{t-1} + u_3 \ln r_t + u_4 \ln r_{t-1} + u_5 \ln K_{t-1} + u_6 \ln K_{t-2} + u_7 \ln P_t \quad , \quad (35)$$

where $u_0 = \delta^2 \ln A$;

$$u_1 = \delta m \quad ;$$

$$u_2 = -\delta(1 - \delta) m \quad ;$$

$$u_3 = \delta n \quad ;$$

$$u_4 = -\delta(1 - \delta) n \quad ;$$

$$u_5 = 2(1 - \delta) \quad ;$$

$$u_6 = -(1 - \delta)^2 \quad ; \text{ and}$$

$$u_7 = \delta^2 \quad .$$

In addition, we may take a real terms representation, allowing the above equation to be written as

$$\ln k_t = \eta_0 + \eta_1 \ln q_t + \eta_2 \ln q_{t-1} + \eta_3 \ln r_t + \eta_4 \ln r_{t-1} + \eta_5 \ln k_{t-1} + \eta_6 \ln k_{t-2} + \eta_7 \pi_t \quad , \quad (36)$$

where $\eta_0 = u_0$;

$$\eta_1 = u_1 ;$$

$$\eta_2 = u_2 ;$$

$$\eta_3 = u_3 ;$$

$$\eta_4 = u_4 ;$$

$$\eta_5 = u_5 ;$$

$$\eta_6 = u_6 ; \text{ and}$$

$$\eta_7 \pi_t = (\delta^2 - 1) \ln P_t + 2(1 - \delta) \ln P_{t-1} - (1 - \delta)^2 \ln P_{t-2} .$$

Note that the coefficient of explanatory variable relative to the speed of adjustment is elasticity of itself. Again, we see that only the price variable is not lagged. Other variables, like output and capital cost, should be lagged one period in the estimated equation, while the capital stock variable appears with a two-period lag.

Comparing case A, real adjustment, to case B, nominal adjustment, the role of price is determined by the squared speed of adjustment restriction. This is quite different from the above generalized expectation model, where price should combine with various time structures.

Checking both nominal and real representations of the estimated regression, the role of the price is different in its time pattern. As the real terms shows the price combination may be included in the current and lagged periods since the capital stock variable has the same dynamics.

VI. IMPLICATIONS OF THE STUDY

It is generally argued through estimated regressions that the time structure of price, output, and capital cost, as well as capital services, plays important roles in investment behavior or the demand for capital. According to various models derived, we have different estimated equations including real and nominal adjustments and expectations which are static, adaptative, or extrapolative presented here. Especially we have introduced the Waud General Model, which combines both Nerlove and Cagan models allowing an analysis of the price pattern.

The time pattern between investment behavior and its underlying determinants is found to depend on desired quantities and expectations. The form of time lag is different in each estimated equation in Table 1. To characterize the lag structure underlying investment behavior for the price situation, including time pattern and its linear combination, we have derived the

lag formation of output price expectations in Table 2. Table 2 shows various comparisons of the price lag structure, as well as various combinations.

Owing to different expectations models, we get a more appropriate specification for the estimated regressions. But, in testing the estimated regressions, one may encounter autocorrelated errors, thus biasing the coefficient estimates.

Table 1

The Numbers (Periods) of Lags in Estimated Regression Equation

Model	Variable		
	k	q	r
Static Expectation	1	0	0
Adaptive Expectation	0	1	1
Extrapolative Expectation	2	0	0
Partial Adjustment Expectation	1	0	0
Generalized Expectation Adjustment			
A. k_t / N & q_t / C	0	2	1
B. k_t / C & q_t / N	2	0	1
C. K_t / N & Q_t / C	0	2	0
D. K_t / C & Q_t / N	2	0	1

Note:(1) N is the Nerlove setting, and C is the Cagan setting.

(2) Figures: 0 is the current time, i.e., no lags; 1 is a lag of one period, i.e., K_{t-1} , Q_{t-1} , etc.; 2 is a lag of two period, i.e., K_{t-2} , Q_{t-2} , etc.

Table 2
The Lag Formation of Output Price Expectations

Model	Real Adjustment	Nominal Adjustment
Static Expectation		
K_t form	$\pi_K^R = \ln P_t + \ln P_{t-1}$	$\pi_K^N = \ln P_t$
k_t form	$\pi_k^R = \ln P_t$	$\pi_k^N = \ln P_t - \ln P_{t-1} = \ln(P_t / P_{t-1})$
Adaptive Expectation		
K_t form	$\pi_K^R = \ln P_t$	$\pi_K^N = \frac{1}{\lambda} \ln P_t - \frac{1-\lambda}{\lambda} \ln P_{t-1}$
k_t form	$\pi_k^R = 0$	$\pi_k^N = \frac{1-\lambda}{\lambda} \ln(P_t / P_{t-1})$
Extrapolative Expectation		
K_t form	$\pi_K^R = \ln P_{t-1} - \frac{\lambda}{1-\lambda} \ln P_{t-2}$	$\pi_K^N = \frac{1}{1-\lambda} \ln P_t$
k_t form	$\pi_k^R = 0$	$\pi_k^N = \frac{1}{1-\lambda} \ln P_t + \ln P_{t-1} - \frac{\lambda}{1-\lambda} \ln P_{t-2}$
Partial Expectation		
K_t form	$\pi_K^R = \ln P_t - (1-\lambda) \ln P_{t-1}$ $= \ln(P_t / P_{t-1}) + \lambda \ln P_{t-1}$	$\pi_K^N = \lambda \ln P_t$
k_t form	$\pi_k^R = 0$	$\pi_k^N = (1+\lambda) \ln P_t - (1-\lambda) \ln P_{t-1}$ $= \ln(P_t / P_{t-1}) + \lambda(\ln P_t + \ln P_{t-1})$

Note: π_K^R denotes nominal K with nominal adjustment and π_k^r real k with real adjustment. So, π_K^r is nominal K with real adjustment, and the π_K^N is the nominal K with nominal adjustment.

VII. CONCLUSION

It is generally argued that an arbitrary lag distribution may be approximated to any desired degree of accuracy by any kinds of expectations. In this paper, we explore the adjustment of expectations of output price and capital as relates to the firm's investment decision. We consider the firm's problem, assuming various dynamic expectation models in the face of the adjustment of capital

accumulation and output price.

This paper reconciles previous works by showing that with a more appropriate specification, the expected or actual rate of inflation is redundant and should not be significant. Certainly, from theoretical considerations in real stock adjustment models with dynamic expectations, there is little role for the expected rate of inflation. Herein lies the paradox.

We allow for the adjustment expectations not only in different model settings, including adaptative, extrapolative, and partial adjustment expectations, but also in its general model settings in the real and nominal adjustment's factor of the investment demand functions. We get a more appropriate specification for the estimated regressions.

NOTES

1. See Jorgenson and Stephenson [21].
2. The equation is a demand for capital and is close to the accelerator theory of capital, i.e., $K_t = a(P_t Q_t / C_t)$. See note 1.
3. The various expectation settings can see Cagan (1956) and Nerlove (1958) for adaptive expectation and partial adjustment model respectively, and others can see Intriligator (1978) [13].
4. This Section's idea is derived from Milbourne [39].
5. This definition, P_t / P_{t-1} , is not inflation but an intertemporal relative price. In fact, $\pi = (P_t / P_{t-1}) / P_{t-1}$ is the true definition of inflation. If we take the logarithm, then $\ln(1 + \pi)$ approximates to π .
6. For the asymptotic properties of distributed lag models and models settings, see Theil H. [52].
7. Herein, we use the nominal terms, K_t and Q_t , to compare with the real terms, k_t and q_t , but it is the same model setting from case A.
8. Herein we also use the nominal terms, K_t and Q_t , to compare with the real terms, k_t and q_t , but it is the same model setting from case B. See footnote 7.

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