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Foreign Exchange Market Efficiency: Fractional Cointegration Approach

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ABSTRACT

In the present study, we investigate the market weak efficiency hypothesis (MEH) in the case of the Tunisian exchange market. For this aim, we use fractional cointegration tests based essentially on estimation of an error correction bivariate ARFIMA model.

The cointegration tests are conducted using spot and 1- month forward daily exchange rate of the Tunisian Dinar (TND) vis-à-vis the US dollar (USD), the Euro and the Japanese Yen (JPY) during the period between January 1999 and December 2003.

For this, an error correction bivariate ARFIMA model (VECFM) was estimated. The results indicate evidence of fractional cointegration between the one-month forward rate and the spot rate relative to these parities (TND/USD) and (TND/Euro).

JEL: F30, F31, G14

Keywords: *Informational efficiency; Exchange rates; Standard cointegration tests; Fractional integration; Long memory; Bivariate ARFIMA model; Vectoriel error correction fractional model (VECFM)*

I. INTRODUCTION

The informational efficiency of the foreign exchange market is a subject that has been a topic of renewed attention by empirical and theoretical analysis in the field of international finance. One of the reasons underlying the regeneration of interest in testing the efficient markets hypothesis (EMH) within the framework of the foreign exchange market lies in the importance of the role of this market in the determination of exchange rates particularly in a world economy characterized by an increasing integration. Consequently, the determination of the exchange rates is not simple, considering that they are today sensitive to any event affecting the domestic as well as the international markets. A second reason relates to the development of the practical and sophisticated time-series techniques, in particular those relating to the theory of cointegration often used to test the efficiency of the foreign exchange market.

The objective of this study is to investigate the weak efficiency (Fama (1984, 1998) of the Tunisian foreign exchange market by means of fractional cointegration tests and estimation of an error correction bivariate ARFIMA model.

This paper is organized as follows: we proceed initially by the presentation of some concepts relating to the theory of cointegration. A particular interest will be granted to the concept of fractional cointegration. For better clarifying this concept, we start with a presentation of the characteristics of ARFIMA processes before analyzing the interest of the taking into account of fractional integration by the tests of cointegration. A third section of this paper will be devoted to report the empirical investigations.

II. THEORETICAL CONCEPTS

A. The Cointegration

The concept of cointegration is viewed as a long term equilibrium relationship which can be defined between nonstationary variables. This theory allows, in fact, specifying stable relations in the long run while jointly analyzing the short term dynamics of the considered variables.

A definition of the concept of cointegration was presented by Granger (1981) and Granger and Weiss (1983) such as:

The components of a vector X_t are said to be co-integrated of order, d, b, denoted $X_t \sim CI(d,b)$ if : (1) Each series of X_t is integrated of order *d* noted $x_{jt} \approx I(d)$, and (2) There is a set of constants β_j such as: $z_t = \sum_j \beta_j x_{jt} \sim I(d_z)$ with $d_z = d - b$ and $d_z < d$. β is called the co-integrating vector of dimension $r \times 1$. Thus, the idea of cointegration is that the linear combination z_t has a lower order of integration than its components.

Usually, in standard cointegration tests, studied series are considered to be nonstationnary in levels ($I(1)$), and the cointegrating linear combination is $I(0)$. But this distinction between process $I(1)$ or $I(0)$ is arbitrary, since the conditions of cointegration stipulate only that the equilibrium error z_t must be stationary. The cointegration also applies when the series are I (d) with d fractional. Dueker and Startz (1998) propose a definition of the fractional cointegration in which the initial series of X_t can also be I(d) with $0 < d < 1$, the relation of cointegration being $I(d-b)$ with $0 < d-b < d < 1$. However, if $\frac{1}{2} < d-b < 1$, the relation of cointegration will not be stationary and the long-term equilibrium is never reached even if the process is "mean-reverting".

B. Fractional Integration and the Cointegration

The Autoregressive Fractionally-Integrated Moving Average (ARFIMA) model of Granger and Joyeux (1980) and Hosking (1981) is an extension of the ARIMA models of Box Jenkins. A stationary process $\{X_t\}$ follows an ARFIMA (p,d,q) is written in this form:

$$\Phi(L)(1-L)^d X_t = \theta(L)\,\varepsilon_t \tag{1}$$

$(1-L)^d$ indicates the binomial development and can be written in the following form:

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)}{\Gamma(d)\Gamma(k+1)} \tag{2}$$

where d is the differencing operator and can take on integer and non-integer values; $\Phi(L)$ et $\theta(L)$ refer to a finite polynomial in the lag operator L; and ε_t is a white-noise disturbance term. Processes ARFIMA (p,d,q) are with long memory when $d \in \left] -\frac{1}{2}, \frac{1}{2} \right[$ and $d \neq 0$. They are invertible if $d > -\frac{1}{2}$ and stationary if $d < \frac{1}{2}$.

When the equilibrium residuals are fractionally integrated and mean-reverting, the studied series are fractionally cointegrated (Marmol 1998; Olekalns and Wilkins 1998). Generally, the study of the stationarity of the series is done via the standard tests of stationarity like that of Dickey-Fuller (1979,1981), of Phillips-Perron (1988) or that of Kwiatkowski and al. (KPSS) (1992). Diebold and Rudbusch (1991) examined by methods of simulation the power of the standard tests of unit root when the true generating process data is a white noise fractionally integrated. They concluded that the tests tend to exhibit low power against fractional alternatives.

III. EMPIRICAL STUDY

We examine Tunisian exchange market efficiency with respect to 3 bilateral exchange rates of the Tunisian Dinar.

A. Efficiency of the Tunisian Spot Market

Daily exchange rates of Tunisian Dinar against the American Dollar, the Euro and the Japanese Yen (TND/USD, TND/euro, and TND/JPY) are used for the period 04 January 1999 to 31 December 2003 (1242 observations).

1. Statistical properties of exchange rate series

a. Results of the tests of stationarity

The results of three types of unit root tests realized on the series of logarithm of exchange rates are given in Table 1.

Table 1

Results of the tests of stationarity on the series of logarithm of exchange rates

		$H_0: I(1)$				$H_0: I(0)$
	lags	τ	τ_μ	$Z(t_\alpha)$	$Z(t_{\alpha\bullet})$	$\hat{\eta}_\mu$
LUSD	1	-0.026***	-2.233***	-0.004***	-2.236***	21.239***
	4	-0.047***	-2.110***	-0.016***	-2.234***	8.543***
	8	-0.080***	-2.094***	-0.033***	-2.234***	4.778***
	10	-0.069***	-2.056***	-0.035***	-2.234***	3.922***
	15	-0.0856***	-2.077***	-0.045***	-2.234***	2.720***
	24	-0.152***	-1.911***	-0.046***	-2.234***	1.768***
LEUR	1	2.265***	1.413***	2.185***	1.325***	50.809***
	4	2.401***	1.564***	2.325***	1.494***	20.407***
	8	2.553***	1.785***	2.413***	1.606***	11.393***
	10	2.670***	1.894***	2.467***	1.675***	9.344***
	15	2.628***	1.788***	2.540***	1.774***	6.463***
	24	2.340***	1.470***	2.485***	1.740***	4.179***
LYEN	1	-0.763***	-2.378***	-0.766***	-2.372***	9.552***
	4	-0.710***	-2.107***	-0.786***	-2.354***	3.845***
	8	-0.703***	-2.092***	-0.794***	-2.349***	2.152***
	10	-0.752***	-2.247***	-0.797***	-2.347***	1.767***
	15	-0.761***	-2.283***	-0.800***	-2.347***	1.225***
	24	-0.678***	-2.269***	-0.801***	-2.351***	0.797***

Notes: τ (resp. $Z(t_\alpha)$) and τ_μ (resp. $Z(t_{\alpha\bullet})$) are the statistics of ADF (resp. PP) tests for the models without and with constant. The breaking values for tests ADF and PP are those tabulated by Mckinnon (1991). For the first model, they are –2.56 and –1.94 respectively to 1% and 5%. Those for the second are –3.43 and –2.86 respectively to 1% and 5%. $\hat{\eta}_\mu$ is the statistics of test KPSS where the residues result from the regression with a constant. The critical values are 0.739, 0.463 and 0.347 respectively at 1%, 5% and 10% level. ***, ** and * indicates significance respectively at 1%, 5% and 10% level.

If we refer to the statistics of tests ADF and PP, we will accept without ambiguity the null of unit root whatever the model retained (with or without constant) at a level of significance of 1%. This result is confirmed by KPSS test, where we reject the null of stationarity, for all the series, and at a level of significance of 1%. Thus, all series are *I*(1).

In addition, the results of the application of these tests of unit root on the series of variation of exchange rates (r_t) are given in the following table.

Table 2
Results of the stationarity tests on the variations of the exchange rates

		$H_0: I(1)$				$H_0: I(0)$
	lags	τ	τ_μ	$Z(t_\alpha)$	$Z(t_{\alpha\bullet})$	$\hat{\eta}_\mu$
DLUSD	1	-24.260***	-24.261***	-32.903***	-32.899***	0.996***
	4	-15.012***	-15.015***	-32.905***	-32.900***	0.956***
	8	-11.418***	-11.427***	-32.975***	-32.969***	0.899***
	10	-9.791***	-9.798***	-32.979***	-32.972***	0.895***
	15	-8.609***	-8.617***	-33.041***	-33.031***	0.863***
	24	-6.362***	-6.363***	-33.122***	-33.107***	0.830***
DLEUR	1	-27.662***	-27.792***	-37.393***	-37.495***	0.570**
	4	-16.275***	-16.450***	-37.475***	-37.598***	0.633**
	8	-12.536***	-12.765***	-37.536***	-37.698***	0.672**
	10	-11.436***	-11.712***	-37.591***	-37.782***	0.697**
	15	-8.380***	-8.701***	-37.635***	-37.892***	0.728**
	24	-9.520***	-6.262***	-37.466***	-37.744***	0.692**
DLYEN	1	-25.662***	-25.665***	-34.990***	-34.992***	0.259
	4	-16.441***	-16.451***	-34.999***	-35.000***	0.273
	8	-11.871***	-11.891***	-35.009***	-35.011***	0.279
	10	-10.516***	-10.535***	-35.015***	-35.017***	0.282
	15	-8.784***	-8.813***	-35.020***	-35.025***	0.284
	24	-6.285***	-6.312***	-35.018***	-35.026***	0.285

Referring to the statistics of tests ADF and PP, we can reject the null of nonstationarity at a significance of 1% level. However, while referring to the statistics of KPSS test, the null of stationarity is also rejected in the case of TND/USD and TND/euro series and this respectively at significance level of 1% and 5%. This situation suggests that these series are fractionally integrated. As for TND/JPY serie, we can say that it is stationary I(0).

b. Conditional Heteroscedasticity of the variations of rate of exchange

The statistics of Box-Pierce and the multiplier of Lagrange tests are highly significant (1%), which allows us to reject the null of absence of ARCH effect.

	DLUSD	DLEUR	DLYEN
$Q_{\varepsilon^2}(50)$	95.0584	67.778	77.108
	[0.000]***	[0.000]***	[0.000]***
Lm-arch	106.128	98.528	100.662
	[0.000]***	[0.000]***	[0.000]***

c. Test of normality of the exchange rate variations

The coefficients of asymmetry are 1% significantly different from zero only for the Yen variation series. There exists an excess of Kurtosis for the three series. In particular the excess of Kurtosis is significantly higher than zero, which means that the nonconditional density function of the series is leptokurtic.

	DLUSD	DLEUR	DLYEN
Skewness $(\hat{\tau}_1)$	-0.0394	0.097	0.406
t statistics	(0.566)	(1.395)	(5.847)***
Kurtosis $(\hat{\tau}_2)$	3.544	3.870	8.312
t statistics	(3.915)***	(6.270)***	(38.272)***
Jacques-Bera	$\chi^2(2)$= 15.588	$\chi^2(2)$= 41.092	$\chi^2(2)$= 1492.96
	[0.000]***	[0.000]***	[0.000]***

2. Results of the Cointegration tests

a. Johansen cointegration test results

The trace statistic suggests that one – cointegrating vector is present, which implies that there is a long-run relationship between all three bilateral exchange rates (see Table 3). The existence of such relation of cointegration between these spot rates can be explained, for example by the presence of a variable risk premium (see Barkoulas and al. (2003)), foreign market interventions or transaction costs.

Table 3
Results for the Johansen (1988) test
Series: LDOLLAR LEURO LYEN
Orders of delay: 1 to 4

Tendency:	None	None	Linear	Linear	Quadratic
Model	Without constant	With constant	With constant	With constant	With constant
	Without tendency	Without tendency	Without tendency	With tendency	With tendency
cointegration relation(s) Number (level of 5%)	1	0	0	0	0

The following table gives the results of the identification of the cointegration relation.

Table 4
Estimation of cointegration relation

Equation of cointegration	Log of the M.L.	15263.61
Standardized coefficients of cointegration:		
LDOLLAR	LEURO	LYEN
1.000000	1.381896	0.030095
	(0.56799)	(0.39333)
Adjusted coefficients:		
D(LDOLLAR)	D(LEURO)	D(LYEN)
-0.004174	0.001828	-0.002230
(0.00127)	(0.00068)	(0.00189)

The results of the VECM estimation are presented in Table 5. By analyzing the estimated coefficients of the lagged residuals one period, we notice that for the dollar and the Euro, this coefficient is negative and significantly different from zero. Thus, these parities are characterized by a return towards the long term target.

Thus, the presence of cointegration between series of exchange rates implies the existence of Granger-causal orderings among cointegrated exchange rates. Hence, it is possible to predict one exchange rate given observations of the other exchange rates (Granger (1986)). Thus, the exchange market agents can carry out beneficial transactions in the long run as well in the short run, Which contradicts the weak informational efficiency hypothesis and can lead to the inefficience of the Tunisian spot exchange market.

Table 5
Estimate of the VECM

	Equation of Cointegration		
Ldollar(-1)	1.000000		
Leuro(-1)	3.156743		
	(2.98527)		
	[1.05744]		
Lyen(-1)	0.391940		
	(0.18915)		
	[2.07211]		
	D(LDOLLAR)	D(LEURO)	D(LYEN)
Eq Cointegration	-0.000366	-0.000174	-0.000371
	(0.00022)	(0.00012)	(0.00032)
	[- 1.67927]	[- 1.47833]	[- 1.14196]
D(ldollar(-1))	0.030341	-0.021048	0.152221
	(0.03668)	(0.01978)	(0.05470)
	[0.82714]	[- 1.06426]	[2.78305]
D(ldollar(-2))	-0.010123	-0.027222	-0.008397
	(0.03648)	(0.01967)	(0.05439)
	[- 0.27749]	[- 1.38410]	[- 0.15437]
D(leuro(-1))	-0.107276	-0.093328	-0.051597
	(0.06708)	(0.03617)	(0.10003)
	[- 1.59911]	[- 2.58039]	[- 0.51584]
D(leuro(-2))	-0.000753	-0.111100	-0.233599
	(0.06691)	(0.03608)	(0.09977)
	[- 0.01125]	[- 3.07957]	[- 2.34128]
D(lyen(-1))	-0.014507	-0.009294	-0.019714
	(0.01926)	(0.01038)	(0.02872)
	[- 0.75325]	[- 0.89508]	[- 0.68648]
D(lyen(-2))	0.007939	0.011810	-0.041291
	(0.01917)	(0.01034)	(0.02859)
	[0.41407]	[1.14242]	[- 1.44429]

Table 6
Integration orders of spot exchange rate series

	LUSD	LEUR	LYEN
$\hat{d}'_{ML}$	1.0458	0.9906	0.9985
t- Student	57.11	185.18	707.67
Prob	[0]	[0]	[0]
$\hat{d}'_{GPH}$	1.0430	1.0068	1.0079
t- Student	54.40	132	688
Prob	[0]	[0]	[0]

b. Standard fractional cointegration test results

The estimates of the order of integration using of Maximum Likelihood (see Lyhagen (1999)) and GPH (see Geweke and Porter – Hudak (1983)) tests are all insignificantly different from one. Thus, we can say that it is not possible to have a fractional cointegration between these series only when the cointegrating residuals are fractionally integrated (0 <d'<1) (see for example Baillie and Bollerslev (1994)).

The series of the residuals are stationary I(0) since they seem to fluctuate in a random way around zero. We can also note periods of strong volatility, which is primarily with ARCH effects. Indeed, we note that the autocorrelations decrease at an exponential rate when lags increase.

The following table sums up results of long memory tests on the residuals:

Table 7
Estimate of the orders of integration of the series of the residues

	R USD	R EUR	R YEN
$\hat{d}'_{ML}$	0.0100	0.0077	-0.0126
T Student	0.462	0.356	-0.561
Prob	[0.644]	[0.722]	[0.575]
$\hat{d}'_{GPH}$	0.0095	0.0041	-0.0132
T Student	0.438	0.195	-0.586
Prob	[0.661]	[0.846]	[0.558]

The estimated values of the integration orders of residuals are no significantly different from zero. Therefore residuals are I(0). Consequently, we can say that there is no fractional cointegration between the three spot exchange rate series.

B. Efficiency of Tunisian Forward Market

In order to test the Tunisian forward market efficiency, we use the Engle-Granger (1987) cointegration test. We initially, apply the standard fractional cointegration test that involves of comparing the order of integration of the residuals (d') from OLS estimates of the cointegrating equation and those of original series (d). It should be noted, that OLS estimates of the cointegrating vector are only consistent if $d > \frac{1}{2}$ (Robinson and Marinucci (1998)). Thus, if it is found that the order of integration of the series is $d \leq \frac{1}{2}$, and then we will apply another procedure of fractional cointegration: The bivariate ARFIMA model.

Cheung and Lai (1993) provide two inequality bounds for the speed of adjustment parameters that comes from fractional cointegrated system with error correction representation of the form:

$$H(B)(1-B)^{d} X_t = -\alpha[(1-(1-B)^{b}](1-B)^{d-b} Y_t + G(B)Z_t \quad (3)$$

In a two-dimensional case, the data generation process relating to the two series, X_t and Y_t, integrated of the same order *d*, is presented by:

$$H(B)\begin{pmatrix}(1-B)^{d} & 0\\ 0 & (1-B)^{(d-b)}\end{pmatrix}\begin{pmatrix}1 & 0\\ -\alpha & 1\end{pmatrix}\begin{bmatrix}X_t\\ Y_t\end{bmatrix} = G(B)\ Z_t \quad (4)$$

In the ARFIMA bivariate model $(0, d, d-b, 0)$ we estimate at the same time, the order of integration (d) of the spot and forward exchange rate series, and that of the cointegrating vector (d') such as $d' = d - b$.

Table 8
Results of stationarity tests on the spot and forward exchange rate series

		$H_0: I(1)$				$H_0: I(0)$
	lags	τ	τ_μ	$Z(t_\alpha)$	$Z(t_{\alpha\bullet})$	$\hat{\eta}_\mu$
LFUSD	1	-0.220	-1.639	-0.073	-2.043	0.981***
	4	-0.211	-1.352	-0.085	-2.049	0.436*
LSUSD	1	-0.225	-1.614	-0.103	-2.071	0.978***
	4	-0.209	-1.316	-0.102	-2.073	0.435*
LFEUR	1	2.531	1.752	2.670	1.754	2.568***
	4	3.162	2.427	2.972	2.360	1.107***
LSEUR	1	2.392	1.530	2.613	1.568	2.583***
	4	3.091	2.289	2.988	2.201	1.113***
LFYEN	1	-0.597	-2.550	-0.690	-2.488	0.455*
	4	-0.559	-2.724	-0.694	-2.489	0.213
LSYEN	1	-0.772	-2.598	-0.791	-2.500	0.457*
	4	-0.580	-2.652	-0.792	-2.505	0.213

1. Empirical results

a. Stationarity test Results

Stationarity tests were initially carried out on the logarithm of the spot and forward exchange rate series reported in Table 8.

- For TND/USD parity: ADF and PP statistics are higher than their critical values. Therefore, we can accept the null of unit root at a significance level of 1%. From KPSS test, we notice that for the two series, we can reject the null of stationnarity at a 1% significance level (for lag 1) and of 10% (for lag 4).
- For TND/euro parity: While referring to the ADF and PP statistics, we accept the null of unit root at significance level of 1% that is confirmed by KPSS test, since we reject the null of stationnarity for the two series at a 1% significance level.
- For TND/JPY parity: From the results of ADF and PP tests, we accept the null of unit root at 1% significance level. Concerning KPSS test, we notice that for a lag of 1 and at 10% significance level, we can reject the null of stationarity.

Thus, we can say that for all parities, the spot and forward exchange rate series share the same statistical characteristics. They are I(1).

Table 9
Results of stationarity tests on the first differences of the spot and forward exchange rate series

		$H_0: I(1)$				$H_0: I(0)$
	lags	τ	τ_μ	$Z(t_\alpha)$	$Z(t_{\alpha\bullet})$	$\hat{\eta}_\mu$
DLFUSD	1	-5.065	-5.008	-6.393	-6.330	0.747***
	4	-3.713	-3.642	-6.347	-6.278	0.723**
DLSUSD	1	-5.029	-4.968	-6.831	-6.762	0.734**
	4	-3.554	-3.476	-6.807	-6.732	0.745***
DLFEUR	1	-4.219	-4.832	-6.993	-7.553	0.600**
	4	-2.622	-3.632	-7.048	-7.550	0.650**
DLSEUR	1	-4.435	-5.057	-6.616	-7.133	0.532**
	4	-2.740	-3.791	-6.642	-7.103	0.614**
DLFYEN	1	-5.729	-5.725	-6.777	-6.759	0.299
	4	-3.256	-3.260	-6.752	-6.728	0.302
DLSYEN	1	-5.560	-5.569	-7.533	-7.537	0.295
	4	-3.286	-3.306	-7.534	-7.538	0.295

- <u>For TND/USD parity:</u> From the results of ADF and PP tests, we can reject the null of unit root at significance level of 1%. That does not seem to be confirmed by KPSS test since the null of stationarity is rejected at significance level of 1 %. Thus, we can say a priori that the two series relating to TND/USD parity are fractionally integrated.
- <u>For TND/Euro parity:</u> Based on ADF and PP tests, we reject the null of unit root for the two series, at significance level of 1%. In addition, from KPSS test, the null of stationarity can also be rejected at significance level of 5% and that for the two series. Consequently, we can suggest that these series are controlled by a long memory process.
- <u>For TND/JPY parity</u>: while referring to ADF and PP statistics, we reject the null of nonstationarity at significance level of 1%. This result is confirmed by KPSS test, where we accept the null of stationarity at significance level of 10%. Consequently, we can say that the spot and forward exchange rate series relating to TND/JPY parity follow a stationary process $I(0)$.

The results of stationarity tests on the residuals are illustrated in Table 10.

Table 10
Results of stationarity tests on the residuals

		$H_0: I(1)$				$H_0: I(0)$
	lags	τ	τ_μ	$Z(t_\alpha)$	$Z(t_{\alpha\bullet})$	$\hat{\eta}_\mu$
D resid	1	-5.222***	-5.174***	-7.709***	-7.644***	0.0538
	4	-2.978***	-2.944***	-7.709***	-7.644***	0.0550
E resid	1	-6.099***	-6.060***	-7.772***	-7.709***	0.172
	4	-3.693***	-3.679***	-7.852***	-7.787***	0.208
Y resid	1	-5.342***	-5.281***	-6.264***	-6.205***	0.391*
	4	-2.719***	-2.665***	-6.174***	-6.111***	0.400*

Referring to ADF and PP statistics, we reject the null of unit root at 1% significance level for all parities of exchange rate. That is confirmed by KPSS test, since we accept the null of stationnarity at 10% level for the dollar and euro series and a 5% significance level for the yen series. Thus, all the residuals are stationary. Consequently, for each parity of exchange rate, the spot and forward rates admit a long-run relationship, which is the necessary condition for forward exchange market efficiency (Hakkio and Rush; 1989).

b. Estimate of the error correction models

Table 11
Error correction model for TND/USD parity
Dependent variable: D(LSUSD)

Variable	Coefficient	Standard deviation	Stat(Student)	Prob.
D(LFUSD)	1.0032	0.0278	36.0075	0.0000
Rd(-1)	-1.0228	0.1339	-7.6370	0.0000
R^2	0.9623			
R^2 adjusted	0.9616			
stat Durbin-Watson	1.9610			

Table 12
Error correction model for TND/euro parity
Dependent variable: D(LSEUR)

Variable	Coefficient	Standard deviation	Stat(Student)	Prob.
D(LFEUR)	0.9372	0.0262	35.7671	0.0000
Re(-1)	-1.0013	0.1270	-7.8830	0.0000
R^2	0.9549			
R^2 adjusted	0.9541			
stat Durbin-Watson	1.8917			

Table 13
Error correction model for TND/JPY parity
Dependent variable: D(LSYEN)

Variable	Coefficient	Standard deviation	Stat(Student)	Prob.
D(LFYEN)	0.8776	0.0394	22.2638	0.0000
Ry(-1)	-0.9941	0.1267	-7.8408	0.0000
R^2	0.9287			
R^2 adjusted	0.9275			
stat Durbin-Watson	2.0532			

The above tables give, for all parities, the results of modelling of the change of spot exchange rate as a function of changes in the forward exchange rate as well as the level of disequilibrium in the cointegrating relationship. It is noticed that the coefficient of the error-correction term is significantly negative. Then, the spot exchange rate moves to restore the long-run equilibrium relationship implied by the cointegration relationship.

Thus, we can say that there exists for each parity, co-movements between the spot and forward series which converge together to a long-run stable equilibrium. Consequently, we can say that the Tunisian forward exchange market is efficient.

2. Fractional cointegration Results

a. Results of the standard test of fractional cointegration

Table 14

Estimate of the orders of integration of the series in level

	LFUSD	LSUSD	LFEUR	LSEUR	LFYEN	LSYEN
$\hat{d}'_{ML}$	0.4982	0.4981	0.4993	0.4994	0.9939	0.9940
Test–Wald	205.146 [0]		2377.14 [0]		–	

- For TND/USD and TND/euro parities: The estimated values of the order of integration of the series show that those have a long memory. Moreover, the test of Wald does not reject the null assumption ($d_1 = d_2$), which is the necessary condition for fractional cointegration.
- For TND/JPY parity: The estimated values of the order of integration of the series are close to the unit. Therefore, one can say that these series are nonstationary.

The existence of a long memory on the series in level of the dollar and euro could not be detected by the standard tests of unit root used in the preceding section. Indeed, Diebold and Rudebush (1989) and Sowell (1990) showed that ADF tests had a low power to distinguish the null assumption from process I(1) against the alternative of fractional process I(d). In the same way, Hassler and Wolters (1995) stressed that the performance of PP tests was not better than that of ADF tests to distinguish the fractional alternative of integration.

The application of long memory tests on the residuals gives the following results:

Table 15

Estimate of the orders of integration of the residuals series

	R USD	R EUR	R YEN
$\hat{d}'_{ML}$	-0.0904	-0.0896	0.0809
t- Student	-0.710	-0.751	0.675
Prob	[0.480]	[0.456]	[0.502]
$\hat{d}'_{GPH}$	-0.1393	0.0297	0.2779
t- Student	-1.09	0.225	1.770
Prob	[0.280]	[0.823]	[0.083]

For the three parities, the estimated values of integration orders of the residuals are not significantly different from zero. Therefore, one can say that these residues follow a process I(0).

We can see that the spot and forward exchange rate series relating to the TND/USD and TND/euro parities have the same fractional orders of integration (Wald test). Moreover, OLS estimates residuals from the cointegrating equation are stationary and their orders of integration are lower than the original series.

So, we can say that there is a fractional cointegration between the series. Hence there exists a co-movement between the two series of exchange rates that converge towards an equilibrium long run relation. Thus we can conclude that the Tunisian forward exchange market is efficient.

The estimate of error correction models of filtered TND/USD and TND/euro series leads to the following results reported in Tables 16 and 17.

Table 16
Estimate of (MCE) for the filtered TND/USD series
Endogenous variable: FLSUSD

Variable	Coefficient	Standard deviation	Stat(Student)	Prob.
FLFUSD	1.0151	0.0188	53.8597	0.0000
Resusd(-1)	-0.5139	0.1357	-3.7869	0.0004
R^2	0.9777			
R^2 adjusted	0.9773			
stat Durbin-Watson	1.9101			

Table 17
Estimate of (MCE) for the filtered TND/euro series
Endogenous variable: FLSEUR

Variable	Coefficient	Standard deviation	Stat(Student)	Prob.
FLFEUR	0.9782	0.0152	64.1867	0.0000
Reseur(-1)	-0.5283	0.1334	-3.9579	0.0002
R^2	0.9788			
R^2 adjusted	0.9785			
stat Durbin-Watson	1.9179			

For the TND/USD and TND/euro parities: we note that the spot rate depends on a positive and significant way of the forward rate. The coefficient of the error-correction term is negative and significantly different from zero. Then, the spot exchange rates move to restore the long-run relationship implied by the equilibrium relationship. We note that for the two parities, the speed adjustment coefficients are lower then those of previous results. This implies slower speed of return to equilibrium. It is to be recalled that the standard test of fractional cointegration is based on the OLS estimate. But, according to Robinson and Marinucci (1998) this method gives unbiased estimates only when $d > \frac{1}{2}$. Thus, since it was found that the orders of integration of the rough series are lower than 1/2, estimating a vectorial error correction bivariate ARFIMA model will be justified.

b. Estimate of the vectorial error correction fractional model (VECFM)

Table 18

	d	Test-Wald	d′	Coefficient	α
LSUSD	0.4126 [0.001]		0.0841 [0.512]		
LFUSD	0.4138 [0.004]	185.396 [0]	0.1526 [0.369]	1.3856 [0]	-0. 4870 [0.007]
$R^2 = 0{,}9146$					
Log Max Probability:	455.137				
LSEUR	0.4876 [0.006]		0.1194 [0.402]		
LFEUR	0.4885 [0.009]	2452.97 [0]	0.1069 [0.372]	1.2085 [0]	-0.4784 [0.003]
$R^2 = 0{,}8635$					
Log Max Probability:	429.176				

The estimates of the orders of integration of spot and forward series have significant and fractional values (< 1/2). Moreover, the test of Wald accepts the null assumption, which implies equality between the orders of integration of the two exchange rate series. This answers the necessary condition of fractional cointegration since the estimate of bivariate ARFIMA model supposes a common fractional parameter d in studied series.

In addition, the orders of integration of the residues d' relating to the series of all parities, have given values that are not significantly different from zero. Therefore, the linear combination of these series is stationary and has orders of integration lower than the two parent series. Consequently, one can conclude that there is a fractional cointegration between the series of spot and forward exchange rates. Moreover, the estimated coefficient of the speed of adjustment is negative and significantly different from zero.

For each of the two parities, there exists a mechanism of adjustment, which restores the equilibrium between spot and forward exchange rates, defined by the stable relationship of cointegration.

IV. CONCLUSION

Within the framework of this paper, we apply panoply of cointegration tests in order to investigate the weak form of the Tunisian exchange market efficiency. We started our analysis by standard cointegration tests, which usually assume, that taking integer differences leads to a stationary $I(0)$ linear combination. But this distinction between process $I(1)$ or $I(0)$ is arbitrary, since cointegration conditions require only that the equilibrium error be stationary. The cointegration can also be considered when the series are I(d) with d fractional.

The use of the fractional cointegration process allowes us:

- To better model the financial series relating to the rates of exchange (process of long memory).
- To avoid problems particularly related to the consequences of under or overdifferencing of the original series.
- To provide an evaluation of the long-run relationships (slower speeds of adjustment).

The estimate of a fractional vectorial error correction model of (FVECM) gives evidence to a fractional cointegration relationship between the spot and forward exchange rate series relating to the two parities (TND/USD) and (TND/euro). This allows us to decide about the Tunisian forward exchange market efficiency.

REFERENCES

Baillie, R.T., and T. Bollerslev, 1994, "Cointegration, Fractional Cointegration and Exchange Rate Dynamics", *The Journal of Finance*, XLIX(2), 737-745.

Barkoulas, J.T., Baum, C.F., and A. Chakraborty, 2003, "Forward Premiums and Market Efficiency", *Journal of Macroeconomics*, 25, 109-125.

Cheung, Y., and K. Lai, 1993, "A Fractional Cointegration Analysis of Purchasing Power Parity", *Journal of business and Economic Statistics*, 11, 103-112.

Diebold, Francis X., and Glenn D. Rudebusch, 1991, "On the Power of Dickey Fuller Test Against Fractional Alternatives", *Economics Letters*, 35, 155-160

Diebold, Francis X., and Glenn D. Rudebusch, 1989, "Long Memory and Persistence in Aggregate Output", *Journal of Monetary Economics,* 24, 189-209.

Dickey, D.A., and W.A. Fuller, 1979, "Distribution of the Estimators for Autoregressive Time Series with Unit Root", *Journal of The American Statistical Association*, 74, 427-431.

Dickey, D.A., and W.A. Fuller, 1981, "Likelihood Ratio Statistics for Autoregressive Time Series With Unit Root", *Econometrica*, 49, 1057-1072.

Dueker, M., and R. Statz, 1998, "Maximum-likelihood Estimation of Fractional Cointegration With an Application to US and Canadian Bond Rates", *The Review of Economic and Statistics*, 80, 762-769.

Engel, R.F., and C.W.J. Granger, 1987, "Cointegration and Error Correction: Representation, Estimation and Testing", *Econometrica*, 55(2), 250-276.
Fama, E.F., 1998, "Market Efficiency, Long-term Returns and Behavioural Finances", *Journal of Financial Economics*, 49, 283-306.
Fama, E.F., 1984, "Forward and Spot Exchange Rates", *Journal of Monetary Economics*, 14, 319-338.
Geweke, J., and S. Porter-Hudak, 1983, "The Estimation and Application of Long Memory Times Series Model", *Journal of Time Series Analysis*, 4, 221-237.
Granger, C.W.J., 1986, "Developments in the Study of cointegrate economic variables", *Oxford Bulletin of Economics and Statistics, 48 (3), 213*-220.
Granger, C.W.J., and A.A. Weiss, 1983, "Time Series Analysis of Error-Correction Models", in S. Karlin, T. Amemiya and L.A. Goodman (edit)., *Studies in Econometrics, Time Series, and Multivariate Statics (New York, Academic Press)* 255-278.
Granger, C.W.J.,1981, "Some Properties of Time Series Data and Their use in Econometric Model Specification", *Journal of Econometrics*, 16, 121-130.
Granger, C.W.J, and R. Joyeux, 1980, "An Introduction to Long Memory Time Series Models and Fractional Differencing", *Journal Of Time Series Analysis* (1), 15-39.
Hakkio, C.S., and M. Rush, 1989, "Market Efficiency and Cointegration: An Application to the Sterling and Deutshemark Exchange Markets", *Journal of International Money and Finance*, 9, 78-89.
Hassler, U., and J. Wolters, 1995, "Long memory in inflation rates international evidence", *Journal of Business and Econornic Statistics,* 13, 37-46.
Hosking, J.R.M., 1981, "Fractional Differencing", *Biometrika*, 68, 165-176.
Johansen, S., 1988, "Statically Analysis of Cointegration Vectors", *Journal of Dynamics and Control* 12, 231-254.
Kwiatkowski, D., P. Phillips, P. Schmidt, and Y. Shin, 1992, "Testing the Null Hypothesis of Stationary against the Alternative of Unit Root", *Journal of Econometrics*, 54, 159-178.
Lyhagen, J., 1999, "Maximum Likelihood Estimation of the Multivariate Fractional Cointegrational Model", *Working Paper Series in Economics and Finance n°233, Stockholm School of Economics.*
Marmol, F, 1998, "Spurious Regression Theory With Nonstationary Fractional Integrated Processes" *Journal of Econometrics*, 84, 233-250.
Olekalns, N., and N. Wilkins, 1998, "Re-examining the Evidence For Long-run Purchasing Power Parity", *Economic Record*, 74, 54-61.
Phillips, P.C., and P. Perron, 1988, "Testing For a Unit Root in Time Series Regression", *Biometika,* 75, 335-346.
Robinson, P.M., and D. Marinucci, 1998, "Semiparametric Frequency Domain Analysis Of Fractional Cointegration", *Discussion paper No EM/98/348, LSE*, London
Sowell, F, 1990, "The Fractional Unit Root Distribution", *Econometrica,* 58, n°2, 495-505.