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An Exponential Extrapolation Approach for the Valuation and Hedging of American Options

Chuang-Chang Chang
Department of Finance, National Central University, Chung-Li,Taiwan

ABSTRACT

In this paper, we propose an extrapolation approach for the valuation and hedging of American options. The approach developed in this paper is very simple and computationally efficient. In particular, it can be used to value the long-term American options and American exotic options, such as lookback options and Asian options, to a high degree of accuracy.

JEL classification: G13

Keywords: Exponential extrapolation approaches; Long-term American options; American exotic options

I. INTRODUCTION

In an important contribution, Geske and Johnson [11] showed that it was possible to value an American option by using a series of compound options. They demonstrated that the estimated values using the compound option approach with Richardson extrapolation were very close to the values obtained using other numerical techniques. The Geske and Johnson [11] approach is of interest since it allows evaluation of options for which the possible exercise dates are relatively large or the values of the options are affected by more than one variable.

There have been numerous extensions of the Geske and Johnson [11] approach. These extensions can be divided into three broad categories. The first extension is the one first proposed by Kim [17] and extended by Jacka [16], Car, Jarrow and Myneni [7], Kim and Yu [18] and Huang, Yu, and Subrahmanyam [14]. In a sense, this is the continuous-time extension of the Geske and Johnson [11] approach. The second extension is the one proposed by Breen [3]. His approach is a hybrid of the Cox, Ross, and Rubinstein [9] binomial method (henceforth referred to as CRR) and the Geske and Johnson [11] approach.

The third extension is the one proposed by Bunch and Johnson [5] and Ho, Stapleton and Subrahmanyam [13] (henceforth referred to as HSS). Bunch and Johnson [5] modified the Geske and Johnson [11] approach to approximate the American option values, using only the value of a European option (P_1) permitting exercise at maturity date, T, and the value of a twice exerciseable option (P_2) permitting exercise at time $T/2$ and T. However, they used the maximum value of P_1 and P_2 rather than the simple P_1 and P_2 of Geske and Johnson [11]. Bunch and Johnson [5] showed that their approach worked well for the non-deep in-the-money options. Moreover, they showed that their approach was much more computationally efficient than the Geske and Johnson [11] method. Following similar lines, HSS [13] derived an exponential formula that was based on an observed relationship between the American option value and the number of exercise points allowed up to the maturity date. Although their method made significant improvements on the valuation of futures options and currency options with low carrying cost, it incurred large pricing biases for deep in-the-money American put options with short-term maturities and for long-term American options.[2]

In this paper, we propose an exponential extrapolation approach that not only is very simple and computationally efficient but also significantly reduces the large pricing biases incurred by the Bunch and Johnson [5] approach and the HSS [13] method for the deep in-the-money American put options with short-term maturities and for long-term American options.[3] Additionally, we show that our exponential extrapolation approach can compute the values of American exotic options to a high degree of accuracy.

The rest of this paper is organized as follows. In Section 2, we review the Bunch and Johnson [5] approach and the HSS [13] method. In Section 3, we propose an exponential extrapolation approach for the valuation and hedging of American options. In Section 4, we compare the American put and call values estimated by our exponential extrapolation approach with those estimated by the Bunch and Johnson [5] approach and the HSS [13] method. We also demonstrate that our exponential extrapolation approach can compute the values of American exotic options and the option hedging parameters accurately. In Section 5, we state our conclusions.

II. LITERATURE REVIEW

In this section we briefly review the Bunch and Johnson [5] approach and the HSS [13] method.

A. The Bunch and Johnson [5] Approach

Bunch and Johnson [5] argued which there is nothing in the Geske and Johnson [11] approach that requires the exercise points to be equally spaced. Therefore, they proposed a model that has the optimal exercise points, i.e., they chose the exercise points as a wealth maximizer would. In the case of out-of-the-money or at-the-money put options, if a wealth maximizer is allowed to buy a put that would be exercised at only one point in time and he could choose the allocation of the point at the time of purchase, in virtually all realistic cases, he would place the exercise at maturity date T, at which the value of the put is denoted as $P_1(T)$. Suppose that the wealth maximizer has the opportunity to choose two exercise points, t_1 and t_2, with a view to maximize $P_2(t_1,t_2)$. The second point, t_2, would again be frequently placed at maturity date T. The intermediate date, t_1, can be located by performing a maximization of $P(t_1, T)$ over the time interval $(0, T)$ to find the intermediate date, t^* , $0 \leq t^* < T$. Hence, the point yields the highest value $P_2(t^*, T)$. One example of the one-dimensional maximization method suggested by Bunch and Johnson [5] to allocate t^* is to examine seven trial values for t^* spaced at $T/8, 2T/8, 3T/8, 4T/8, 5T/8, 6T/8, 7T/8$. Then

$$P_2^{\max} = \max[P_2(T/8, T), P_2(2T/8, T), .., P_2(7T/8, T)], \tag{1}$$

Employing the two-point Geske and Johnson [11] formula (see Appendix) to estimate the value of American put option using $P_1(T)$ and P_2^{max} , we have

$$\hat{P} = P_2^{\max} - P_1(T), \tag{2}$$

where $\hat{P}$ is the estimated American put option value.

As far as a (deep) in-the-money put option is concerned, Bunch and Johnson [5] claimed that there is a very high probability for a wealth-maximizing put buyer to place the exercise point for P_1 at time 0 (immediate exercise), especially when the European option value is less than the immediate exercise option value. Similarly, optimal assignment for P_2 will also lead to immediate exercise. Therefore, the estimated American put value would be equal to K-S, where K is the exercise price and S is the current stock price. This is why the Bunch and Johnson [5] approach sometimes incurs large pricing biases for the deep in-the-money American put options. From the simulation results of Bunch and Johnson [5] study, their approach worked well for the out-of-the-money and at-the-money American put options. However, the Bunch and Johnson [5] approach sometimes performed poorly for the deep in-the-money American put options.

B. The HSS [13] Exponential Approach

HSS [13] plotted the values of the multiple exerciseable options and the corresponding exercise points. They found that there is an exponential relationship between the n-times-exerciseable option value, P_n , and the number of exercise points, n, as shown in Figure 1. Therefore, they suggested an exponential formula to describe the relationship between the American option value, *P*, and the number of exercise points as follows:

$$P = P_n \exp(\frac{\alpha}{n}), \tag{3}$$

where α is a constant.

The American option value, *P*, in equation (3) can be estimated given any two observations of P_n. To use only P_1 and P_2 to estimate *P*, we let n equal 1 and n equal 2 in equation (3) and obtain

$$P_1 \exp(\alpha) = P_2 \exp(\frac{\alpha}{2}), \tag{4}$$

and

$$\alpha = 2\ln(\frac{P_2}{P_1}). \tag{5}$$

Therefore, the estimated value of *P* given P_1 and P_2 is obtained by using equation (3) with n=1, and then substituting α from equation (5),

$$\hat{P} = P_1 (\frac{P_2}{P_1})^2, \tag{6}$$

Equation (6) relates the estimated American option value, $\hat{P}$, to the European option value, P_1, and the twice exerciseable option value, P_2. Furthermore, from equation (6), the estimated American option value only depends on the magnitudes of the European option value, the ratio of P_2 to P_1 , and the grown rate, 2.

Figure 1

European option values and asympotic values of American put options

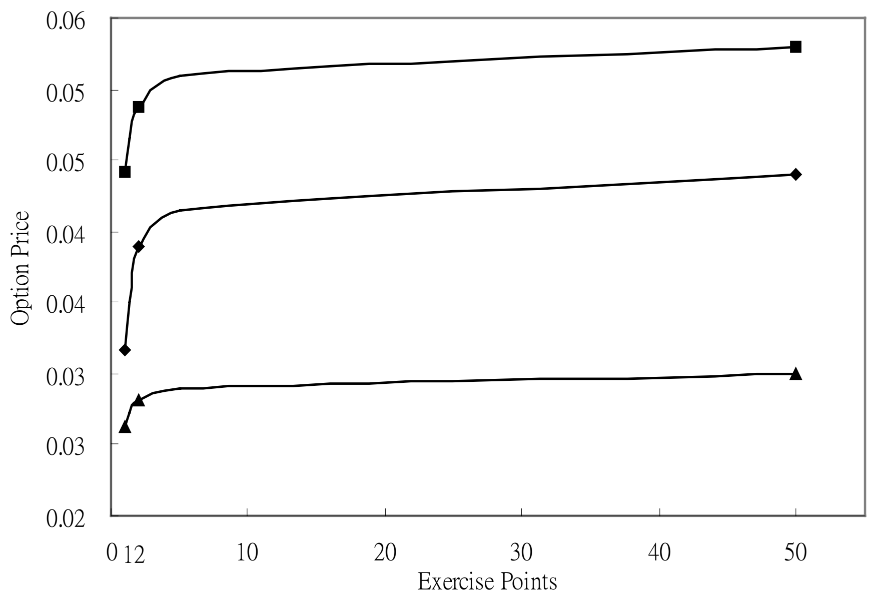

Figure 1 plots once, twice, and n-times exercisable option values P_1, P_2, and P_n, where n=50, for three American puts. The P_1 values are the Black and Scholes European option prices, and the P_{50} values are the Parkinson .put prices determined by numerical methods, as reported in Geske and Johnson (1984) Table 1. All three options have stock price, S=40, time to maturity, T=1, and the exercise price, K=40. The top curve (■) pots the options with a riskless rate, r=0.08, and an asset volatility, σ =0.2. The middle curve (♦) plots the options with a riskless rate, r=0.12, and an asset volatility, σ =0.2. The bottom curve (▲) plots the options with a riskless rate, r=0.03, and an asset volatility, σ =0.1.

To prove the n-times-exerciseable option value with an asymptotic value $P_1 exp(\alpha)$, under the assumption of equation (3), HSS [13] let n equal 1 and n equal n in equation (3), which gives

$$P_n = P_1(\frac{\alpha(n-1)}{n}). \tag{7}$$

Rearranging equation (7), we obtain

$$P_n = P_1 \exp(\alpha) \exp\left(\frac{-\alpha}{n}\right). \tag{8}$$

From equation (8), we observe that P_n is a positive, increasing function of n and has an asymptotic value

$$P = \lim_{n \to \infty} P_n = P_1 \exp(\alpha). \tag{9}$$

HSS [13] used equation (6) to estimate the American option value. From their simulation results, their method worked well for the out-of-the-money and at-the-money American options, but performed poorly for deep in-the-money American options. Additionally, as reported in HSS [13], their method also has limitations in estimating American options with long-term maturities.

III. AN EXPONENTIAL EXTRAPOLATION APPROACH

From Table 1 of the HSS [13] study, we observe that the difference between the option values of P_1 and P_2 is very small for the out-of-the-money and at-the-money options. Therefore, the estimated American option values are not very sensitive to the change of the grown rate, which is 2 in the HSS [13] method. However, the difference between the option values of P_1 and P_2 is large for the in-the-money options. Thus, the estimated deep in-the-money American option values computed by equation (6) are very sensitive to the change of the growing rate. From the simulation results of HSS [13] study, equation (6), in general, over-estimates the in-the-money American options. To reduce the large pricing biases computed by equation (6) for valuing the in-the-money American options, we try to find an exponential formula that has a lower grown rate than that in equation (6) but does not significantly reduce the values of the out-of-the-money and at-the-money American options. Moreover, the new formula must also preserve the properties of the HSS [13] method.

Based on the above argument, we propose an exponential extrapolation approach that is not only able to preserve the properties of the HSS [13] exponential formula but also can reduce the pricing biases computed by the Bunch and Johnson [5] approach and the HSS [13] method for the deep in-the-money American options. We assume that the American option value and the exercise points have the following exponential relationship[4]

$$P = P_n \exp\left(\frac{\alpha(3(n-1)+1)}{2n^2(n-1)+1}\right), \tag{10}$$

where α is a constant. Setting n = 1 and n = 2, we have

$$P_1 \exp(\alpha) = P_2 \exp\left(\frac{4\alpha}{9}\right), \tag{11}$$

and

$$\alpha = \frac{9}{5}\ln\left(\frac{P_2}{P_1}\right). \tag{12}$$

Therefore the estimated value of P given P_1 and P_2 can be obtained by using equation (10) with n = 1 and then substituting α from equation (12),

$$\hat{P} = P_1\left(\frac{P_2}{P_1}\right)^{\frac{9}{5}}, \tag{13}$$

Equation (13) is similar to equation (6) except for the different grown rate. The grown rate in the HSS [13] method is 2 whereas the grown rate is 9/5 in our exponential extrapolation approach.

To show the n-times exerciseable option values computed by our exponential extrapolation approach also with an asymptotic value, $P_1 exp(\alpha)$, we let n equal 1 and n equal n in equation (10) to obtain

$$P_n = P_1\left(\frac{\alpha\left(2n^2(n-1)-3(n-1)\right)}{2n^2(n-1)+1}\right). \tag{14}$$

Equation (14) can be rewritten as:

$$P_n = P_1 \exp(\alpha) \exp\left(\frac{-\alpha(3(n-1)+1)}{2n^2(n-1)+1} \right). \tag{15}$$

From equation (15), we observe that P_n is a positive, increasing function of n and has an asymptotic value

$$P = \lim_{n \to \infty} P_n = P_1 \exp(\alpha). \tag{16}$$

Equation (16) shows that our exponential extrapolation approach can preserve the properties of the HSS [13] exponential formula.

Our exponential extrapolation approach has several advantages. Firstly, our approach only needs to compute simple P_1 and P_2 whereas the Bunch and Johnson [5] approach has to compute the maximum value of P_1 and P_2. Secondly, since our approach only depends on the number of exercise points, it can be applied to any pattern of time steps. The pattern of the time steps, however, must be equally spaced when Richardson extrapolation method is used to derive the two-point or three-point Geske and Johnson [11] formula. Nevertheless, in general, the time steps are not equally spaced when the Bunch and Johnson [5] formula is used. Therefore extra estimation error occurs when the Bunch and Johnson [5] used the two-point Geske and Johnson [11] formula based on equal time space to estimate the American option values. Thirdly, since we only use P_1 and P_2 , our approach does not encounter the non-uniform convergence problem pointed out by Omberg [20]. Finally, our approach is computationally more efficient than the Bunch and Johnson [5] approach.[5]

VI. NUMERICAL RESULTS

In this section, we compare the American put and call values estimated by our exponential extrapolation approach with those estimated by the Bunch and Johnson method [5] and the HSS [13] method. We also demonstrate that our approach can compute the values of American exotic options and the option hedging parameters accurately.

A. Estimating the Values and Hedging Ratios of the Short-Term Maturity American Put Options

For comparison purpose, we carry out simulations using the data of Tables 1 to 3 reported in the HSS [13] study. Further, for compactness, we drop the option values computed by the Geake and Johnson [11] approach, the finite-difference method and the Barone-Whaley [1] quadratic-approximation method reported in the HSS [13] study. From Table 1, we find that our exponential extrapolation approach can compute all the option values to a high degree of accuracy. The estimated errors range from one cent to three cents. Unlike the Bunch and Johnson [5] approach and the HSS [13] method, our approach estimates the deep in-the-money American put options very accurately. These results show that our approach can reduce the large pricing biases computed by the Bunch and Johnson [5] approach and the HSS [13] method for the deep in-the-money American put options with short-term maturities.

From Table 2, we find that our approach can also compute the delta of the American put options accurately. In no case is the hedge ratio off by more than 0.015. The difference is even less significant for the total portfolio of options. Our approach can compute the delta values for the deep in-the-money put options with short-term maturities more accurately than the HSS [13] method. Additionally, the results of Table 3 are similar to those of Table 1.

As argued by HSS [13], the case of the computations using the HSS [13] method and our approach is readily apparent in these calculations. The usual methods of computing the values are laborious, since they generally involve perturbing the current parameter inputs. Both the HSS [13] method and our approach substantially economize the computational effort.

Table 1
Comparison of short-term American put option values using alternative approaches: zero-dividend case (S=40)

K	σ	T	P_{CRR}	P_{BJ}	P_{HSS}	P_{EEA}	P_{BJ}-P_{CRR}	P_{HSS}-P_{CRR}	P_{EEA}-P_{CRR}
35	0.2	0.0833	0.01	0.01	0.01	0.01	0.00	0.00	0.00
35	0.2	0.3333	0.20	0.20	0.20	0.20	0.00	0.00	0.00
35	0.2	0.5833	0.43	0.43	0.42	0.42	0.00	-0.01	-0.01
40	0.2	0.0833	0.85	0.85	0.85	0.85	0.00	0.00	0.00
40	0.2	0.3333	1.58	1.58	1.57	1.57	0.00	-0.01	-0.01
40	0.2	0.5833	1.99	1.99	1.99	1.98	0.00	0.00	-0.01
45	0.2	0.0833	5.00	5.00	5.00	5.00	0.00	0.00	0.00
45	0.2	0.3333	5.09	5.00	5.14	5.10	-0.09	0.05	0.01
45	0.2	0.5833	5.27	5.25	5.34	5.29	-0.02	0.07	0.02
35	0.3	0.0833	0.08	0.08	0.08	0.08	0.00	0.00	0.00
35	0.3	0.3333	0.70	0.70	0.69	0.69	0.00	-0.01	-0.01
35	0.3	0.5833	1.22	1.22	1.20	1.20	0.00	-0.02	-0.02
40	0.3	0.0833	1.31	1.31	1.31	1.30	0.00	0.00	-0.01
40	0.3	0.3333	2.48	2.48	2.47	2.46	0.00	-0.01	-0.02
40	0.3	0.5833	3.17	3.17	3.16	3.16	0.00	-0.01	-0.01
45	0.3	0.0833	5.06	5.07	5.08	5.07	0.01	0.02	0.01
45	0.3	0.3333	5.71	5.73	5.74	5.72	0.02	0.03	0.01
45	0.3	0.5833	6.25	6.28	6.29	6.26	0.03	0.04	0.01
35	0.4	0.0833	0.25	0.25	0.25	0.24	0.00	0.00	-0.01
35	0.4	0.3333	1.35	1.35	1.34	1.34	0.00	-0.01	0.01
35	0.4	0.5833	2.16	2.16	2.13	2.13	0.00	-0.03	-0.03
40	0.4	0.0833	1.77	1.76	1.76	1.76	-0.01	-0.01	-0.01
40	0.4	0.3333	3.38	3.38	3.37	3.36	0.00	-0.01	-0.02
40	0.4	0.5833	4.35	4.34	4.33	4.32	-0.01	-0.02	-0.03
45	0.4	0.0833	5.29	5.30	5.30	5.29	0.01	0.01	0.00
45	0.4	0.3333	6.51	6.52	6.52	6.50	0.01	0.01	-0.01
45	0.4	0.5833	7.39	7.41	7.40	7.38	0.02	0.01	-0.01

Notes: Columns 1, 2, 3 and 4 are reprinted from Table 1 of HSS [13]. For these put options, the risk free rate r = 0.05 and the stock price S=$40; k, σ, and T are the exercise price (in dollars), the standard deviation of the rate of return on the stock, and the time to maturity of options (in years), respectively. Column 4 (P_{CRR}), presents the American option values computed by CRR [9] binomial model with time steps equal to 150. Column 5 (P_{BJ}), column 6 (P_{HSS}), and column 7 (P_{EEA}) show the American option values computed by the Bunch and Johnson [5] approach, the HSS [13] method, and our exponential extrapolation approach, respectively. Columns 8, 9, and 10 present the estimated errors caused by the Bunch and Johnson [5] approach, the HSS [13] method, and our exponential extrapolation approach, respectively.

Table 2
Comparison of short-term American put option hedge ratios using alternative approaches: zero-dividend case (S=40)

r	K	σ	T	Δ_{GJ}	Δ_{HSS}	Δ_{EEA}	Δ_{HSS}-Δ_{GJ}	Δ_{EEA}-Δ_{GJ}
0.0488	35	0.2	0.0833	-0.008	-0.008	-0.007	0.000	0.001
0.0488	35	0.2	0.3333	-0.090	-0.089	-0.085	0.001	0.004
0.0488	35	0.2	0.5833	-0.134	-0.132	-0.127	0.002	0.007
0.0488	40	0.2	0.0833	-0.470	-0.470	-0.467	0.000	0.003
0.0488	40	0.2	0.3333	-0.443	-0.449	-0.436	-0.006	0.007
0.0488	40	0.2	0.5833	-0.427	-0.439	-0.412	-0.012	0.015
0.0488	45	0.2	0.0833	-1.000	-0.997	-0.995	0.003	0.005
0.0488	45	0.2	0.3333	-0.888	-0.878	-0.875	0.010	0.013
0.0488	45	0.2	0.5833	-0.805	-0.801	-0.790	0.004	0.015
0.0488	35	0.3	0.0833	-0.052	-0.051	-0.049	0.001	0.003
0.0488	35	0.3	0.3333	-0.174	-0.172	-0.169	0.002	0.005
0.0488	35	0.3	0.5833	-0.213	-0.211	-0.206	0.002	0.007
0.0488	40	0.3	0.0833	-0.470	-0.469	-0.465	0.005	0.001
0.0488	40	0.3	0.3333	-0.442	-0.445	-0.440	-0.003	0.002
0.0488	40	0.3	0.5833	-0.425	-0.431	-0.420	-0.006	0.005
0.0488	45	0.3	0.0833	-0.926	-0.926	-0.922	0.000	0.004
0.0488	45	0.3	0.3333	-0.726	-0.739	-0.719	-0.013	0.007
0.0488	45	0.3	0.5833	-0.651	-0.667	-0.640	-0.016	0.011
0.0488	35	0.4	0.0833	-0.106	-0.106	-0.102	0.000	0.004
0.0488	35	0.4	0.3333	-0.226	-0.225	-0.220	0.001	0.006
0.0488	35	0.4	0.5833	-0.254	-0.253	-0.246	0.001	0.008
0.0488	40	0.4	0.0833	-0.467	-0.466	-0.460	0.001	0.007
0.0488	40	0.4	0.3333	-0.437	-0.438	-0.430	-0.001	0.007
0.0488	40	0.4	0.5833	-0.418	-0.421	-0.410	-0.003	0.008
0.0488	45	0.4	0.0833	-0.835	-0.842	-0.833	-0.007	0.002
0.0488	45	0.4	0.3333	-0.646	-0.656	-0.640	-0.010	0.006
0.0488	45	0.4	0.5833	-0.580	-0.593	-0.565	-0.013	0.015

Notes: The first five columns are reprinted from Table 2 of HSS [13]. Column 5 shows the Geske and Johnson [11] put option hedge ratios,Δ_{GJ} . Column 6 (Δ_{HSS}), and 7 (Δ_{MHSS}), represent the hedge ratios computed by the HSS [13] method and our exponential extrapolation approach, respectively. Columns 8 and 9 show the estimated errors of hedge ratios computed by the HSS [13] method and our approach, respectively.

Table3

Comparison of short-term American put option values using alternative approaches: zero-dividend case (K=100)

r	σ	T	S	P_{CRR}	P_{BJ}	P_{HSS}	P_{EEA}	P_{BJ}-P_{CRR}	P_{HSS}-P_{CRR}	P_{EEA}-P_{CRR}
0.08	0.2	0.25	80	20.00	20.00	20.00	20.00	0.00	0.00	0.00
0.08	0.2	0.25	90	10.04	10.00	10.12	10.01	-0.04	0.08	-0.03
0.08	0.2	0.25	100	3.22	3.22	3.22	3.20	0.00	0.00	-0.02
0.08	0.2	0.25	110	0.67	0.66	0.65	0.65	-0.01	-0.02	-0.02
0.08	0.2	0.25	120	0.09	0.09	0.09	0.09	0.00	0.00	0.00
0.12	0.2	0.25	80	20.00	20.00	20.00	20.00	0.00	0.00	0.00
0.12	0.2	0.25	90	10.20	10.00	10.00	10.00	0.00	0.00	0.00
0.12	0.2	0.25	100	2.92	2.93	2.95	2.91	0.01	0.03	-0.01
0.12	0.2	0.25	110	0.56	0.55	0.54	0.53	-0.01	-0.02	-0.03
0.12	0.2	0.25	120	0.11	0.07	0.07	0.07	0.00	0.00	0.00
0.08	0.4	0.25	80	20.32	20.12	20.48	20.37	-0.20	0.16	0.05
0.08	0.4	0.25	90	12.57	12.63	12.62	12.58	0.06	0.05	0.01
0.08	0.4	0.25	100	7.10	7.10	7.08	7.06	0.00	-0.02	-0.04
0.08	0.4	0.25	110	3.70	3.69	3.66	3.65	-0.01	-0.04	-0.05
0.08	0.4	0.25	120	1.80	1.79	1.77	1.77	-0.01	-0.03	-0.03
0.08	0.2	0.50	80	20.00	20.00	20.00	20.00	0.00	0.00	0.00
0.08	0.2	0.50	90	10.29	10.00	10.46	10.29	-0.29	0.17	0.00
0.08	0.2	0.50	100	4.19	4.20	4.22	4.17	0.01	0.03	-0.02
0.08	0.2	0.50	110	1.41	1.40	1.38	1.38	-0.01	-0.03	-0.03
0.08	0.2	0.50	120	0.40	0.39	0.38	0.38	-0.01	-0.02	-0.02

Note: The first four columns are reprinted from Table 3 of HSS [13]. Columns 1 to 4 represent the parameter input for r (the risk-free rate), σ (the standard deviation of the rate of return on the stock), T (the time to time to maturity of options) and S (the stock price), respectively. Column 5 (P_{CRR}), presents the American option values computed by the CRR [9] binomial model with time steps equal to 150. Column6 (P_{BJ}), column 7 (P_{HSS}) , and column 8 (P_{EEA}) show the American option values computed by the Bunch and Johnson [5] approach, the HSS [13] method and our exponential extrapolation approach, respectively. Columns 9, 10 and 11 present the estimated errors for the Bunch and Johnson [5] approach, the HSS [13], method, and our approach, respectively.

B. Estimating the Values of the Short-Term American Call and Put Options on Dividend-Paying Stocks

In the case of the short-term American call and put on dividend-paying stocks, for comparison, we carry out simulations using the data of Table 4 reported in the HSS [13] study. From Table 4, we find that our approach can compute all the option values to a high degree of accuracy. The estimated errors range only from one cent to four cents. Unlike the Bunch and Johnson [5] approach and the HSS [13] method, our approach estimates the deep in-the-money American put option values very accurately. These results indicate that our approach can be applied to value currency options in which the foreign interest rate is similar to a continuous dividend yield.

C. Estimating the Values of the Long-Term Maturity Call an Put Options on Dividend-Paying Stocks

There are more and more options that have time to maturity longer than one year. For example, bond warrants, currency warrants, swaptions, and commodity trust units are traded with maturities of two years or more. The need for a computationally tractable model for these options has recently increased. Thus, in this subsection, we will test the accuracy of the Bunch and Johnson [5] approach, the HSS [13] method, and our exponential extrapolation approach for valuing the long-term American call and put options on dividend paying stocks.

For comparison, we carry out simulations using the data of Table 5 and 6 reported in the HSS [13] study. From Table 5, we find that the Bunch and Johnson [5] method and the HSS [13] method perform marginally better than our approach for the out-of-the-money American put options with three-year maturities. However, our approach performs significantly better than the Bunch and Johnson [5] method and the HSS [13] exponential formula for the in-the-money American put options with three-year maturities. From Table 6, we obtain similar results for the American call options with three-year maturities.

Table 4
Comparison of short-term American put option values using Alternative approaches: continuous-dividend case (K=100)

y	r	σ	T	S	P_{CRR}	P_{BJ}	P_{HSS}	P_{EEA}	P_{BJ}-P_{CRR}	P_{HSS}-P_{CRR}	P_{EEA}-P_{CRR}
0.04	0.08	0.2	0.25	80	20.00	20.00	20.00	20.00	0.00	0.00	0.00
0.04	0.08	0.2	0.25	90	10.23	10.16	10.31	10.26	-0.07	0.08	0.03
0.04	0.08	0.2	0.25	100	3.54	3.54	3.53	3.52	0.00	-0.01	-0.02
0.04	0.08	0.2	0.25	110	0.79	0.79	0.78	0.78	0.00	-0.01	-0.01
0.04	0.08	0.2	0.25	120	0.11	0.11	0.11	0.11	0.00	0.00	0.00
0.08	0.12	0.2	0.25	80	20.00	20.00	20.00	20.00	0.00	0.00	0.00
0.08	0.12	0.2	0.25	90	10.20	10.09	10.29	10.23	-0.11	0.09	0.03
0.08	0.12	0.2	0.25	100	3.52	3.52	3.51	3.50	0.00	-0.01	-0.02
0.08	0.12	0.2	0.25	110	0.78	0.78	0.77	0.77	0.00	-0.01	-0.01
0.08	0.12	0.2	0.25	120	0.11	0.11	0.11	0.11	0.00	0.00	0.00
0.04	0.08	0.4	0.25	80	20.59	20.73	20.69	20.63	0.14	0.10	0.04
0.04	0.08	0.4	0.25	90	12.96	12.98	12.97	12.95	0.02	0.01	-0.01
0.04	0.08	0.4	0.25	100	7.45	7.45	7.43	7.42	0.00	-0.02	-0.03
0.04	0.08	0.4	0.25	110	3.95	3.94	3.92	3.92	-0.01	-0.03	-0.03
0.04	0.08	0.4	0.25	120	1.95	1.95	1.93	1.93	0.00	-0.02	-0.02
0.04	0.08	0.2	0.50	80	20.00	20.00	20.00	20.00	0.00	0.00	0.00
0.04	0.08	0.2	0.50	90	10.76	10.95	10.89	10.80	0.19	0.13	0.04
0.04	0.08	0.2	0.50	100	4.76	4.77	4.76	4.73	0.01	0.00	-0.03
0.04	0.08	0.2	0.50	110	1.74	1.73	1.71	1.70	-0.01	-0.03	-0.04
0.04	0.08	0.2	0.50	120	0.52	0.52	0.52	0.51	0.00	0.00	-0.01

Note: The first five columns are reprinted from Table 4 of HSS [13]. Columns 1 to 5 represent the parameter input for y (the dividend yield), r (the risk-free rate), σ (the standard deviation of the rate of return on the stock), T (the time to time to maturity of options), and S (the stock price), respectively. Column 6 (P_{CRR}) presents the American option values computed by the CRR [9] binomial model with time steps equal to 150. Column7 (P_{BJ}), column 8 (P_{HSS}) , and column 9 (P_{EEA}) show the American option values computed by the Bunch and Johnson [5] approach, the HSS [13] exponential formula and the modified HSS exponential formula, respectively. Columns 10, 11 and 12 present the estimated errors for the Bunch and Johnson [5] approach, the HSS [13] exponential formula, and the modified HSS exponential formula, respectively.

Table 5
Comparison of long-term American put option values using alternative approaches: continuous-dividend case (K=100)

y	r	σ	T	S	P_{CRR}	P_{BJ}	P_{HSS}	P_{EEA}	P_{BJ}-P_{CRR}	P_{HSS}-P_{CRR}	P_{EEA}-P_{CRR}
0.12	0.08	0.2	3.00	80	25.65	25.94	25.96	25.83	0.29	0.31	0.18
0.12	0.08	0.2	3.00	90	20.08	20.20	20.18	20.12	0.12	0.10	0.04
0.12	0.08	0.2	3.00	100	15.48	15.53	15.50	15.45	0.05	0.02	-0.03
0.12	0.08	0.2	3.00	110	11.81	11.83	11.78	11.77	0.02	-0.03	-0.04
0.12	0.08	0.2	3.00	120	8.89	8.90	8.86	8.85	0.01	-0.03	-0.04
0.08	0.08	0.2	3.00	80	22.20	22.70	22.88	22.53	0.50	0.68	0.33
0.08	0.08	0.2	3.00	90	16.20	16.53	16.59	16.39	0.33	0.39	0.19
0.08	0.08	0.2	3.00	100	11.69	11.81	11.83	11.72	0.12	0.14	0.03
0.08	0.08	0.2	3.00	110	8.38	8.42	8.36	8.33	0.04	-0.02	-0.05
0.08	0.08	0.2	3.00	120	5.94	5.87	5.87	5.87	-0.07	-0.07	-0.07
0.04	0.08	0.2	3.00	80	20.34	20.00	20.80	20.58	-0.34	0.46	0.24
0.04	0.08	0.2	3.00	90	13.49	14.01	14.06	13.64	0.22	0.57	0.15
0.04	0.08	0.2	3.00	100	8.93	9.12	9.24	8.99	0.19	0.33	0.06
0.04	0.08	0.2	3.00	110	5.92	5.94	5.99	5.88	0.02	0.07	-0.04
0.04	0.08	0.2	3.00	120	3.90	3.89	3.86	3.83	-0.01	-0.04	-0.07
0.00	0.08	0.2	3.00	80	20.00	20.00	20.00	20.00	0.00	0.00	0.00
0.00	0.08	0.2	3.00	90	11.68	10.21	12.09	11.44	-1.47	0.41	-0.24
0.00	0.08	0.2	3.00	100	6.92	6.92	7.29	6.94	0.00	0.37	0.02
0.00	0.08	0.2	3.00	110	4.15	4.14	4.30	4.11	-0.01	0.15	-0.04
0.00	0.08	0.2	3.00	120	2.51	2.47	2.52	2.46	-0.04	0.01	-0.05

Notes: The first five columns are reprinted from Table 5 of HSS [13]. Columns 1 to 5 represent the parameter input for y (the dividend yield), r (the risk-free rate), σ (the standard deviation of the rate of return on the stock), T (the time to time to maturity of options), and S (the stock price), respectively. Column 6 (P_{CRR}) presents the American option values computed by the CRR [9] binomial model with time steps equal to 150. Column7 (P_{BJ}), column 8 (P_{HSS}) , and column 9 (P_{EEA}) show the American option values computed by the Bunch and Johnson [5] approach, the HSS [13] exponential formula and the modified HSS exponential formula, respectively. Columns 10, 11 and 12 present the estimated errors for the Bunch and Johnson [5] approach, the HSS [13] exponential formula, and the modified HSS exponential formula, respectively.

Table 6
Comparison of long-term American call option values using alternative approaches: continuous-dividend case (K=100)

y	r	σ	T	S	P_{CRR}	P_{BJ}	P_{HSS}	P_{EEA}	P_{BJ}-P_{CRR}	P_{HSS}-P_{CRR}	P_{EEA}-P_{CRR}
0.12	0.08	0.2	3.00	80	2.34	2.33	2.31	2.29	-0.01	-0.03	-0.05
0.12	0.08	0.2	3.00	90	4.76	4.81	4.87	4.75	0.25	0.11	-0.01
0.12	0.08	0.2	3.00	100	8.48	8.73	8.91	8.61	0.25	0.33	0.13
0.12	0.08	0.2	3.00	110	13.79	14.14	14.55	13.98	0.35	0.76	0.19
0.12	0.08	0.2	3.00	120	20.87	20.00	21.66	20.74	-0.87	0.79	0.13
0.08	0.08	0.2	3.00	80	3.97	3.97	3.93	3.92	0.00	-0.06	-0.05
0.08	0.08	0.2	3.00	90	7.25	7.28	7.24	7.21	0.03	-0.01	-0.04
0.08	0.08	0.2	3.00	100	11.69	11.81	11.83	11.72	0.12	0.14	0.03
0.08	0.08	0.2	3.00	110	17.32	17.64	17.71	17.49	0.32	0.39	0.17
0.08	0.08	0.2	3.00	120	24.02	24.82	24.73	24.36	0.80	0.71	0.34
0.04	0.08	0.2	3.00	80	6.86	6.86	6.88	6.86	0.00	0.02	0.00
0.04	0.08	0.2	3.00	90	11.49	11.49	11.49	11.48	0.00	0.00	-0.01
0.04	0.08	0.2	3.00	100	17.19	17.20	17.20	17.18	0.01	0.01	-0.01
0.04	0.08	0.2	3.00	110	23.85	23.86	23.83	23.83	0.01	-0.02	-0.02
0.04	0.08	0.2	3.00	120	31.17	31.18	31.15	31.15	0.01	-0.02	-0.02

Notes: The first five columns are reprinted from Table 5 of HSS [13]. Columns 1 to 5 represent the parameter input for y (the dividend yield), r (the risk-free rate) , σ (the standard deviation of the rate of return on the stock), T (the time to time to maturity of options), and S (the stock price), respectively. Column 6 (P_{CRR}) presents the American option values computed by the CRR [9] binomial model with time steps equal to 150. Column7 (P_{BJ}), column 8 (P_{HSS}) , and column 9 (P_{EEA}) show the American option values computed by the Bunch and Johnson [5] approach, the HSS [13] exponential formula and the modified HSS exponential formula, respectively. Columns 10, 11 and 12 present the estimated errors for the Bunch and Johnson [5] approach, the HSS [13] exponential formula, and the modified HSS exponential formula, respectively.

For American call and put options with time to maturities longer than three years, as shown in Tables 7 and 8, we find that our approach performs significantly better than the Bunch and Johnson [5] method and the HSS [13] method for all the options. Surprisingly, our approach still performs very well for those options with maturities longer than three years. For most of the options with maturities longer than three years, our approach has estimated biases of less than one percent. These results show that our approach is also powerful for estimating the values and hedge ratios of long-term American options.

D. Estimating the Values of Exotic Options

In the last decade, interest in exotic options has been growing, especially in the over-the-counter market. A lot of research has been devoted to the pricing and hedging these products, not only within the continuous-time framework of Black and Scholes [2] but increasingly also within the discrete-time economy of the CRR [9] method. The advantages of using the discrete-time economy of the CRR [9] method are its simplicity and flexibility. However, the feasibilty of a discrete-time model depends heavily on the number of nodes involved. The usefulness of a discrete-time model for a specific kind of exotic option contract depends not only on the numerical accuracy, but also on the computer time needed for calculations.

Hull and White [15] developed an efficient procedure for valuing American lookback options and Asian options. In this subsection, following Breen's [3] idea, we combine the Hull and White [15] efficient procedure with the HSS [13] method and our exponential extrapolation approach to value American lookback options and Asian options. The combination of these two approaches can significantly reduce the number of nodes in calculations, and hence can save a lot of time for computing the option values.

As reported in Table 9, we find that both the HSS [13] method and our approach can compute values for the American lookback options to a high degree of accuracy. The estimated errors with these two methods are within one percent for most of the options using the Hull and White [15] model as a benchmark. For the case of American Asian options, as shown in Table 10, the results are similar to those of American lookback options. These results validate that the HSS [13] method and our approach are also useful for estimating the values of American exotic options.

Table 7
Comparison of long-term American put option values using alternative approaches: continuous-dividend case (S=40)

y	r	σ	T	K	P_{CRR}	P_{BJ}	P_{HSS}	P_{EEA}	P_{BJ}-P_{CRR}	P_{HSS}-P_{CRR}	P_{EEA}-P_{CRR}
0.08	0.08	0.2	3.00	35	2.53	2.54	2.52	2.51	0.01	-0.01	-0.02
0.08	0.08	0.2	3.00	40	4.68	4.72	4.73	4.69	0.04	0.05	0.01
0.08	0.08	0.2	3.00	45	7.55	7.72	7.74	7.65	0.17	0.20	0.10
0.08	0.08	0.2	4.00	35	2.99	3.01	2.99	2.97	0.02	0.00	-0.02
0.08	0.08	0.2	4.00	40	5.15	5.25	5.27	5.20	0.10	0.12	0.05
0.08	0.08	0.2	4.00	45	7.97	8.23	8.27	8.12	0.26	0.29	0.15
0.08	0.08	0.2	5.00	35	3.35	3.39	3.39	3.34	0.04	0.04	-0.01
0.08	0.08	0.2	5.00	40	5.52	5.65	5.70	5.59	0.13	0.18	0.08
0.08	0.08	0.2	5.00	45	8.29	8.66	8.68	8.45	0.37	0.39	0.16
0.08	0.08	0.3	3.00	35	4.57	4.60	4.56	4.54	0.03	-0.01	-0.03
0.08	0.08	0.3	3.00	40	6.97	7.05	7.06	7.00	0.08	0.09	0.03
0.08	0.08	0.3	3.00	45	9.89	10.07	10.09	9.99	0.18	0.20	0.10
0.08	0.08	0.3	4.00	35	5.25	5.30	5.30	5.25	0.05	0.05	0.00
0.08	0.08	0.3	4.00	40	7.67	7.81	7.85	7.74	0.14	0.18	0.07
0.08	0.08	0.3	4.00	45	10.55	10.82	10.88	10.70	0.27	0.33	0.15
0.08	0.08	0.3	5.00	35	5.77	5.85	5.88	5.79	0.08	0.09	0.03
0.08	0.08	0.3	5.00	40	8.20	8.41	8.49	8.32	0.21	0.29	0.12
0.08	0.08	0.3	5.00	45	11.04	11.46	11.53	11.26	0.42	0.49	0.22
0.00	0.08	0.3	3.00	35	2.95	2.97	3.00	2.97	0.02	0.05	0.02
0.00	0.08	0.3	3.00	40	4.97	5.09	5.16	5.00	0.12	0.19	0.03
0.00	0.08	0.3	3.00	45	7.67	7.96	7.99	7.74	0.29	0.32	0.07
0.00	0.08	0.3	4.00	35	3.29	3.31	3.39	3.28	0.02	0.10	-0.01
0.00	0.08	0.3	4.00	40	5.31	5.41	5.60	5.38	0.10	0.29	0.07
0.00	0.08	0.3	4.00	45	7.97	8.14	8.39	8.02	0.17	0.42	0.05
0.00	0.08	0.3	5.00	35	3.53	3.56	3.70	3.56	0.03	0.17	0.03
0.00	0.08	0.3	5.00	40	5.56	5.59	5.90	5.62	0.03	0.35	0.06
0.00	0.08	0.3	5.00	45	8.19	8.13	8.65	8.21	-0.06	0.46	0.02

Notes: The first five columns are reprinted from Table 6 of HSS [13]. Columns 1 to 5 represent the parameter input for y (the dividend yield), r (the risk-free rate) , σ(the standard deviation of the rate of return on the stock), T (the time to time to maturity of options), and S (the stock price), respectively. Column 6 (P_{CRR}) presents the American option values computed by the CRR [9] binomial model with time steps equal to 150. Column7 (P_{BJ}), column 8 (P_{HSS}) , and column 9 (P_{EEA}) show the American option values computed by the Bunch and Johnson [5] approach, the HSS [13] exponential formula and the modified HSS exponential formula, respectively. Columns 10, 11 and 12 present the estimated errors for the Bunch and Johnson [5] approach, the HSS [13] exponential formula, and the modified HSS exponential formula, respectively.

Table 8

Comparison of long-term American call option values using alternative approaches: continuous-dividend case (S=40)

y	r	σ	T	K	P_{CRR}	P_{BJ}	P_{HSS}	P_{EEA}	P_{BJ}-P_{CRR}	P_{HSS}-P_{CRR}	P_{EEA}-P_{CRR}
0.08	0.08	0.2	3.00	35	7.03	7.20	7.20	7.12	0.17	0.17	0.09
0.08	0.08	0.2	3.00	40	4.68	4.72	4.73	4.69	0.04	0.05	0.01
0.08	0.08	0.2	3.00	45	3.07	3.08	3.06	3.05	0.01	-0.01	-0.02
0.08	0.08	0.2	4.00	35	7.37	7.65	7.65	7.50	0.28	0.28	0.13
0.08	0.08	0.2	4.00	40	5.15	5.25	5.27	5.20	0.10	0.12	0.05
0.08	0.08	0.2	4.00	45	4.01	4.05	4.07	4.02	0.04	0.06	0.02
0.08	0.08	0.2	5.00	35	7.64	7.98	8.01	7.82	0.24	0.37	0.18
0.08	0.08	0.2	5.00	40	5.52	5.65	5.70	5.59	0.13	0.18	0.08
0.08	0.08	0.2	5.00	45	4.01	4.05	4.06	4.02	0.04	0.05	0.01
0.08	0.08	0.3	3.00	35	9.04	9.21	9.24	9.14	0.17	0.20	0.10
0.08	0.08	0.3	3.00	40	6.97	7.05	7.06	7.00	0.08	0.09	0.03
0.08	0.08	0.3	3.00	45	5.52	5.65	5.70	5.59	0.13	0.18	0.08
0.08	0.08	0.3	4.00	35	9.62	9.89	9.94	9.77	0.27	0.32	0.15
0.08	0.08	0.3	4.00	40	7.67	7.81	7.85	7.74	0.14	0.18	0.07
0.08	0.08	0.3	4.00	45	6.18	6.24	6.26	6.19	0.06	0.08	0.01
0.08	0.08	0.3	5.00	35	10.05	10.44	10.50	10.25	0.39	0.45	0.20
0.08	0.08	0.3	5.00	40	8.20	8.41	8.49	8.32	0.21	0.29	0.12
0.08	0.08	0.3	5.00	45	6.76	6.86	6.92	6.79	0.10	0.16	0.03

Notes: The first six columns are based on the option parameters in Table 6 of HSS [13]. Columns 1 to 5 represent the parameter input for y (the dividend yield), r (the risk-free rate) , σ (the standard deviation of the rate of return on the stock), T (the time to time to maturity of options), and S (the stock price), respectively. Column 6 (P_{CRR}) presents the American option values computed by the CRR [9] binomial model with time steps equal to 150. Column7 (P_{BJ}), column 8 (P_{HSS}) , and column 9 (P_{EEA}) show the American option values computed by the Bunch and Johnson [5] approach, the HSS [13] exponential formula and the modified HSS exponential formula, respectively. Columns 10, 11 and 12 present the estimated errors for the Bunch and Johnson [5] approach, the HSS [13] exponential formula, and the modified HSS exponential formula, respectively.

Table 9
Comparison of American lookback put option values using alternative approaches: zero dividend case (K=40)

S	σ	T	r	P_{HW}	P_{HSS}	P_{EEA}	$P_{HSS}-P_{CRR}$	$P_{EEA}-P_{CRR}$
35	0.2	0.0833	0.05	1.53	1.52	1.52	-0.01	-0.01
35	0.2	0.3333	0.05	3.00	2.97	2.96	-0.03	-0.04
35	0.2	0.5833	0.05	3.93	3.88	3.88	-0.05	-0.05
40	0.2	0.0833	0.05	1.74	1.73	1.73	-0.01	-0.01
40	0.2	0.3333	0.05	3.43	3.39	3.39	-0.04	-0.04
40	0.2	0.5833	0.05	4.49	4.44	4.43	-0.05	-0.06
45	0.2	0.0833	0.05	1.96	1.95	1.95	-0.01	-0.01
45	0.2	0.3333	0.05	3.86	3.82	3.83	-0.04	-0.05
45	0.2	0.5833	0.05	5.05	4.99	4.98	-0.06	-0.07
35	0.3	0.0833	0.05	2.33	2.32	2.32	-0.01	-0.01
35	0.3	0.3333	0.05	4.67	4.63	4.63	-0.04	-0.04
35	0.3	0.5833	0.05	6.19	6.13	6.13	-0.06	-0.06
40	0.3	0.0833	0.05	2.67	2.65	2.65	-0.02	-0.02
40	0.3	0.3333	0.05	5.34	5.30	5.29	-0.04	-0.05
40	0.3	0.5833	0.05	7.07	7.01	7.00	-0.06	-0.07
45	0.3	0.0833	0.05	3.00	2.99	2.99	-0.01	-0.01
45	0.3	0.3333	0.05	6.00	5.96	5.95	-0.04	-0.05
45	0.3	0.5833	0.05	7.96	7.89	7.88	-0.07	-0.08

Note: Columns 1 to 4 represent the parameter inputs for S (the stock price), σ (the standard deviation of the rate of return on the stock), T (the time to maturity of options), and r (the risk-free rate), respectively. Column 5 (P_{HW},) presents the American lookback put option values computed by the Hull and White [15] binomial method with steps equal to 300, which we use as a benchmark. Column 6 (P_{HSS}) and column 7 (P_{EEA}) show the American option values computed by the HSS [13] method and the modified HSS exponential formula, respectively. Columns 8 and 9 present the estimated errors for the HSS [13] method, and our approach, respectively.

Table 10
Comparison of American Asian put option values using alternative approaches: zero-dividend case (K=40)

S	σ	T	r	P_{HW}	P_{HSS}	P_{EEA}	$P_{HSS}-P_{CRR}$	$P_{EEA}-P_{CRR}$
35	0.2	0.0833	0.05	0.00	0.00	0.00	0.00	0.00
35	0.2	0.3333	0.05	0.04	0.04	0.04	0.00	0.00
35	0.2	0.5833	0.05	0.15	0.15	0.15	0.00	0.00
40	0.2	0.0833	0.05	0.67	0.65	0.64	-0.02	-0.03
40	0.2	0.3333	0.05	1.38	1.35	1.34	-0.03	-0.04
40	0.2	0.5833	0.05	1.88	1.84	1.83	-0.04	+0.05
45	0.2	0.0833	0.05	5.54	5.48	5.47	-0.06	-0.07
45	0.2	0.3333	0.05	6.09	6.02	6.00	-0.07	-0.09
45	0.2	0.5833	0.05	6.48	6.40	6.38	-0.08	-0.10
35	0.3	0.0833	0.05	0.00	0.00	0.00	0.00	0.00
35	0.3	0.3333	0.05	0.21	0.20	0.20	-0.01	-0.01
35	0.3	0.5833	0.05	0.54	0.52	0.52	-0.02	-0.02
40	0.3	0.0833	0.05	0.97	0.95	0.94	-0.02	-0.03
40	0.3	0.3333	0.05	1.98	1.94	1.93	-0.04	-0.05
40	0.3	0.5833	0.05	2.66	2.61	2.60	-0.05	-0.06
45	0.3	0.0833	0.05	5.76	5.70	5.69	-0.06	-0.07
45	0.3	0.3333	0.05	6.54	6.45	6.44	-0.08	-0.09
45	0.3	0.5833	0.05	7.08	6.98	6.96	-0.10	-0.12

Note: Columns 1 to 4 represent the parameter inputs for S (the stock price), σ (the standard deviation of the rate of return on the stock), T (the time to maturity of options), and r (the risk-free rate), respectively. Column 5 (P_{HW}) presents the American Asian put option values computed by the Hull and White [15] binomial method with steps equal to 80, which we use as a benchmark. Column 6 (P_{HSS}) and column 7 (P_{EEA}) show the American Asian option values computed by the HSS [13] method and the modified HSS exponential formula, respectively. Columns 8 and 9 present the estimated errors for the HSS [13] method, and our approach, respectively.

V. CONCLUSION

In this paper, we propose an exponential extrapolation approach for the valuation and hedging of American options. Our exponential extrapolation approach, developed in this paper, not only is very simple and computationally efficient but also significantly reduces the large pricing biases with the use of the HSS [13] method and the Bunch and Johnson [5] approach for the deep in-the-money American options with short-term maturityies and long-term American options. Additionally, we also show that our approach can compute the values of American exotic options and option hedging parameters to a high degree of accuracy.

Option practitioners have to deal with a book of American options ranging from options on stocks to fixed-income securities. Recently, option practitioners also have had to deal with a book of American exotic options. The computation of option values and hedging ratios in these cases is quite laborious. Therefore, our approach is potentially applicable to price a book of American options efficiently and accurately and hence can solve the computational problem encountered by option practitioners. Moreover, our approach is also applicable to the pricing of American options under stochastic interest rates and stochastic volatility. We will leave these interesting topics to future research.

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NOTES

1. Apart from the papers mentioned in this section, there are many other important papers that use different approaches to value American options. (See, for example, Carr and Faguet [7] and Brodie aird Demple [4].
2. Over the last few years, more and more long-term derivatives have begun trading in the exchange and over-the-counter markets. Equity options (such as contingent value rights), long-term currency options, currency warrants, and swaptions have maturities longer than two years. Therefore, the need for a computationally tractable model for these options has recently increased.
3. Using Bunch and Johnson's approach and the HSS approach to value short-term in-the-money and long-term American options, the pricing errors will be as large as five percent, which is much larger

than the transactions costs.

4. We did simulations to find the optimal exponential relationship between the American option value and the exercise point.
5. Our approach is computationally as fast as the Geske and Johnson [11] two-point extrapolation formula. Hence, as reported in the Bunch and Johnson [5], our approach will be 6.5 times faster than the Bunch and Johnson [5] approach.

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APPENDIX

The valuation and delta formula under alternative approaces

This appendix is adopted from the appendix of the HSS [13] method. For a twice-exerciseable option (exerciseable at time t_1 and t_2) on a dividend paying stock. Geske and Johnson [11] show that the value of the option is given by

$$p_2 = \delta(Kw_2 - Sw_1), \tag{17}$$

where

$$w_1 = e^{-yt_1} N_1\left(-\delta d_1^{'}\right) + e^{-yt2} N_2\left(\delta d_1^{'}, -\delta d_1^{''}; -\rho\right), \tag{18}$$

$$w_2 = e^{-rt_1} N_1\left(-\delta d_2^{'}\right) + e^{-rt2} N_2\left(\delta d_2^{'}, -\delta d_2^{''}; -\rho\right), \tag{19}$$

with δ= 1 for a put option and δ= -1 for a call option, and where *y* is the continuous dividend yield, *r* is the riskless interest rate, and N_1 and N_2 are the standard cumulative univariate and bivariate normal distributions, respectively, with parameters

$$d_1^{'} = \frac{\ln\left(S/S_1^*\right) + \left(r + \frac{1}{2}\sigma^2\right)t_1}{\sigma\sqrt{t_1}} \tag{20}$$

$$d_1^{''} = \frac{\ln(S/K) + \left(r + \frac{1}{2}\sigma^2\right)t_2}{\sigma\sqrt{t_2}} \tag{21}$$

$$d_2^{'} = d_1^{'} - \sigma\sqrt{t_2} \tag{22}$$

$$d_2^{''} = d_1^{''} - \sigma\sqrt{t_2} \tag{23}$$

$$\rho = \sqrt{t_1/t_2} \tag{24}$$

where S_1* is the critical price.

Using Richardson's extrapolation, Geske and Johnson [11] get the two-point extrapolation formula as follows:

$$\hat{P} = 2P_2 - P_1 \tag{25}$$

where P_1 is the European option value.
The delta for a twice-exerciseable option can be obtained as follows

$$\Delta_2 = \frac{\partial P_2}{\partial S} = -\delta w_1 \tag{26}$$

It follows from equation (18) that the delta for a twice-exerciaeable option is

$$\Delta_2 = \frac{\partial P_2}{\partial S} = -\delta\left[e^{-yt1} N_1\left(-\delta d_1^{'}\right) + e^{-yt2} N_2\left(\delta d_1^{'}, -\delta d_1^{''}; -\rho\right)\right] \tag{27}$$

Hence, the estimated delta using the two-point Geske and Johnson [11] extrapolation formula is obtained by differentiating equation (17) to get

$$\Delta_{GJ} = \frac{\partial \hat{P}}{\partial S} = 2\Delta_2 - \Delta_1 \tag{28}$$

where Δ_1 is the delta of a European options.

Similarly, we can obtain the estimated delta using the HSS (1994) method and our exponential extrapolation approach by differentiating equations (6) and (13) respectively to get

$$\Delta_{HSS} = \frac{\partial \hat{P}}{\partial S} = \hat{P}\left(\frac{2\Delta_2}{P_2} - \frac{\Delta_1}{P_1}\right) \tag{29}$$

$$\Delta_{EEA} = \frac{\partial \hat{P}}{\partial S} = \hat{P}\left(\frac{9/5\,\Delta_2}{P_2} - \frac{\Delta_1}{P_1}\right). \tag{30}$$