INTERNATIONAL JOURNAL OF BUSINESS, 11(2), 2006 ISSN: 1083–4346

Sensitivity of Interest Rate Caps to the Elasticity of Forward Rate Volatility

Jean-Michel Sahut[a] and Mehdi Mili[b*]
[a] *Dean for Research & Professor of Finance*
sahutjm@esc-larochelle.fr
Sup de Co La Rochelle, CEREGE – University of Poitiers (France)
[b] *PhD Student*
Sup de Co La Rochelle, CEREGE – University of Poitiers (France)
University of Sfax (Tunisia)
milimehdi@gmail.com

ABSTRACT

This paper examines the pricing performance of interest rate option pricing models in the Euribor interest rate cap markets. We investigate the sensitivity of the prices of cap derivatives to alternative specifications of the forward interest rate derivatives. We use a term structure-constrained model that allows us to modify the volatility structure without altering their initial values. Consequently, any differences in prices can be directly attributed to alternative assumptions on the term structure of forward rate volatilities rather than to the initial conditions. Our results show that cap prices are significantly sensitive to the structure of forward interest rate volatility and the inelasicity of this volatility function may lead to significant pricing errors.

JEL Classification: *G13*

Keywords: *Term structure; Interest rate; Volatility; HJM model; Cap pricing; Kalman Filter*

* We are particularly grateful for the comments and suggestions of Associate Professor N. Bellamy- University of Evry - in revising this article. The authors would like to thank Professor Fathi Abid of University of Sfax (Tunisia) for his helpful comments. We also would like to acknowledge the European Social Fund for its support.

I. INTRODUCTION

Demand for risk control, due to unfavourable changes in the shape of yield curves, has made over-the-counter interest rate derivatives increasingly popular in the last two decades. Thus, the pricing of interest rate contingent claims has been subject to a significant amount of research in finance in order to help financial institutions to establish their speculative and hedging strategies. Theoretical work in the area of interest rate derivatives has produced a variety of models and techniques to values these derivatives, some of which are widely used by experts[1]. The major problem encountered by researchers when evaluating interest rate derivatives is the exiting of the direct relationship between option prices and the interest rate term structure.

Two major approaches to term structure models may still be distinguished for pricing interest-rate contingent claims. For one there is the "equilibrium-based" approach, according to which one must specify one or more factors that are jointly Markov and drive the term structure. Given the process for these factors under the real measure, P, and some specification for the "market price of risk" of each of these factors, one can define the so-called risk-neutral measure, Q, under which all discounted-asset-price processes are martingales (Harrison and Kreps, 1979). Notice that the "market price of risk" specification may either be arbitrarily imposed (Vasicek, 1977) or derived under some restrictive preferences and economic-environment assumption (Cox, Ingersoll and Ross, 1985) (CIR).

More recently, however, a second strand of literature has been developed that avoids the crux of explicitly having to specify the "market price of risk" when pricing interest-rate derivatives. The "arbitrage" approach initiated by Ho and Lee (1986) and generalized by Heath-Jarrow-Morton (1992) (HJM), takes the initial term structure as given and, using the no-arbitrage condition, derives some restrictions on the drift term of the process of the forward rates under the risk-neutral probability measure Q. In essence, HJM show that if there exists a set of traded interest-rate-dependent contracts then the dynamics of their prices under the risk-neutral measure are fully specified by their volatility structure. Under the Q measure, lack of arbitrage places restrictions on the drifts of the contracts.

Despite the advantages of HJM over the short-rate models, it was found out it does also have drawbacks; some practical and some theoretical. Firstly, in general, those models are non-Markovian and consequently PDE theory techniques no longer apply. Secondly, many volatility term structures $\sigma(t, T)$ result in dynamics of the forward interest date $f(t, T)$, which are non-Markov (i.e: with a finite state space). This introduces path dependency to pricing problems, which increases, significantly, computational times. Third, generally there are no simple formulas or methods for pricing commonly-traded derivatives such as caps and swaptions. This is again a significant problem from the computational perspective. Finally, if we model forward rates as log-normal processes the HJM model will explode[2]. This last theoretical problem can be avoided by modelling Libor and swap rates as log-normal (market models)[3] rather than instantaneous forward rates.

So, by construction, the HJM model fits the initial term structure exactly. Unfortunately, according to this approach, no finite dimensional set of state variables exists, in general, that captures the information necessary for pricing. This feature makes it difficult to describe the dynamics of the term structure in terms of a reduced set of state variables and to obtain closed-form pricing formulas (or even to use numerical pricing methods).

The most important ingredient of the HJM term structure models is the choice of a volatility structure for forward interest rates. The aim of this paper is to explore the importance of forward rate volatility structures in pricing interest rate cap options. Many researchers have focused on this problematic issue for different kinds of interest rate derivatives or different term structure models.

Moraleda and Pelsser (2000) tested three alternative spot-rate models and two Markovian forward-rate models on cap and floor data from 1993 to 1994, and found that spot rate models outperform the forward-rate models in pricing interest rate claims. However, as they acknowledge, their empirical tests are not very formal. Bühler, Uhing, Walter and Weber (1999) tested different one-factor and two-factor models in the German fixed-income warrants market. They report that the one-factor forward rate model with linear proportional volatility outperforms all other models. However, their study has the same limitations. First, they use options data with less than 3 years of maturity. Second, the underlying asset for these options is not homogeneous. Ritchken and Sankarasubramanian (1995) study the sensitivity of the prices of interest rate claims to alternative specifications of the volatility of forward interest rates. They use a term structure-constrained model that allows change in volatility structure for forward rates without altering their initial values or the set of initial bond prices. They consider the following volatility structure:

$$\sigma(t,T) = \sigma_0 r^{\gamma}(t) e^{-\lambda(T-t)}$$

Our paper extends the Ritchken and Sankarasubramanian (1995) study on two levels. First, they arbitrarily consider values for σ_0 and λ. More particularly, they consider a range of σ_0 from 0.005 to 0.015 and a range of λ of 0.01 and 0.05. In our case we propose to estimate those two values from market data using the Kalman filter technique. This is in order to limits option pricing errors to depend only upon the elasticity degree of the forward rate volatility. Second, contrary to Ritchken and Sankarasubramanian who test the sensitivity of interest rate options to volatility specification, we propose to test the effect of elasticity of the forward rate volatility on cap prices. Furthermore, our study extends Amin and Morton (1994) and Klassen, Dressien and Pelsser (1999) by considering that the volatility structure depends on the level of the spot interest rate.

In order to price Euribor interest rate caps, we estimate a restrictive HJM model via the Kalman filter. The filtering technique will be used to estimate a system of unobserved state variables where the observed variables are linked to the unobserved state variables via a measurement equation. The transition equation of the unobserved state variables can be specified as a system of linear equations with Gaussian innovations.

The state-space and Kalman Filter framework has a long tradition in applied econometrics. Pennacchi (1991), for instance, used this approach to obtain estimates of the real interest rate and inflation dynamics based on survey data. Ball and Torous (1996) used this framework to estimate parameters of the one-factor CIR model in a simulation study. Their study focuses on the problems associated with estimating the mean reversion parameter if interest rates are close to a non-stationary process. They conclude that the properties of estimates are remarkably improved when cross-sectional information is included by way of the state-space framework. In the next section, we show how the one-factor HJM model can be expressed in a state space form and can be estimated by the Kalman filter technique.

In this paper, the empirical performance of analytical models is evaluated along their pricing accuracy conditional on the term structure. The pricing accuracy of a model is useful in picking out deviation from arbitrage-free pricing. The HJM models are estimated with specific volatility functions to ensure that the interest rate process is Markovian, i.e. path independent. Except for special cases, path dependence renders the implementation of a term structure model unfeasible.

The paper is organized as follows. Section II briefly presents the HJM term structure framework and its implementation with the Kalman filter technique. In section III, empirical methodology and market data are described. Section IV reports and discusses the results of the study. A conclusion is contained in Section V.

II. IMPLEMENTIONTATION OF THE HJM (1992) MODEL WITH THE KALMAN FILETER

Let f(t,T) be the forward interest rate at date t for instantaneous riskless borrowing or lending at date T. The one-factor model specifies the evolution of the entire instantaneous forward rate curve by:

$$df(t,T) = \mu(t,T)dt + \sigma(t,T)dW(t) \quad (1)$$

where W(t) is one dimensional Brownian motion and $\mu(t,T)$ and $\sigma(t,T)$ are the instantaneous mean and volatility coefficients for the forward interest rate of maturity T. HJM (1992) point out that for each choice of volatility functions $\sigma(t,T)$, the drift of the forward rates under the risk-neutral measure, is uniquely determined by the no-arbitrage condition:

$$\mu(t,T) = \sigma(t,T)\int_t^T \sigma(t,T)ds \quad (2)$$

The choice of the volatility function $\sigma(t,T)$ determines the interest rate process that describes the stochastic evolution of the entire term structure. The object of this paper is to test the effect of this volatility structure on pricing the interest rate-sensitive claims. We will use the extended Kalman filter to estimate a system of unobserved state variables where the observed variables are linked to the unobserved state variables via a measurement equation. The transition equation for the unobserved state variables can be specified as a system of linear equations with Gaussian innovations[4].

A. The State Space Formulation

Our first step when applying the Kalman filter is to specify the state space formulation. In this context, the observable or measurable interest rates are assumed to be related to unobservable state variables via a *measurement equation*. The unobservable state variables are, in turn, assumed to follow a Markov process described by the *transition equation*.

Our object consists in implementing the Kalman filter in order to estimate the parameters of the generalized CIR (1985) model expressed by:

$$\sigma(t,T) = \sigma_0 r^{\gamma}(t) e^{-\lambda(T-t)} \tag{3}$$

Equation (3) shows that we set the volatility forward rates to decay exponentially with maturity. This representation has been used in many other studies, including Vasicek (1977), Jamshidian (1989) and Trunbull and Milne (1991). Positive signs of λ imply that shocks to the term structure have an exponentially dampened effect across maturities. When T converges to t, near-term forward rates will have volatilities "close" to the volatility structure considered. Therefore, the structure captures the notion that distant forward rates are less volatile than near-term rates.

The parameter γ indicates the elasticity parameter of the volatility function. Thus if γ=0, we retrieve the generalized Vasicek model which is inelastic to interest rate level.

Ball and Torous (1993) suggest that when ë is near zero, the interest rate process resembles a non-stationary process and that estimates will generally not be precise.

As shown by Bhar and Chiarella (1995.a. 1996) and Carvehill (1994) the Markovian stochastic dynamics system formed by the differential of the state variables r(t), ψ(t) [5] and the Logarithm of the pure discount bond P(t,T) noted F(t,T)=lnP(t,T) can be expressed by the following state space stochastic differential system :

$$dX(t) = [A(t) + B(X(t),t)X(t)]dt\sigma_x(X(t),t)d\widetilde{W}(t) \tag{4}$$

$$\text{Where } X(t) = [F(t,T), r(t), \psi(t)]^T \tag{5}$$

$$A(t) = [0, f_2(0,t) + \lambda f(0,t), 0]^T$$

where $f_2(0,t)$ is the second derivative of f(0,t).

$$B(t)=\begin{bmatrix} 0 & 1-\frac{1}{2}r(t)^{2\gamma-1}[e^{-\lambda(T-t)}-1]^2/\lambda^2 & 0 \\ 0 & -\lambda & \sigma_0^2 \\ 0 & r(t)^{2\gamma-1} & -2\lambda \end{bmatrix} \tag{6}$$

$$\sigma_X(X(t),t)=[\sigma_0 r(t)^{\gamma}(e^{-\lambda(T-t)}-1)/\lambda, \sigma_0 r(t)^{\gamma}e^{-\lambda(T-t)}, 0] \tag{7}$$

Since Log bond price, F(t,T), is the only observable element of the state vector X(t), and the two parameters to be estimated are $\theta=[\sigma_0,\lambda]$. In order to maintain the analytical tractability of the model, we presume that:

$$dX(t)=[F(X(t),t)]dt+\sigma_x(X(t),t)d\widetilde{W}(t) \tag{8}$$

The two parameters of the volatility structures will be estimated via the state space formulation of the model.

B. Measurement equation

The measurement equation relates the vector of observable variables to the vector of non observable variable. Since r(t) and ψ(t) are both unobservable and the only observable variable is F(t.T), the measurement equation can be expressed by:

$$Y(t)=HX(t)+\varepsilon_t \qquad \varepsilon_t \to N(0,\sigma_\varepsilon) \tag{9}$$

Where Y(t)=(3×1), X(t)=(3×1), and H=[1, 0, 0]. In this case, the measurement equation is non-linear, which leads us to apply the extended Kalman filter method.

C. Transition Equation

The essence of the Kalman filter is to optimally calculate $\hat{X}_{k/k-1}$ which is the best forecast of X_k given all the information available up to k-1. By using the additional information up to time k, we can then estimate $\hat{X}_k$.

In line with Bhar and Chiarella (1997), we will use the Milstein scheme to transform the stochastic differential equation (4) into discreet time as:

$$X_k=[X_{k-1}+F(X_{k-1},\theta)\Delta_{k-1}]+\sigma(X_{k-1},\theta)\sqrt{\Delta_{k-1}}\xi^2+\frac{1}{2}\sigma(X_{k-1},\theta)\sigma(X_k,\theta)\Delta_k[\xi_k^2-1] \tag{10}$$

where $\zeta^2 \to N(0,1)$. Equations (8) and (9) define state space representation.

D. The prediction step

The prediction of the unobservable state variable X_k at time k is made based on its value at time k-1.

$$\hat{X}_{k/\ k-1} = E_{k-1}(X_k) = \hat{X}_k + F(X_k, \theta)\Delta_k \tag{11}$$

E. The updating step

The updating step consists in using the additional information at time k to obtain an updated estimator of X_k:

$$\hat{X}_k = E_k(X_k) = \hat{X}_{k/k-1} + \Sigma_{k/k-1} H^T \sigma_k^{-1} D_k$$

$\hat{X}_k$ is the filtered estimate where $\Sigma_{k/k-1} H^T \sigma_k^{-1}$ is the Kalman gain matrix. And D_k is the estimation error over $[t_{k-1}, t_k]$ given by: $D_k = Y_k - H\hat{X}_{k/k-1}$

F. The quasi-Likelihood function

Harvey (1994) provides the prediction error decomposition form of the Likelihood function as:

$$\text{LogL} = -\frac{n}{2}\text{Log}2\pi - \frac{1}{2}\sum_{k=1}^{n}\text{Log}|\sigma_k| - \frac{1}{2}\sum_{k=1}^{n}\sigma_k^2\sigma_k^{-1} \tag{12}$$

All the parameters of the volatility function of the forward rates will be estimated by maximizing the Likelihood optimisation function.

III. EMPIRICAL METHODOLOGY AND DATA

Our empirical study consists of two steps. Primarily, we implement the Kalman filter to estimate parameters of three different volatility structures inspired from the general form:

$$\sigma(t, T) = \sigma_0 r^{\gamma}(t) e^{-\lambda(T-t)} \tag{13}$$

Note that the form of the volatilities is completely characterized by the selection of the three parameters, σ, γ and λ. The parameter λ captures the dampening effect of the volatilities across the term structure, and γ is the elasticity measure.

First we consider the case of γ=0 which corresponds to the generalized Vasicek (1977) model. In this case the volatility structure becomes deterministic and the spot interest rate becomes the only state variable. Second, we consider the case of

γ=1/2 which corresponds to the generalized Cox Ingersoll and Ross (1985). Finally, we assume that γ=1, allowing us to consider the Dothan (1978) model.

Secondly, we compare the performance of these three volatility structure models in pricing interest rate caps, in order to characterize the best model that capture the behaviour of the interest rate. Therefore, we use parameters estimated from the Kalman filter to price caps on Euribor interest rates, then we can compare these prices to caps market prices.

The three models considered in this paper have a common term structure, which permits us to explore the sensitivity of options prices to changes in the parameter γ without altering the set of prices of the underlying bonds. In this case, differences in interest rate options prices result directly from the difference in assumptions regarding the volatilities structures.

For this purpose we need two kinds of datasets. On the one hand, data on money market interest rate and on the other hand, data on interest rate caps market prices. Our money market interest rate consists in retrieving monthly Euribor interest rate for maturities of 1, 3, 6 and 12 months. These data series cover the period from January 1994 to November 2003. Figure 1 depicts the four interest rate series.

Basic summary statistics are contained in Table 1. According to this table the unconditional mean yields increase monotonically with maturity from 4.29 at three-months to 4.45 at 12 months. It is also shown that unconditional volatilities go down monotonically with maturity from 1.41 to 1.32.

Figure1
Euribor Interest rate curve

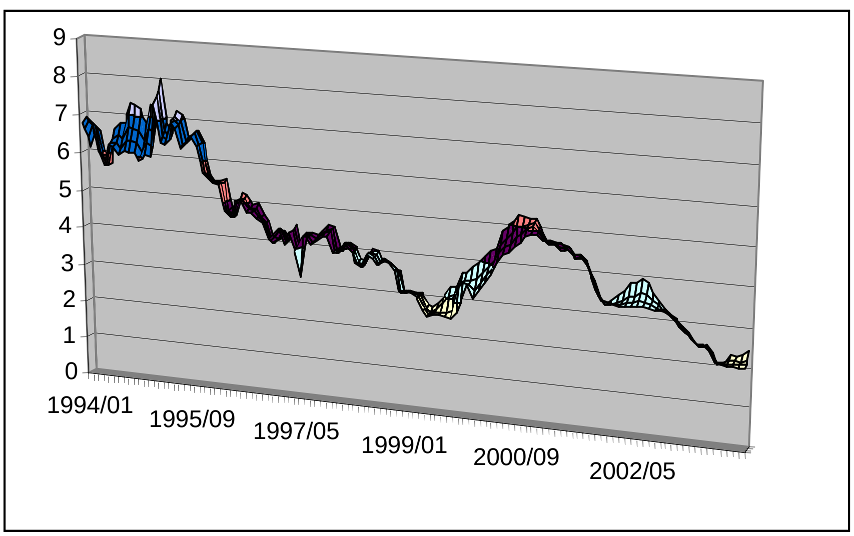

Table 2 presents descriptive statistics of the derivatives data set. The data consists on Euribor caps market prices with maturities 1, 2, 3, 5, 7 and 10 years. The sample period is from January 1994 to November 2003. The prices of the contracts are expressed in basis point of the notional principal contract. The mean, minimum, maximum and standard deviation price of the respective contracts over the sample period are reported in this table. It's shown that the prices of caps increase on average with maturity from 1 year to 4 years then it decrease from maturity of 5 years to 10 years. Market prices of caps express decreasing volatilities across maturities.

Table1
Summary statistics of Euribor interest rates

	1 Month	**3Month**	**6Month**	**12Month**
Mean	4.2992	4.3356	4.3612	4.4511
Maximum	7.4300	7.5900	7.7400	8.0200
Minimum	2.0900	2.1300	2.0800	2.0100
Std.Dev	1.4140	1.3597	1.3346	1.3279

Table 2
Descriptive statistics of market at-the money caps prices

Maturity (year)	Mean	Maximum	Minimum	Std. Dev
1	18.19	28.00	10.10	4.05
2	18.53	25.80	12.30	3.08
3	18.82	29.00	13.50	3.48
4	18.04	26.30	14.00	2.78
5	17.42	24.30	14.20	2.32
7	16.31	20.90	13.40	1.76
10	16.31	20.90	13.40	1.76

IV. ESTIMATION RESTULTS

The estimation of the parameters σ_0 and λ of forward rate will be performed through maximizing the Likelihood function in (12) with respect to θ. Table 3 shows the estimated coefficients and their standard deviation along with the value of the Log likelihood function of each model.

Table 3
Kalman-filter parameters model estimates

	Vasicek (1977)			**CIR (1985), γ=1/2**			**Dothan (1978), γ=1**		
	Coeff.	**Std.Dev**	**Log L**	**Coeff.**	**Std.Dev**	**Log L**	**Coeff.**	**Std.Dev**	**Log L**
σ_0	0.02508	0.00629	1425	0.05386	0.03365	1505	0.06425	0.01250	2135
λ	-0.00062	0,01985		0.06192	0.05669		0.16432	0.00135	

This table presents summary statistics for the parameter estimates of the three different volatility structures. Concerning the Generelized Vasicek (1977) model, the estimate for the mean-reversion parameter λ is small and negative. This is the result of the hump shape of the variance of forward rate changes. The standard deviations of parameter estimates are a little higher for the CIR model compared to the other models.

The next step is to focus on the models' performance in pricing caps. To measure how well a given model conditionally predicts derivatives, our procedure is as follows. Given the estimated parameters of each model and the term structure at any trading date we value the caps and compare the implied prices with the observed marked prices. This procedure is then repeated over all the months in the dataset.

A cap can be regarded as a portfolio of European call options on interest rates. Caps price is the sum of caplets of different maturities. Given the specification of the HJM models, the pricing formula for caps is readily available.

Thus the price of a caplet at time t, Caplet_t, that pays off $\zeta \text{Max}(0, \text{Eur}_\zeta(T,T) - k)$ at time T+ζ, where $\text{Eur}_\zeta(t.T)$ is the ζ-period forward Euribor rate.

By means of the estimated parameters and the term structure of the interest rate, we price caps on a monthly scale. The observed market price is then subtracted from the model-based price to calculate both the absolute pricing error and the percentage pricing error. This procedure is repeated for each cap in the sample.

The summary statistics of the errors are presented in Table 4. These give an idea about the empirical quality of the models.

Table 4
Pricing results for cap prices

	Gen. Vasicek (1977)	Dothan (1978)	Gen. CIR (1985)
Average error	-9.55%	-8.78%	-9.06%
Average absolute error	15.60%	11.32%	12.36%

The average percentage error is defined as the (model price – market price)/market price.

This table illustrates the degree of potential mispricing to approximate option prices with different degrees of elasticity. The major remark to be drawn from the table is that almost all models under price caps on average. The Dothan model has the lowest absolute prediction errors, which are on average around 11.32%, whereas the Generalized Vasicek has the highest prediction errors, which are on average equal to 15.60%. Our results support the fact that forward term structure with volatility depends on their levels, such that Generalized CIR (1985) and Dothan (1978), outperform the Generelized Vasicek one. This is in line with the findings of Ritchken and Sankarasubramanian (1999).

From the same table we can also notice that deviations in the value of caps from their market prices appear to expand with the elasticity parameter γ. This is because the largest average absolute error of 15.6% appears in the Vasicek model and the lowest one of 12.36% appears in the Dothan (1978) model. In line with Gupta and Subrahmanyam (2002), these results indicate that adding more parameters to the model improves its ability to forecast interest rate derivatives.

Furthermore we can deduce that using a simple generalized Vasicek model to price interest rate caps can lead to significant mispricing if interest rate volatilities do indeed depend on their levels.

V. CONCLUSTION

The object of this paper is to investigate the sensitivity of option contracts to alternative volatility specification of the forward rate. By applying the Kalman filter we estimate three kinds of volatility structure, which are then used to price interest rate caps. In line with the findings of Ritchken and Sankarasubramanian (1999) our results show, first, that interest rate options prices are quite sensitive to the elasticity parameter in the volatility structure. Second, we find that the Vasicek model estimates caps prices better than the generalized Vasicek one, allowing the lowest average error.

The considered volatility structure does not incorporate all possible forms and supposes that only one-factor determines all the interest rate dynamics. For further research, we can explore the impact of including other stochastic factors governing the term structure in pricing interest rate-sensitive claims. The results obtained in this empirical framework are of significant value in implementing the models in practice.

ENDNOTES

1. Such as Black (1976), Vasicek (1977), Cox, Ingersoll and Ross (1985), Ho and Lee (1986), Black, Derman and Toy (1990), etc.
2. For example, see Sandmann and Sondermann (1997).
3. See Brace and Musela (1995).
4. This formulation is the same as used by Bahr and Chiarella (1997).
5. Bahr and Chiarella (1997) suggest that $\psi(t)$ is defined by :

$$\psi(t) = \int_0^t \sigma(\mu, t)^2 d\mu \text{ and } d\psi(t) = [\sigma(\mu, t)^2 - 2\lambda\psi(t)]d(t)$$

They also support that this variable plays a central role in allowing the transformation of the original non-Markovian dynamics to Markovian form. Similar subsidiary variables appear in the reduction to Markovian form of Ritchken ans Sankarasubramanian (1995), Bahr and Chiarella (1997), Inui and Kijima (1998) and Chiarella and Kwon (1999).

REFERENCES

Amin, K. and A. Morton, 1994, "Implies Volatility Functions in Arbitrage-free Term Structure Models", *Journal of Financial Economics* 35, 141-180.

Ball, C., and W. Torous, 1985, "On Jumps in Common Stock Prices and Their Impact on Call Option Pricing", *Journal of Finance,* 40, 155-173.

Black, F., 1976, "The Pricing of Commodity Contracts", *Journal of Financial Economics* 3, 67-179.

Bhar, R and Carl Chiarella, 1995. "The Estimation of the Heath-Jarrow-Morton Model by Use of Kalman Filtering Techniques," Working Paper Series 54, School of Finance and Economics, University of Technology, Sydney.

Bhar, R and Carl Chiarella, 1995. "Transformation of Heath-Jarrow-Morton Models to Markovian Systems," Working Paper Series 53, School of Finance and Economics, University of Technology, Sydney.

Bhar , R and Carl Chiarella, 1996. "Construction of Zero-Coupon Yield Curve From Coupon Bond Yield Using Australian Data," Working Paper Series 70, School of Finance and Economics, University of Technology, Sydney.

Bhar, R. and C. Chiarella, 1997, "Transformation of Heath-Jarrow-Morton Models to Markovian Systems", *European Journal of Finance* 3, 1–26.

Carverhill, A. 1994, "When Is the Short Rate Markovian?", *Mathematical Finance* 4(4), 305–312.

Chiarella, C. and O. Kwon, 1998a, "A Class of Heath-Jarrow-Morton Term Structure Models with Stochastic Volatility", *Working Paper No. 34*, School of Finance and Economics, University of Techonology Sydney.

Chiarella, C. and O. Kwon, 1999, "Square Root Affine Transformations of the Heath-Jarrow-Morton Term Structure Model and Partial Differential Equations", *Working Paper*, School of Finance and Economics, University of Techonology Sydney.

Chiarella, C. and O. Kwon, 2001a, "Formulation of Popular Interest Models under the HJM Framework", *Asia Pacific Financial Markets* (Forthcoming).

Collin. P, and R.S. Goldstein, 2002, "Pricing Swaptions in An Affine Framework," *Journal of Deivatives*, 10:9–26.

Cox, J., J. Ingersoll, and S. Ross, 1985, "An Intertemporal General Equilibrium Model of Asset Prices", *Econometrica* **5**3(2), 386–384.

Dai, Q. and K.J. Singleton, 2000, "Specification Analysis of Affine Term Structure Models," *Journal of Finance*, 55:1943–1978.

Dai, Q. and K.J. Singleton, 2002, "Fixed Income Pricing," *Working paper*, Stanford University.

Duffie, D. and R. Kan, 1996, "A Yield Factor Model of Interest Rates", *Mathematical Finance* 6(4), 379–406.

Goldstein, R. S., 2000, "The Term Structure of Interest Rates as a Random Field", *Review of Financial Studies* 13, 365-384.

Harrison, M. and D. Kreps, 1979, "Martingales and Arbitrage in Multiperiod Securities Markets", *Journal of Economic Theory* 20, 381-408.

Heath, D., R. Jarrow, and A. Morton, 1992, "Bond Pricing and the Term Structure of Interest Rates: A New Methodology for Contingent Claim Valuation", *Econometrica* 60(1), 77–105.

Ho, T. and S. Lee, 1986, "Term Structure Movements and Pricing Interest Rate Contingent Claims", *Journal of Finance*, 41, 1011-1029.

Inui, K. and M. Kijima, 1998, "A Markovian Framework in Multi-Factor Heath-Jarrow-Morton Models", *Journal of Financial and Quantitative Analysis* 33(3), 423–440.

Jamshidian, F., 1990, "The Preference-Free Determination of Bond and Option Prices From the Spot Interest Rate", *Advances in Futures and Options Research*, 4, 51-67.

Moraleda, J. M. and A. Pelsser, 2000, "Forward versus Spot Interest Rate Models of the Term Structure: An Empirical Comparison", *The Journal of Derivatives* 7, 9-21.

Ritchken, P. and L. Sankarasubramanian, 1995, "Volatility Structures of Forward Rates and the Dynamics of the Term Structure", *Mathematical Finance* 5(1), 55–72.

Sandmann, K. and D. Sondermann, 1997, "A Note on the Stability of Lognormal Interest Rate Models and the Pricing of Eurodollar Futures," *Mathematical Finance* 7 (2), 119-125.

Vasicek, O., 1977, "An Equilibrium Characterisation of the Term Structure", *Journal of Financial Economics* 5, 177–188.