On Arbitrage, Information Costs, Compound Options and the Valuation of the Firm and Its Assets

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ABSTRACT

This paper presents a simple framework for the valuation of compound options within a context of incomplete information. Information costs are linked to the theory of signaling, agency models and generic stocks in the spirit of Merton's (1987) model of capital market equilibrium with incomplete information. We propose some ideas to explain arbitrage in financial markets in the presence of information costs. The use of these costs is important in the valuation of equity of some firms in the "new economy" like Internet stocks. Equity in these firms cannot be valued in an appropriate way by a model ignoring information uncertainty.

When deriving the compound call option formula, we consider a call option on a stock, which is itself an option on the assets of the firm. Our methodology incorporates shadow costs of incomplete information on the firm's assets as well as the effects of leverage in the capital structure. The compound option formula is derived using two approaches: the standard Black and Scholes approach and the martingale method. The formula can be useful in the valuation of several corporate liabilities in the presence of information uncertainty about the firm and its cash flows. Our analysis can be used for the valuation of several real options.

JEL: *G3, G31, G32, G33*

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I.   INTRODUCTION

The compound option or an option on an option has been studied in a context of complete information by several authors including Black and Scholes (1973), Geske (1979), Triantis and Hodder (1990), Briys-Bellalah et al. (1998), etc . The concept of an option on an option is important in the study of several opportunities with a sequential nature where some of them are available only if earlier opportunities are undertaken. For a survey of this literature on real standard and complex options, the reader can refer to Dixit and Pindyck (1994), Triantis and Hodder (1990) and Grenadier and Weiss (1997) among others. Black and Scholes (1973), Black and Cox (1976), Galai and Masulis (1976) and Geske (1979) show that several corporate liabilities may be considered as options. In a context of complete information, they study the pricing of a firm's common stock and bonds by considering the stock as an option on the firm's value. They show that corporate investment opportunities may be analyzed as options and compound options. However, their analysis does not account for information uncertainty.

Since the acquisition of information and its dissimination are central activities in finance, and especially in capital markets, Merton (1987) develops a model of capital market equilibrium with incomplete information, CAPMI, to provide some insights into the behavior of security prices. He also studies the equilibrium structure of asset prices and its connection with empirical anomalies in financial markets. In this spirit, Bellalah (1990) and (1999) provides a valuation formula for stock options and commodity options in a context of incomplete information. The formula is derived in an equilibrium approach by a simple extension of the main results in Merton's model.

In this paper, we use arbitrage arguments rather than an equilibrium approach to derive the formula in a Black and Scholes (1973) economy. Such a formula might be applied to the valuation of equity in the capital structure of the firm. The use of information costs regarding the firm and its cash flows might help to understand why Black and Scholes model leads to theoretical prices, which are systematically biased. The information uncertainty about the firm and its cash flows reflects the agency costs and the asymmetric information problems. By assuming the stock as an option on the value of the firm, the value of the call as a compound option can be derived as a function of the firm's value by accounting for information costs and the effects of leverage. We present two alternative derivations for the value of the firm's stock as a compound option in the presence of information costs. The first derivation parallels that of Geske (1979) using the Black and Scholes approach. The second derivation relies on the martingale approach a la Harrison and Kreps (1979) and Harrison and Pliska (1981).

The structure of the paper is as follows. Section II reminds the main results in Merton's model regarding information costs and applies the non-arbitrage approach to derive the call option formula. Section III proposes two approaches for the derivation of the compound option formula in the presence of information uncertainty. The formula corresponds to the value of a stock in a levered firm as a function of the firm's asset value. The first approach relies on the same arguments as those in Geske (1979). The second approach parallels the martingale approach. The continuous-time approach can

be for example obtained by convergence arguments from a modified standard Cox, Ross and Rubinstein (1979) model to account for information costs. Section IV proposes the option price sensitivities and some comparisons with respect to standard models in the absence of information costs.

II.   THE VALUATION EQUATIONS FOR STANDARD OPTIONS WITH INFORMATION COSTS

This section states Merton's (1987) model and justifies the use of information costs in the recent literature for the explanation of liquidity effects, corporate risks and some anomalies observed in capital and real markets.

A.   Shadow Costs of Incomplete Information and the Pricing of Financial Assets

Merton's model is a two period model of capital market equilibrium in an economy where each investor has information about only a subset of the available securities. The key behavioral assumption of the model is that an investor considers including security V in his portfolio only if he has some information on this security. Information costs have two components: the costs of gathering and processing data, and the costs of information transmission. The main results in Merton's model are recently applied in a different context by Shleifer and Vishny (1997) and Orosel (1997). This problem is related to the literature on the principal-agent problem, to the signaling models, to the differential information models and to the theory of generic and neglected stocks (see Arbel and Strebel (1982), Barry, and Brown (1986) among others).
Merton's   model may be stated as follows:

$$\overline{R}_V - r = \beta_V\left[\overline{R}_m - r\right] + \lambda_V - \beta_V\lambda_m$$

where :
$\overline{R}_v$: the equilibrium expected return on security V,
$\overline{R}_m$: the equilibrium expected return on the market portfolio,
r: the riskless rate of interest,
β_V: the beta of security V , that is the covariance of the return on that security with the return on the market portfolio, divided by the variance of market return,
λ_V: the equilibrium aggregate "shadow cost" for the security V. It is of the same dimension as the expected rate of return on this security V,
λ_m: the weighted average shadow cost of incomplete information over all securities.
        From Merton's model (1987), it appears that taking into account the effect of incomplete information on the equilibrium price of an asset is similar to applying an additional discount rate to this asset's future cash flows. In fact, the expected return on the asset is given by the appropriate discount rate that must be applied to its future cash flows. Then, relying on Merton's model to derive a valuation formula should lead to more accurate theoretical prices since information costs are included for options and their underlying securities.

Kadlec and McConnell (1994) study the effect of market segmentation and illiquidity on asset prices. They document the effect on share value of listing on the New York Stock Exchange.

The results in Kadlec and McConnell (1994) provide support for investor recognition as a source of value from exchange listing, supporting Merton's model. They also provide support for superior liquidity as a source of value from exchange listing, supporting Amihud and Mendelson's (1986) model.
Several studies have shown how share prices increase for firms that list on the NYSE from the Nasdaq OTC market. These studies attributed this outcome to increased investor recognition or superior liquidity (see Merton [1987], Foerster and Karolyi (1999)).

Beneish and Gardner (1995) show that stock prices decrease when the quantity of available information decreases. This result is consistent with the evidence in Merton (1987) that investors demand higher returns for holding stocks with less available information. They find also consistent evidence with that in Amihud and Mendelson (1986) where investors demand higher expected returns for higher trading costs.

Coval and Moskowitz (1999) show that investment managers exhibit a strong preference for locally-headquartered firms, particularily small, highly-levered firms, that produce non-traded goods. Stulz (1999) shows that globalization decreases the cost of capital under some assumptions. In this context, following Merton (1987), Stulz (1999) assumes that some investors do not hold some securities because they do not know about them.

Amihud and Mendelson (1988) consider several observed corporate policies that can be viewed as increasing the liquidity of investments. The use of Merton's (1987) model in our analysis is based on its simplicity as well as its explicative power in explaining some anomalies in financial markets.

B.   Arbitrage and Information Costs

Arbitrage involves simultaneously at least two transactions in different markets giving the investor a riskless profit. Consider for example a stock traded on both the New York Stock Exchange and the Paris Bourse.

Assume that the stock is worth 200 dollars in New York and 201 euro in Paris. The exchange rate is one dollar for an euro. An arbitrageur could enter simultaneously in two transactions: buy the stock in New York and sell it in Paris. He would realize a profit of one euro per share bought.

In this case, this arbitrage opportunity is attractive in the absence of transaction costs and information costs. This arbitrage opportunity is eliminated if informed arbitrageurs buy the stock in New York and sell it in Paris. The market forces will cause an equivalence between the prices in Paris and New York by acting on exchange rates. The actions of arbitrageurs eliminate the major disparity between the prices of the stock in different currencies.

For the ease of notation, we denote the information cost by λ. A clear analysis of arbitrage implies the presence of an information cost for each asset and market. Hence, there are information costs for stocks, S, λ_S, for bonds, B, λ_B, etc. For a portfolio of N

assets, there is an information cost for each asset and market. If the funds used in arbitrage are borrowed or the investor has the opportunity to lend them at the riskless rate, the return from arbitrage must be at least $(r + \lambda)$. Therefore, we expect information costs to appear in all arbitrage operations in bond markets, foreign exchange markets, equities transactions, derivatives, etc.

C.   The Pricing of Assets Under Incomplete Information

Let us denote by S(V,t) the value of the option as a function of the underlying asset and time. Assume that the price of the underlying asset follows a geometric Wiener process:

$$dV/V = \alpha dt + \sigma dZ \ dt$$

where α and σ refer to the instantaneous rate of return and the standard deviation of the underlying asset, and dz is a Brownian motion. Using the same assumptions as in the seminal paper of Black and Scholes and the additional assumption of information uncertainty on the option S(V,t) and its underlying asset, V, it is possible to construct a hedged position which contains one share of stock long and $1/S_V$ options short. The term S_V is the first derivative of the option with respect to its underlying asset. The value of equity in this position is: $V - S(V,t)/S_V$.

Over a short interval Δ_t, the change in the value of the equity is: $\partial V - \Delta S(V,t)/S_V$. Assuming a hedged position is constructed and "continuously" re-balanced, and since $\partial S(V,t)$ is a continuous and differentiable function of two variables, it is possible to use Taylor series expansion to expand $\Delta S(V,t)$. When limiting arguments are used and second order terms ignored, we obtain:

$$\Delta S = 1/2 S_{VV}\sigma^2 V \Delta t + S_V \Delta V + S_t \Delta dt$$

This is just an extension of simple results to obtain Ito's lemma. The application of this lemma gives:

$$\Delta S = 1/2 S_{VV}\sigma^2 V^2 \Delta t + S_V \Delta V + S_t \Delta dt$$

Substituting from this equation into the previous one, the change in the value of the equity in the hedged position is:

$$-\left(1/2 S_{VV}\sigma^2 V^2 + S_t\right)\Delta t/S_V$$

Since the return in the hedged position is certain, the return must be equal to $(r + \lambda_V)$ Δt for the underlying asset and $(r + \lambda_S)$ Δt for the option where λ_V and λ_S refer

respectively to the information costs on the underlying asset and the option. In this context, we have:

$$-\left(1/2 S_{VV}\sigma^2 V^2 + S_t\right)\Delta t/S_V = V(r+\lambda V)\Delta t - S(V,t)/S_V\,(r+\lambda_S)\Delta t \qquad (1)$$

Dropping the ∂t and rearranging the terms, we get the differential equation for the value of the option:

$$1/2\,\sigma^2 V^2 S_{VV} + (r+\lambda V)VS_V - (r+\lambda S)S + S_t = 0 \qquad (2)$$

The effect of the shadow costs of incomplete information on an asset equilibrium price appears to be similar to applying an additional discount rate to this asset's future cash flows. Note that if λ_V and λ_S are set equal to zero, this partial differential equation reduces to the Black and Scholes valuation equation. This equation appears in Bellalah (1990) and (1999).

Let T be the maturity date of the call and M be its strike price. Equation (2) subject to the following boundary condition at maturity:

$$S(V,T) = V_T \text{ - } M \text{ if } V_T \geq M$$
$$S(V,T) = 0 \text{ if } V_T < M$$

is solved using standard methods for the price of a European call, which is found to be equal to :

$$S(V,T)= Ve^{-(\lambda_s-\lambda_v)T}N_1(d_1) - Me^{-(r+\lambda s)T}N_1(d_2) \qquad (3)$$

with: $d_1 = [\ln(V/M) + (r+1/2\,\sigma^2+\lambda_v)T]/\sigma\sqrt{T}$; $d_2 = d_1 - \sigma\sqrt{T}$; and where $N_1(.)$ is the univariate cumulative normal density function. Appendix 2 presents the derivation of the same formula by applying the martingale techniques. In both cases, we obtain the same results. If the information costs on the option and its underlying asset are set equal to zero, this formula collapses to the standard Black and Scholes formula. The presence of information costs on the call and the assets of the firm reflect the costs incurred by investors to get informed about the characteristics of the claims on the firm. These costs appear in the form of an additional return required by investors. They are reflected in the discounting factors, which have the same dimension as an incremental interest rate

III. THE VALUATION OF EQUITY AS A COMPOUND OPTION IN THE PRESENCE OF SHADOW COSTS OF INCOMPLETE INFORMATION

If the stock is considered as an option on the value of the firm, V, then the value of the call as a compound option can be expressed as a function of the firm's value. This

analysis follows from the setting in Geske (1979) and Galai and Masulis (1976). Following Geske (1979), consider a levered firm for which the debt corresponds to pure discount bonds maturing in T years with a face value M. Under the standard assumptions of liquidating the firm in T years, paying off the bondholders and giving the residual value (if any) to stockholders, the bondholders have given the stockholders the option to buy back the assets at the debt maturity date. In this context, a call on the firm's stock is a compound option, $C(S,t) = f(g(V,t),t)$ where t stands for the current time. If we assume that the return on the firm follows a given diffusion process, then changes in the value of the call can be given as a function of changes in the firm's value and time. The valuation of options in this context is standard since a risk-less hedge can be constructed by choosing an appropriate mixture of the firm and call options on the firm's stock. This is possible under the standard assumptions of perfect and competitive markets, continuous trading, the absence of short sales constraints when the risk-less interest rate and the information costs are known and constant over time. Using the standard dynamics, the return on the firm's assets follows the stochastic differential equation :

$$dV/V = \alpha_V dt + \sigma_V dz_V$$

where α_V and σ_V refer to the instantaneous rate of return and the standard deviation of the return of the firm per unit time, and dZ_V is a Brownian motion. Using the definition of the call $C(V,t)$, its return can be described by the following differential equation :

$$dC/C = \alpha_C dt + \sigma_C dz_C$$

where α_c and σ_c refer to the instantaneous rate of return and the standard deviation of the return on the call per unit time, and dz_c is a Brownian motion. Using Ito's lemma as before, the dynamics of the call can be expressed as:

$$dC = 1/2 C_{VV} \sigma^2_V V^2 dt + C_V dV + C_t dt$$

As we have shown above, it is possible to create a riskless hedge with two securities, in this case, between the firm and a call to get the following partial differential equation:

$$1/2 \sigma^2 V^2 C_{VV} + (r + \lambda V) V C_V - (r + \lambda C) C + C_t = 0 \qquad (4)$$

where λ_v in an information cost relative to the firm's value. At the option's maturity date, $t = T_0$, the value of the call option on the firm's stock must satisfy the following condition:

$$C_{T0} = \max [S_{T0} - K, 0]$$

where K stands for the strike price. Since the stock is viewed as an option on the value of the firm, recall the value of S_{T0}, (see formula (3)):

$$S(V_{T0},T_0)= V_{T_0}e^{-(\lambda_s-\lambda_v)(T-T_0)}N_1(d_1)-Me^{-(r+\lambda s)(T-T_0)}N_1(d_2)$$

with: $d_1 = [\ln(V/M)+(r+1/2\sigma^2+\lambda_v)(T-T_0)]/\sigma\sqrt{T-T_0}$ and $d_2 = d_1-\sigma\sqrt{(T-T_0)}$.

It is convenient to note that equation (4) is more difficult to solve than equation (2) because its boundary condition is a function of the solution to equation (2).

At date T_0, the value of the firm making the holder of the call on the stock indifferent with regard to the exercise decision is solution to the equation: $S_{T0} - K= 0$ where S_{T0} is given by equation (5). Following the methodology in Geske (1979) and using the partial differential equations (2) and (4) with their boundary conditions, the compound call option value with information costs is:

$$C_0 = V_0e^{-(\lambda_c-\lambda_v)T_0}e^{-(\lambda_s-\lambda_v)(T-T_0)}N_2\left(h+\sigma_V\sqrt{T_0},K+\sigma_V\sqrt{T},\sqrt{T_0/T}\right) \quad (6)$$
$$-Me^{-(r+\lambda_s)T}e^{-(\lambda_c+\lambda_s)T_0}N_2\left(h,k,\sqrt{T_0/T}\right)-Ke^{-(r+\lambda c)T_0}N_1(h)$$

with: $h = [\ln(V_0\big/\overline{V})+(r+\lambda_v-1/2\sigma^2V)T_0]/\sigma_v\sqrt{T_0}$ and

$\quad k = [\ln(V_0\big/M)+(r+\lambda_v-1/2\sigma^2V)T]/\sigma_v\sqrt{T}$

The value $\overline{V}$ is determined by the following equation:

$$S_{T0} - K = \overline{V}e^{(-\lambda_s-\lambda_v)(T-T_{0)}}N_1\left(K(\overline{V})+\sigma_V\sqrt{T-T_0}\right)$$
$$Me^{-(r+\lambda_s)(T-T_{0)}}N_1\left(K(\overline{V})\right)-K = 0$$

where: $K(\overline{V}) = [\ln(\overline{V}/M)+(r+\lambda V-1/2\sigma_V^2 T)]/\sigma_v\sqrt{T}$ and $N_2\left(x,y,\sqrt{T_0/T}\right)$ is the bivariate cumulative normal distribution with upper integral limits x and y and $\sqrt{T_0/T}$ is the correlation coefficient. This formula reveals the presence of information costs on the stock, on the firm's assets and on the call. Again, these costs appear as discounting factors, which bring to the present the unknown cash flows. In the absence of information costs, this formula reduces to that in Geske (1979). It is possible to derive the same result using the martingale approach. Moreover, using convergence arguments, we can justify the model in a continuous time setting from the discrete one. The reader can refer to appendix 1 for a justification of the approach in the presence of information costs. For a derivation of the value of the compound option in the martingale approach, see appendix 2.

A first special case is obtained when information costs regarding the call are equal to information costs for the stocks, i.e. $\lambda_c = \lambda_s$. In this case, the investors suffer sunk costs to get informed about the equity and the assets of the firm. The costs

regarding the equity and the firm's cash flows reflect the agency costs and the asymmetric information costs. In this situation, the formula is given by:

$$C_0 = V_0 e^{-(\lambda_c - \lambda_v)T} N_2\left(h + \sigma_V\sqrt{T_0}, K + \sigma_V\sqrt{T}, \sqrt{T_0/T}\right) \tag{7}$$
$$- Me^{-(r+\lambda c)T} N_2\left(h, k, \sqrt{T_0/T}\right) - Ke^{-(r+\lambda c)T_0} N_1(h)$$

The value $\overline{V}$ is determined by the following equation:

$$S_{T0} - K = \overline{V}e^{-(\lambda_c - \lambda_v)(T - T_0)} N_1(k + \sigma_V\sqrt{T - T_0}) - Me^{-(r+\lambda c)(T - T_0)} N_1(K) - K = 0$$

A second special case of the compound option formula is obtained when the incurred information costs are equal to a same value λ for the stock, the firm's value and the call. In this context, the compound option formula
becomes :

$$C_0 = V_0 N_2\left(h + \sigma_V\sqrt{T_0}, K + \sigma_V\sqrt{T}, \sqrt{T_0/T}\right)$$
$$- Me^{-(r+\lambda)T} N_2\left(h, k, \sqrt{T_0/T}\right) - Ke^{-(r+\lambda)T_0} N_1(h)$$

where the value $\overline{V}$ is determined from the following equation:

$$S_{T0} - K = \overline{V}N_1\left(K + \sigma_V\sqrt{T - T_0}\right) - M\ e^{-(r+\lambda)(T - T_0)} N_1(K) - K = 0$$

If the information cost is zero, this compound option pricing formula becomes the one in Geske [1979].

IV.   COMPARISONS TO STANDARD MODELS

Using the above formula, it is easy to see that the value of the call is higher when the value of the firm rises in the absence of information cost. A similar result is obtained when all the information costs are of equal magnitude λ. In this case, we have:

$$C_v = N_2\left(h + \sigma_V\sqrt{\tau_1}, K + \sigma_V\sqrt{T}, \sqrt{T_0/T}\right) = N_2 > 0$$

This result shows that the call value is an increasing function of the value of the firm in the presence of information costs. It is also easy to see that the call value falls when the face value of debt increases:

$$C_M = e^{-(r+\lambda)T} N_2\left(h, k, \sqrt{T_0/T}\right) < 0$$

However, the decrease in the call's value is greater than in standard models as in Geske. In fact, since the discounting factor is greater in the presence of information costs, the call value is less important. When the variance rate increases, the call price rises:

$$C_{\sigma_v^2} = N_2(.)/N_1 \left(K + \sigma_V\sqrt{T}\right) Me^{-(r+\lambda)T} N_1'(K)\sqrt{T/2\sigma_V} > 0$$

where $N_1'(K)$ stands for the normal density function. As the call's exercice price rises, the call price falls:

$$C_k = -e^{-(r+\lambda)T_0} N_1(h)$$

When the risk-less rate increases, the call price rises:

$$C_r = N_2(.)/N_1 \left(K + \sigma_V\sqrt{T}\right) MTe^{-(r+\lambda)T} N_1(K) > 0$$

The same result applies in the presence of information costs where the increase is less than that in Geske's model since the discounting factor is greater. When the information uncertainty increases, the call price rises:

$$C_\lambda = N_2(.)/N_1 \left(K + \sigma_V\sqrt{T}\right) MTe^{-(r+\lambda)T} N_1(K) > 0$$

Since the information costs affect the riskiness of the stock because of incomplete information, they affect the value of the stock and lead to higher option values. The compound option formula takes into account the effects of the capital structure and information costs on the value of the call. When the face value of the debt is zero, i.e. if the option is written on the equity of an unlevered firm, the compound option formula becomes the standard Black and Scholes equation with information costs. When the stock price changes, this affects the debt/equity ratio and the riskiness of the stock. Recall that the instantaneous volatility of the stock is given by:

$$\sigma_S(V,t) = \left(S_V V/S\right)\sigma_V$$

where $\left(S_V V/S\right)$ is the elasticity of the stock with respect to the firm's value.

In the compound option model, the variance of the stock is inversely associated with the stock price. When the stock price falls (rises), the debt/equity ratio falls (rises). The decreased (increased) risk is then reflected by a fall (rise) in the stock's variance. The partial derivative of the stock's standard deviation with respect to the stock price is:

$$\partial\sigma_s/\partial S = -(V/S^2)\partial S/\partial V \sigma_V = -(V/S^2)N_1(K + \sigma_v\sqrt{T})\sigma_V < 0$$

When the debt/equity ratio changes result mainly from changes in the stock price, then the percentage changes in the stock's return will be greater if prices have fallen than when they have risen. Since the option value rises when the volatility is higher, if the stock price has risen (fallen), the decreased (increased) variance of the stock will act to lower (raise) the value of the option on the stock. In this setting, changes in the capital structure induced by variations in the value of the firm are transmitted by the stock's variance to affect the price of the option on the stock. The changes in the firm's value are based on the expectations of the market regarding the prospective cash flows of the firm taking into account the signaling costs and the asymmetric information via the information costs on the firm and its assets. In this model, the stock's variance is a function of all the variables that determine the stock price including information costs and the debt's maturity date. Since the reduction in time to maturity of the option on the stock, reduces the option value, the value of the stock as an option on the value of the firm decreases as well. The decrease in S leads to a higher debt/equity ratio. This increases the riskiness on the firm's stock. The resulting higher variance of the stock will lead to a higher option price for the option on the stock. The effects of the leverage and information costs are clear via the compound option pricing model. In the Black-Scholes model, the option is a function of five parameters: $C_{BS} = f(S, \sigma_s, r, T_0, K)$. In the simple above model, the option has besides two degrees of freedom corresponding to information costs on the option and its underlying asset, $C_{BJ} = f(S, \sigma_s, r, T_0, \lambda_s, \lambda_c, K)$.

The two extra variables are necessary to capture the main messages in the theories of asymmetric information and signaling. The values of the information costs can be estimated using a similar procedure as that in Merton (1987) or an implicit estimation as the one proposed in Bellalah (1999). The compound option model contains four extra parameters with respect to the standard Black and Scholes model to capture the effects of information costs and the effects of leverage via the face value of debt, its maturity and the discounting factors. The value of the compound option is $C_{BJ} = g(S, \sigma_v, r, T_0, T, \lambda_s, \lambda_c, K, M)$. Three of these variables are observable and the others can be computed. The information costs can be estimated as described above. The value of the firm can be computed from the market values of equity and the outstanding debt, $V = S + B$. The face value of debt and its maturity are given by the balance sheet. The following tables present the simulation results of the above models. Table 1 gives the call equity values using the Black and Scholes model and our model. The following parameters are used for the simulations: M = 100, r = 0.08, T = 0.25, σ_V = 0.4. The following information costs are used to generate option values: ($\lambda_s = \lambda_v = 0$) which correspond to the Black and Scholes case, and (λ_s= 0.1%, λ_v= 1 %), (λ_s= 0 %, λ_v= 1 %) and (λ_s= 0.1 %, λ_v = 3 %). The results show that in all cases our option values are less than those reported for the Black and Scholes model. The difference between the two models depends on the magnitude of information costs. Since the Black and Scholes model overvalues call option prices, our model reduces the amount of this mispricing bias.

Table 1
Simulation and comparison of the Black and Scholes model and our model for the
values of European equity options

The following parameters are used:

V	$\lambda_s = \lambda_v = 0$	$\lambda_s = 0.1\%, \lambda_v = 1\%$	$\lambda_s = 0\%, \lambda_v = 1\%$	$\lambda_s = 0.1\%, \lambda_v = 3\%$
70	0.000	0.000	0.000	0.000
80	0.9800	0.9829	0.9775	0.9780
90	4.1206	4.1189	4.1103	4.0984
100	8.9163	8.9085	8.8941	8.8641
110	15.6302	15.6119	15.5912	15.5340
120	23.8003	23.7660	23.7409	23.6489

Table 2
Simulation and comparisons of European equity values as compound options in the
presence of information costs using Geske's model and our model.
The following parameters are used:
K = 20, M = 100, r = 0.08, T =0.25, T_0= 0.125, σ_V = 0.4

V_0	$\lambda_c = 0\%$ $\lambda_s = 0\%$ $\lambda_v = 0\%$	$\lambda_c = 2\%$ $\lambda_s = 2\%$ $\lambda_v = 2\%$	$\lambda_c = 1\%$ $\lambda_s = 1\%$ $\lambda_v = 2\%$	$\lambda_c = 1\%$ $\lambda_s = 2\%$ $\lambda_v = 2\%$
110	6,82	7,13	7,16	7,14
120	15,17	15,65	15,70	15,67
130	26,52	27,16	27,25	27,20

Table 2 gives the simulation results for the compound option formula with
information costs and the Geske's compound call formula for the following parameters:
K = 20 , M = 100 , r = 0.08 , T =0.25 , T_0= 0.125 , σ_V = 0.4 . The parameters used
for information costs in Table 2 are:

case a : ($\lambda_c = \lambda_s = \lambda_v = 0\%$),
case b : ($\lambda_c = \lambda_s = \lambda_v = 2\%$),
case c : ($\lambda_c = \lambda_s = 1\%, \lambda_v = 2$),
case d : ($\lambda_c = 1\%, \lambda_s = \lambda_v = 2\%$).

In case (a), we have exactly the same values as those generated by the formula in
Geske [1979]. This case is a benchmark for the comparisons of our results. The table
shows that the compound option price is an increasing function of the firm's assets V.
This result is independent of the values attributed to information costs. Note also that
the compound option price is an increasing function of the information costs regarding
the firm's assets, λ_v. When λ_v is fixed, this allows the study of the effects of the other
information costs on the option value. In this case, the option price seems to be a
decreasing function of the two information costs λ_c and λ_S. When comparing cases (b)

and (c) on the one hand and the cases (c) and (d) on the other hand, we observe this decreasing feature. We intend to test this model on real data.

V.   SUMMARY

Information costs play a central role in the analysis and the valuation of financial assets, firms and their cash flows. The literature on the valuation of financial claims in the presence of information uncertainty has been formalized by Merton (1987) in his simple model of capital market equilibrium with incomplete information. In that context, information costs include the costs of gathering and processing data and the costs of information transmission from an economic agent to another. The foundations of these costs are linked to the agency theory, the signalling models and the differential information models. These costs affect the cash flows of the firm and its valuation.

Using the notion of information costs, we present an arbitrage argument to derive an option pricing formula within a context of information uncertainty. The formula collapses to that in Black and Scholes in the absence of these costs. By analogy between the option theory and the assets in the capital structure of the firm, the formula can be used to value equity in a levered firm. By considering the call on the firm's stock as a compound option, we develop two alternative approaches for the valuation of compound options in the presence of leverage effects and information costs. The first approach parallels the derivation in Geske [1979]. The second approach uses martingale methods like in Harrison and Kreps [1979]. Convergence arguments from discrete to continuous time models can be applied to give the values of the compound option.

The compound option formulas derived in this paper shed light on the effects of leverage and information uncertainty in the valuation of corporate assets. Since several corporate liabilities can be valued with the compound option approach, our results are useful in the pricing of the capital structure of the firm. Our model can be calibrated to market data. It can be used to estimate implicitly information costs and indirectly, to quantify agency costs and their effects on corporate claims.

The valuation of equity and the firm's assets within information uncertainty is of interest mainly for start up firms, firms facing technological innovations, Internet stocks and firms for which "information" is a main feature in the computation of its value. Our simple context can have different applications and can provide several directions of research in corporate finance and financial markets.   In fact, it is possible to extend our analysis using an appropriate database for the empirical estimation of information costs and firm values. This can be done using the methodologies in Merton (1987), Kadlec and McConnell (1994) and Bellalah (1990, 1999). The purpose of this paper is not the estimation of these costs but the study of the possibility to extend the standard valuation techniques to account for the effects of incomplete information. Our results can offer some new reflexions about the computation of the value of equity and the firm's assets in the presence of information uncertainty. The analysis can be used in the valuation of internet stocks and in the study of investments under uncertainty in the presence of incomplete information as in Bellalah (2 001).

Appendix 1
An alternative derivation of the value of equity as a standard option using the
martingale approach

We introduce a second approach to derive the option formula in the presence of
information uncertainty. Options prices can be calculated using the risk neutral
probability Q defined in the seminal papers of Harrison and Kreps [1979] and Harrison
and Pliska [1981]. The relation is:

$$S_{T_0} = e^{-r(T-T_0)} E_Q[V_T | F_{T0}]$$

where F_{T0} is the available information at time T_0. Following this approach, it is possible
to derive such relations in the presence of information costs. Consider a trinomial
discrete model with a riskless asset denoted by B, a risky asset V and an option S on V.
We assume that there are two dates 0 and 1.

Denote by u, m and d the three random returns of V and by $\tilde{u}$, $\tilde{m}$ and $\tilde{d}$, the
three returns of S. To preclude arbitrage, we must impose an additional condition. For
all possible strategies ($\theta^B, \theta^V, \theta^S$) of investment at time 0:

$$\theta^B(1+r)B_0 + \theta^v uV_0 + \theta^s \tilde{u}S_0 \geq 0$$
$$\theta^B(1+r)B_0 + \theta^v mV_0 + \theta^s \tilde{m}S_0 \geq 0$$
$$\theta^B(1+r)B_0 + \theta^v dV_0 + \theta^s \tilde{d}S_0 \geq 0$$

then necessarily $\theta^B B_0 + \theta^V(1+\lambda_v)V_0 + \theta^S(1+\lambda_v)S_0 \geq 0$. From Farkas' lemma, we
know that it implies the existence of three non-negative constants q_u, q_m and q_d such
that:

$$q_u + q_m + q_d = 1$$
$$q_u u + q_m m + q_d d = (1+\lambda_v)(1+r)$$
$$q_u \tilde{u} + q_m \tilde{m} + q_d \tilde{d} = (1+\lambda_s)(1+r)$$

These coefficients allow to define a new probability that we denote also Q, depending
on the riskless rate r and the information costs λ_V and λ_s. Under this probability, we
get the following properties, which extend the notion of risk-neutral probability to the
case of information costs:

$$E_Q[V_1/V_0] = (1+\lambda_V)(1+r) \text{ and } E_Q[S_1/S_0] = (1+\lambda_s)(1+r)$$

Now, if we extend in the same manner the standard multiperiod model of Cox, Ross and Rubinstein (1979) by discretizing the information costs like the riskless rate (λ_s is divided by n if the periods are equal to 1/n) then, we get analoguous conditions for the limit continuous time model. There exists a probability Q such that $\left(e^{-(r+\lambda_v)t}V_t\right)t$ and $\left(e^{-(r+\lambda_s)t}S_t\right)t$ are martingales under Q. Then, under the probability Q, we have:

$$dV/V = [r + \lambda_V]dt + \sigma_v dz$$

where z is a brownian motion under Q. Since at time T, $S_T = \max[V_T - M, 0]$ then

$$S_t = e^{-(r+\lambda_s)(T-t)}E_Q\left[\max[V_T - M, 0]F_t\right.$$

So we deduce:

$$S_t = e^{-(r+\lambda_s)(T-t)}\left[e^{-(r+\lambda_v)}E_Q[V_T I_{V_{T \geq M}}|F_t] - Me^{-(r+\lambda_v)}Q[V_T \geq M|F_t]]\right.$$

Therefore, S_t is equal to the standard Black and Scholes price with a new riskless rate equal to $(r + \lambda_V)$ multiplied by the discount factor $S_t = e^{-(\lambda_s - \lambda_v)(T-t)}$. So we immediately deduce the relations (3) and (5).

Appendix 2
An alternative derivation of the compound option's formula using the martingale approach

Using similar arguments as before, we obtain:

$$C_0 = e^{-(r+\lambda_c)T_0}E_Q\left[C_{T0}\right]$$

But

$$V_{T0} = V_0 e^{\sigma\sqrt{T_0}\xi + (r+\lambda_v - 1/2\sigma^2)T_0}$$

where the distribution of ξ_i is the standard Gaussian law.

Letting $g(x) = V_0 e^{\sigma\sqrt{T0}x + (r+\lambda_v - 1/2\sigma^2)T_0}$, then:

$$C_0 = e^{-(r+\lambda_c)T0}\left[\int_R \left(e^{-(\lambda_s - \lambda_v)(T-T0)}g(x)N(d_1(x))\right) - Me^{-(r+\lambda_s)(T-T0)}N(d_2(x)) - K)e^{-x^2/2}\middle/\sqrt{2\pi}\right]dx$$

with:

$$d_1(x) = [\ln(V_0/M) + \sigma\sqrt{T_0}\,x + (r + \lambda_v)T + 1/2\sigma^2 T - \sigma^2 T_0]^{\sigma\sqrt{(T-T0)}} \text{ and}$$

$$d_2(x) = d_1(x) - \sigma\sqrt{(T - T_0)}\ .$$

Then by applying standard integral calculation (change of variables for example), we deduce the result. Details about calculations are available on request.

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