

## **Exchange Rate and Risk Premium Conversion on Interest Rate markets**

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### **ABSTRACT**

This paper investigates common factors that jointly determine bond returns across-countries. We study how risk factors deriving bond prices influences exchange rates and we test if parameters of bond price process are fundamental in specifying exchange rate process. We use an arbitrage free international stochastic discount factor (SDF) framework in order to analyse interaction between bond prices and exchange rate process. We show that risk premia are different through countries and exchange rate serve to convert currency-specific risk premia across countries.

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## I. INTRODUCTION

Bond portfolios investors diversify, generally, their portfolio by investing in foreign bonds. The management of an international bond portfolio requires a model that evaluates bond prices of different maturities in diverse countries. Several studies<sup>1</sup> report that premiums of different countries have, generally, similar evolution, which imply the existence of a limited number of common factors that derive the joint yield curve across countries. In order to optimally diversify their assets, portfolio managers are primarily interested in mechanisms that influence premiums and the degree of heterogeneity of premium variations. This involves determination of the number and the nature of common sources of variation for each bond. Despite their crucial role in international portfolio management, the covariance between premiums across countries and the number of underlining common factors are rarely investigated in the theory.

The asset pricing model of Ross (1976) shows that common variation of asset returns can be expressed as linear function of a number of factors. But this model doesn't specify the number and the characteristics of these factors. Despite the ultimate success of this theory, it appears unable to explain high correlations between bond yields across the maturity spectrum and across countries and to identify their common sources of variation.

Solnik (1983) extended the arbitrage pricing theory to an international framework. He shows that if asset returns, expressed on an enumerative money, follow a linear factor model, then the expected return vector are perfectly identified by the principal factor vector. Ikeda (1991) suggests that direct extension of the asset pricing theory to an international context is counter intuitive since exchange risk might induce an additional factor in the arbitrage process on financial markets. The key element of these advancements is to suppose that the generation process of returns is specified on an enumerative currency. Under this precision, exchange risk of assets returns is automatically diversified when constructing a risk free portfolio.

Our study is related to Knez, Litterman and Scheinkman (1994) and Litterman and Scheinkman (1991) who estimate a model for short-term US monetary market returns and long-term USA government bond returns, respectively. Knez et al. (1994) propose a four-factor model. The first two factors correspond to movement in the level and the slope of the term structure, while the other two factors describe credit risk difference of the different money market instruments. Litterman and Scheinkman (1991) use the principal component analysis and show that US bond returns are mainly determined by three factors, which correspond to level movement, slope and curvature movement in the term structure.

This paper extends these two studies by jointly analysing bond returns in four countries, namely the USA, Germany, UK and Japan. The joint analysis of interest rate term structures is particularly useful in studying potential international diversification and international bond portfolios management. In this spirit we model a cross-country covariance matrix including (1) bond return variance of each country, (2) covariance of domestic bond returns across maturities, and (3) covariance of bond return across-countries.

The basic idea we explore here is that there is an important systemic factor that cuts across a wide spectrum of markets. We apply the principal component analysis to determine the number of common factors that influence risk premiums on international bond markets. This empirical method also permits us to deduce in which measure idiosyncratic loading factors affect premiums variations. In this perspective we test two fundamental hypotheses of the international asset pricing theory. First, we test if the expected hedged return of an investor's international portfolio is determined by the factor prices of risk of the investor's home country or by those of the country in which he invests. Second, test if exchange rates serve to convert prices of risk across-countries.

The research methodology we employ is based on the principal component analysis and entails two-stage procedure. First, we apply the principal component analysis separately to the term structure in the following countries; the US, Germany, Japan and the UK, in order to filter principal factors of prices of risk and risk premium relative to each country. In a second stage, we use principal component analysis on the unconditional covariance matrix of bond returns of the different maturities in all countries to estimate the factors that determine these bond returns.

The rest of the paper is organized as follow. Section II develops theoretical framework used to model hedged bond returns and describes the empirical methodology. Section III describes database. In the next section we estimate principal factors of prices of risk specific to each country. Section V estimates and interprets a cross-countries bond return model. We test if exchange rate serves to convert risk premium between countries. Section VI concludes.

## II. THEORETICAL FRAMEWORK

Asset Returns provide information about systematic and idiosyncratic risks. In this section, we extend the international asset pricing model of Ikeda (1991) into a continuous-time framework. We show the importance of the arbitrage free hypothesis in pricing international bond portfolio and we show in what sense the exchange rate can be considered to convert prices of risks across countries.

### A. A linear Factor Model with Exchange Rate Risk

We consider an economy with two countries, a domestic and foreign country. Each country is characterised by domestic currency and an exchange rate  $E_{xg}$  expressing one unit of foreign currency in terms of domestic currency. We suppose  $N$  risky asset and one riskless asset traded for each country. We assume that these assets are freely traded in perfect international financial markets. Denote as  $P_d^i$ ,  $i=\{1, \dots, n\}$ , the price of risky asset  $i$  of the domestic country denominated in domestic currency. Let  $P_f^i$ ,  $i=\{1, \dots, n\}$ , be the price of risky asset  $i$  of the foreign country denominated in foreign currency.

Similarly, we define  $P_d^0$  and  $P_f^0$  as the local currency price of the riskless asset of the domestic and foreign country, respectively. Local currency is defined as the currency of the geographical place where the asset is quoted.

Assume that each risky asset  $i$  is driven by  $K$  international risk factors (undiversifiable) and one specific risk factor (diversifiable). The  $K$  international factors are generated by  $K$  Wiener process  $Z_k$ ,  $k = \{1, \dots, K\}$  which are independent. The local currency dynamics of the prices  $P_d^i$  and  $P_f^i$  is supposed to be given by the following equation:

$$\begin{aligned} \frac{dP_d^i}{P_d} &= \mu_d^i dt + \sum_{k=1}^K b_{dk}^i dZ_k + \varepsilon_d \\ \frac{dP_f^i}{P_f} &= \mu_f^i dt + \sum_{k=1}^K b_{fk}^i dZ_k + \varepsilon_f \end{aligned} \quad (1)$$

where  $\frac{dP_d^i}{P_d}$  is the random local currency return of asset  $i$  and  $\mu_d^i$  is its instantaneous expected value.  $dZ_k$  represents the instantaneous increment of the  $k^{\text{th}}$  international risk factor.  $b_{dk}^i$  and  $b_{fk}^i$  denotes sensitivity of local and foreign assets to fluctuations of the  $Z_k$  factor.  $\varepsilon_d$  and  $\varepsilon_f$  are the non-systematic risk component. The price of risk of these idiosyncratic risk is zero, and  $[dZ_k \varepsilon_d] = 0$  et  $E[dZ_k \varepsilon_f] = 0$ . The number of assets is much larger than the number of international risk factor, i.e.  $2N \gg K$ .

Local returns of riskless asset are given by:

$$\begin{aligned} \frac{dP_d^0}{P_d} &= r^d dt \\ \frac{dP_f^0}{P_f} &= r^f dt \end{aligned} \quad (2)$$

Exchange rate process is given by the following differential stochastic equation:

$$\frac{dS}{S} = \mu_s dt + \sum_{k=1}^K b_{sk} dZ_k + dZ_s \quad (3)$$

where  $\mu_s dt$  is the instantaneous variation of exchange rate,  $b_{sk}$  are the principal factors of exchange rate of the  $K$  international factors and  $dZ_s$  represents the orthogonal idiosyncratic factor of risk that derives the  $K$  asset prices.

Investor that invests in an international portfolio doesn't risk only depreciation of the local value of his investment, but he risks also unfavourable exchange rate evolution when converting the proceeds of his international investment into domestic currency. To convert the value of his foreign portfolio into domestic currency he must multiply it by the spot exchange rate,  $\tilde{P}_f^i = SP_f^i$ , where  $\tilde{P}_f^i$  is the price of risky asset  $i$  of the foreign country measured by domestic investor (in domestic currency).

From this hypothesis, we deduce the return process of the foreign asset in domestic currency.

$$\frac{d\tilde{P}_f^i}{\tilde{P}_f^i} = \frac{d(SP_f^i)}{SP_f^i} = \frac{dP_f^i}{P_f^i} + \frac{dS}{S} + \text{cov}\left(\frac{dP_f^i}{P_f^i}, \frac{dS}{S}\right) \quad (4)$$

where  $\text{cov}\left(\frac{dP_f^i}{P_f^i}, \frac{dS}{S}\right)$  is the instantaneous covariance between foreign asset return and exchange rate variation. From this equation we deduce that foreign asset return measured in domestic currency equal the sum of asset return expressed in domestic currency, variation of exchange rate and the covariance between them. The presence of covariance element imply that domestic investor doesn't convert only the initial amount invested in foreign currency but also return realised in foreign currency. Through equations (1), (3) and (4) we show that foreign asset return evaluated in domestic return depends on the specific factor of risk  $dZ_s$  :

$$\frac{d\tilde{P}_f^i}{P_f^i} = \tilde{\mu}_f^i dt + \sum_{k=1}^K (b_{fk} + b_{Sk}) dZ_k + \varepsilon_f + dZ_S \quad (5)$$

where  $\tilde{\mu}_f^i = \mu_f^i + \sum_{k=1}^K b_{fk} b_{Sk} + \mu_s$  ;  $\tilde{\mu}_f^i$  is the expected value of the foreign asset return expressed in terms of local currency. Similarly, the foreign riskless asset return measured in domestic currency is given by:

$$\frac{d\tilde{P}_f^0}{\tilde{P}_f^0} = \tilde{r}_f dt + \sum_{k=1}^K b_{Sk} dZ_k + dZ_S$$

where  $\tilde{r}_f = r_f + \mu_s$  .

In our framework, its impossible to construct a riskless portfolio through the standard Asset Pricing Theory since shocks of exchange rate  $dZ_s$  are systematic, contrary to residual risk  $\tilde{\varepsilon}_i$  . To study this aspect in methodical ways, we construct a portfolio of  $2N$  risky assets<sup>2</sup> denoted by  $\omega=(\omega_1, \dots, \omega_{2N})'$  where  $\omega_i$  are proportion invested in each asset  $i$ .

The portfolio return,  $\omega$ , is given by:

$$\omega' \left( \frac{d\tilde{P}}{P} \right) = \omega' \tilde{\mu}_f^i dt + \sum_{k=1}^K \omega' (b_k + b_{Sk}) dZ_k + \omega' \varepsilon + \omega' dZ_S \quad (6)$$

$$\begin{aligned} \tilde{\mu} &= (\mu_d^1, \dots, \mu_d^N, \mu_f^1, \dots, \mu_f^N)', \\ \text{where } dZ_S &= (0, \dots, 0, dZ_S, \dots, dZ_S)', \\ b_k &= (b_d^1, \dots, b_d^N, b_f^1, \dots, b_f^N)', \\ \varepsilon &= (\varepsilon_d^1, \dots, \varepsilon_d^N, \varepsilon_f^1, \dots, \varepsilon_f^N) \end{aligned}$$

Exchange risk is eliminated by having short position on riskless bonds in foreign currency. Weights  $\omega_i$  are constrained by:

$$\begin{aligned} \omega' b_k &= 0 \quad k = \{1, \dots, K\} \\ \omega' \varepsilon &= 0 \end{aligned} \quad (7)$$

Applying (7) to equation (6) reveals that:

$$\omega' \left( \frac{d\tilde{P}}{\tilde{P}} \right) \cong \omega' \tilde{\mu} dt + \omega' d\tilde{W}_S \quad (8)$$

where  $d\tilde{W}_S = \sum_{k=1}^K b_{sk} dZ_k + dZ_s$

We remark from this equation that international risk and residual risk ( $dZ_k$ ) are diversified. However, constraints of equation (7) do not lead necessarily to an international risk free portfolio since exchange rate risk  $\omega' dZ_s$  is left undiversified. The next subsection involves this problem by considering an arbitrage portfolio hedged against exchange risk.

## B. Arbitrage Pricing of Hedged Portfolio

Portfolio  $\omega$  isn't hedging against exchange risk. This portfolio hedged against exchange risk by holding short position  $\omega_d$  on riskless domestic bonds for each  $\omega_d$  invested in domestic asset and  $\omega_f$  proportion invested in riskless foreign bond for each  $\omega_f$  invested in foreign asset.

Given this hedging strategy, domestic currency return of an investment in foreign hedged portfolio will be:

$$\begin{aligned} \left[ \frac{d\tilde{P}_f}{\tilde{P}_f} \right] &= \tilde{\mu}_f dt + r^d dt - \tilde{r}^f dt \\ &= \left[ \mu_f dt + \mu_s dt + \text{cov} \left( \frac{dP_f}{P_f}, \frac{dS}{S} \right) \right] + \left[ r^d dt - r^f dt + \mu_s dt \right] \\ &= \mu_f dt + \text{cov} \left( \frac{dP_f}{P_f}, \frac{dS}{S} \right) + r^d dt - r^f dt \end{aligned} \quad (9)$$

The hedged portfolio return, measured in domestic currency, is deterministic and hedged against any type of risk. Since investment in risky assets are totally financed by detention of an opposite position on domestic bonds, the risk generated by international factors ( $dZ_k$ ) and residual factors ( $\varepsilon_d, \varepsilon_f$ ) are diversified and the investor is hedged against specific exchange risk. However, the value of hedged portfolio depends on covariance between exchange rate and risky asset. This is because foreign asset return is not pre-known and in consequence can't be hedged against unfavourable fluctuations of exchange rates.

The portfolio return ( $dP/P$ ) is constituted by domestic and foreign asset through constrains (7) and after be hedged by short positions on riskless bonds is given by:

$$\left[ \frac{dP_f}{P_f} \right]_P = \omega'(\mu^l dt + \text{cov}\left(\frac{dP}{P}, \frac{dS}{S}\right) + \sum b_{fk} dZ_k - rdt) \quad (10)$$

where  $\mu^l$  is a  $2N$ -dimensional vector of local expected rates of return containing  $N\mu_d$  and  $N\mu_f^*$  of domestic and foreign asset; and  $\text{cov}\left(\frac{dP}{P}, \frac{dS}{S}\right)$  is a  $2N$ -dimension vector:

$$\text{cov}\left(\frac{dP}{P}, \frac{dS}{S}\right) = \begin{bmatrix} 0_1 \\ \vdots \\ 0_N \\ \text{cov}\left(\frac{dP_f^1}{P_f}, \frac{dS}{S}\right) \\ \vdots \\ \text{cov}\left(\frac{dP_f^N}{P_f}, \frac{dS}{S}\right) \end{bmatrix} \quad i=1\dots N$$

$$r = \begin{bmatrix} r_d \\ (N \times 1) \\ r_f \\ (N \times 1) \end{bmatrix}$$

Since the portfolio does not require any initial investment due to short position taken on riskless local bonds, its return must be zero to avoid any arbitrage opportunity.

$$\omega'(\mu^l dt + \text{cov}\left(\frac{dP}{P}, \frac{dS}{S}\right) - rdt) = 0 \quad (12)$$

This equation shows that the vector  $\varepsilon$  is also orthogonal to expected hedged returns vector. Hence, Asset Pricing Theory implies that the expected hedged return must be a linear combination of factor loadings  $b_k$ ,  $k = \{1, \dots, K\}$ :

$$\mu^l dt + \text{cov}\left(\frac{dP}{P}, \frac{dS}{S}\right) - r dt = \lambda_1^d b_{d1} + \lambda_2^d b_{d2} + \dots + \lambda_k^d b_{dk} \quad (13)$$

We deduce that expected returns in domestic currency, hedged against exchange risk, are a linear combination of local factor loadings with weights  $\lambda_k^d$ . According to the standard asset pricing theory, weights  $\lambda_k^d$  are the domestic factor risk prices.

In other words, there are  $K$  risk prices that determine expected excess return on hedged assets. Each international factor represents another dimension of systematic risk that cannot be diversified. In consequence, each risk factor must be remunerated. We note that expected hedged excess return of domestic and foreign investment will be determined by the prices of risk of original country of the investor,  $\lambda_k^d$ ,  $k = \{1, \dots, k\}$ . Investor in domestic currency can't apply foreign prices of risk to determine expected return from investing on foreign market, if these conditions are not verified.

### C. Covariance between Asset Returns and Exchange Return

In this subsection we show if exchange rate permit conversion of risk prices across countries. To examine relationship between domestic and foreign factors risk prices as well as asset-exchange rate covariance, we compare expected hedged excess return of foreign asset  $f$  in domestic currency to its expected excess return in foreign currency. Through equations (9) and (13), expected hedged excess return  $E[(dP_f/P_f)]$ , of foreign asset  $f$  in domestic currency is given by:

$$\mu_f dt + \text{cov}\left(\frac{dP_f}{P_f}, \frac{dS}{S}\right) - r_f dt = \lambda_{d1} b_{f1} + \lambda_{d2} b_{f2} + \dots + \lambda_{dk} b_{fk} \quad (14)$$

An arbitrage portfolio constructed through formula (7) can be formed also in foreign currency. The last section shows that prices of factor of risk  $\lambda_{fk}$  that evaluate hedged assets in foreign currency can be determined. The equation measurement of foreign asset excess return can be obtained:

$$\mu_f dt - r_f dt = \lambda_{f1} b_{f1} + \lambda_{f2} b_{f2} + \dots + \lambda_{dk} b_{fk} \quad (15)$$

Comparing equations (14) and (15) shows that presence of term  $\text{cov}\left(\frac{dP_f}{P_f}, \frac{dS}{S}\right)$  convert prices of risks of  $\lambda_{fk}$  to  $\lambda_{dk}$ ,  $k=\{1\dots k\}$ . This conversion propriety of exchange rate is the result of exclusion of arbitrage opportunities.

In this context, comparing these two equations shows that covariance between foreign asset return and exchange return will be given by:

$$\text{cov}\left(\frac{dP_f}{P_f}, \frac{dS}{S}\right) = \sum_{k=1}^K b_{fk} (\lambda_{dk} - \lambda_{fk}) dt$$

This equation shows that covariance between asset return and exchange return depends on factor loadings  $b_{fk}$ . This result shows also that exchange rate serves also to convert the prices of risk of foreign investments prices into domestic ones. However, investor can't convert prices of risk of his original country.

Two major implications are to be noted from these results. First, portfolios returns hedged against exchange risk depend only on factors of risk of domestic country. In other words, domestic and foreign portfolio having the same profile of risk must allow identical expected returns when they are converted into national currency. Second, this result cannot be maintained in case of unhedged portfolio. In fact, exchange rate dynamics and exchange factors intervene entirely. In this case, excess returns will be determined in part by the forward premium, meaning the price of holding additional and orthogonal exchange rate risk.

### III. EMPIRICAL INVESTIGATION OF HEDGED BOND RETURNS

#### A. Databases and Estimation of Unhedged Returns

Our study focuses on the following four bond markets: USA, Germany, Japan and the UK. For each market, we collect daily observations of governmental bond prices for the following maturities: 3 years, 5 years, 7 years, 10 years and 30 years. We cover the time period from 09/10/2000 until 14/10/2005, and have 20 time series of 270 observations. From these series we construct four bond portfolios containing two bonds with different maturities. For each country, we obtain the following portfolio: PF3 to 5 years, PF5 to 7 years, PF7 to 10 years, et PF10 to 30 years.

Summary statistics for local portfolio returns investment are reported in Table 1. Means and standard deviations of returns appear very weak since returns are measured on the basis of one week horizon. Negative means of returns reported on the Japanese market are explained by the slight rise of interest rate in Japan over the period of study but continue to be close to zero. In all countries, average returns and their standard deviations generally increase with maturity.

Expected returns in Table 1 cannot be used to evaluate international investment strategy. Since at the maturity of his investment, the investor not only risks

unfavourable evolution of local interest rates but also risks the exchange when converting his investment into domestic currency. In this context, we need to convert local returns into domestic currency returns of the investor. In this paper, we consider the US as the domestic country and the other ones as foreign country.

**Table 1**  
Descriptive statistics of local returns<sup>3</sup>

Returns are measured weekly over the period 9/10/2000 to 14/10/2005. Every portfolio is formed of two bonds with different maturities for each country.

	<b>Means (%)</b>	<b>Std.Dev (%)</b>	<b>Skew.</b>	<b>Kurtos.</b>	<b>Auto-cor.</b>
<b>USA 3 – 5 years</b>	0.02051	0.0194	7.9555	-1.1962	-0.312
<b>USA 5 – 7 years</b>	0.02307	0.0208	4.1009	0.4025	-0.109
<b>USA 7 – 10 years</b>	0.03614	0.0253	4.5097	0.2002	-0.205
<b>USA 10 - 30 years</b>	0.04510	0.0280	2.5186	-0.8057	-0.482
<b>YEN 3 – 5 years</b>	-0.04584	0.0078	10.1722	0.4764	-0.320
<b>YEN 5 – 7 years</b>	-0.03977	0.0113	10.2341	-0.9217	-0.187
<b>YENP 7 - 10 years</b>	-0.02182	0.0121	5.6137	-1.2058	-0.622
<b>YEN 10 - 30 years</b>	0.06178	0.0287	12.1687	0.6341	-0.455
<b>GBP 3 – 5 years</b>	0.03746	0.0176	49.0120	-2.6231	-0.780
<b>GBP 5 – 7 years</b>	0.03446	0.0211	31.7354	-2.3924	-0.792
<b>GBP 7 – 10 years</b>	0.03667	0.0315	35.4229	-0.1237	-0.806
<b>GBP 10 - 30 years</b>	0.05423	0.0354	24.415	-0.5532	-0.818
<b>EUR 3 – 5 years</b>	0.02300	0.0098	7.2371	-0.0446	-0.543
<b>EUR 5 – 7 years</b>	0.02609	0.0131	5.0173	-0.3781	-0.622
<b>EUR 7 – 10 years</b>	0.03174	0.0158	2.9443	-0.4494	-0.863
<b>EUR 10 - 30 years</b>	0.03633	0.0229	3.1777	-1.1124	-0.766

The one-month money market rate is applied as the risk-free interest rate in order to calculate excess returns on each market. Spot exchange rates between these countries are also sampled for the same period of study. Exchange rates are quoted as the price of foreign currency in units of domestic currency.

Table 2 reports summary statistics of interest rates and first-variations of exchange rates for each country. This table shows that Euro and Pound sterling are appreciated, in means, face to USD over the period of study by 0.082% and 0.124%, respectively. While, the USD is globally appreciated by 0.027% face to the Nippon.

Figure 1, we plot the correlations between bond returns of different maturities and different countries. Since portfolios are regrouped by country in the correlation matrix, high correlations near the diagonal of the matrix indicate high correlations between countries. The majority of correlations are positives.

**Table 2**

Descriptive statistics of exchange returns and one month interest rates

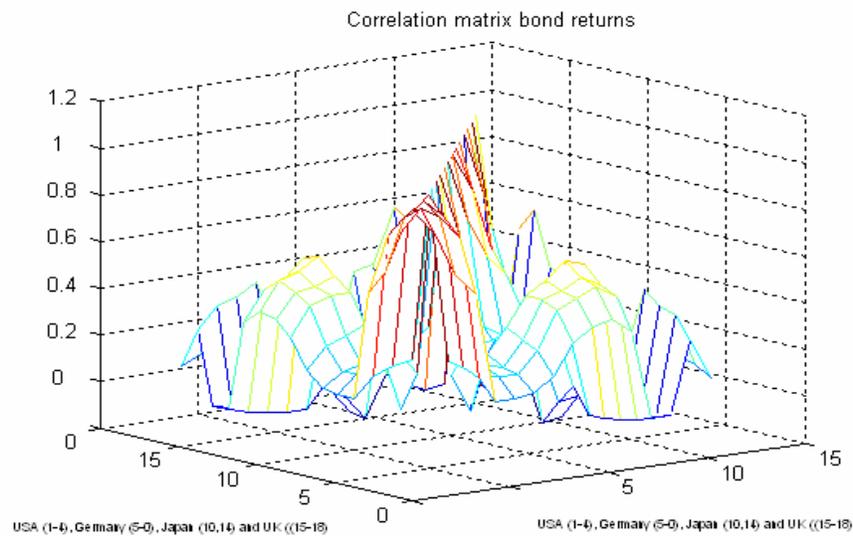
Exchange rates are expressed one dollar in terms of foreign currencies. Les taux de change sont exprimés en termes de dollar par rapport à une unité de monnaie étrangère. Interest rates are one-month monetary market interest rates in each country which will be considered as approximation of spot rates.

	Means. (%)	Std.Dev (%)	Skew.	Kurtos.	Auto-cor.
Rendements des taux de change					
YEN/USD	-0.027	1.038	0.187	1.021	0.875
GBP/USD	0.082	0.951	0.162-	5.046	0,923
EUR/USD	0.124	1.134	-0.046	-0.007	0,967
Taux d'intérêt					
$r^{usd}$	2.425	1.528	1.411	1.117	0,938
$r^{yen}$	0.078	0.126	4.940	30.791	0,840
$r^{gbp}$	4.417	0.706	0.660	-0.354	0,974
$r^{eur}$	2.978	1.006	0.726	-0.869	0,921

**Figure 1**

Correlations of local bond returns

This figure shows the correlation structure of local returns over the period 2003-2005. The numbers 1-4 reported on axes X and Y correspond to US returns, numbers 5-9 correspond to German returns, 10-14 correspond to Japan returns and numbers 15-18 correspond to UK returns.



Returns of unhedged international investment portfolio in domestic currency can be calculated by equation (4). Table 3 reports means and standard deviations of returns of the unhedged portfolio when considering USA as domestic country. This table shows that standard deviations of international investment portfolio are higher than domestic portfolio returns.

We note that when investment occurs in US, expected returns will be the same as those of Table 1, since in this case American investor is not expected to exchange risk. Returns in Table 3 can be compared to those of Table 1.

**Table 3**  
Descriptive statistics of unhedged bond portfolios

	<b>Means (%)</b>	<b>Std.Dev (%)</b>	<b>Skew.</b>	<b>Kurtos.</b>	<b>Auto-cor.</b>
<b>USA 3 – 5 years</b>	0.02051	0.0194	7.9555	-1.1962	-0.312
<b>USA 5 – 7 years</b>	0.02307	0.0208	4.1009	0.4025	-0.109
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<b>USA 10 - 30 years</b>	0.04510	0.0280	2.5186	-0.8057	-0.482
<b>YEN 3 – 5 years</b>	-0.07863	1.0377	0.1891	0.2586	-0.792
<b>YEN 5 – 7 years</b>	-0.07835	1.0380	0.1956	0.2673	-0.785
<b>YENP 7 - 10 years</b>	-0.02601	1.0391	0.1936	0.2617	-0.784
<b>YEN 10 - 30 years</b>	-0.02686	1.0488	0.1957	0.2892	-0.779
<b>GBP 3 – 5 years</b>	0.08148	0.9517	-0.1591	0.5139	-0.788
<b>GBP 5 – 7 years</b>	0.08097	0.9513	-0.1574	0.5115	-0.794
<b>GBP 7 - 10 years</b>	0.08405	0.9541	-0.1665	0.5145	-0.806
<b>GBP 10 - 30 years</b>	0.08582	0.9566	-0.1627	0.5183	-0.816
<b>EUR 3 – 5 years</b>	0.12675	1.1371	-0.0461	-0.3135	-0.884
<b>EUR 5 – 7 years</b>	0.12724	1.1376	-0.0445	-0.3118	-0.876
<b>EUR 7 - 10 years</b>	0.12771	1.1380	-0.0450	-0.3088	-0.884
<b>EUR 10 - 30 years</b>	0.13676	1.1376	-0.0449	-0.3022	0.793

Comparison of Tables 1 and 3 shows that investment in foreign markets increases standard deviation of returns in all considered markets. In fact, standard deviation of unhedged returns in the European and Japanese market lies between 1.037% and 1.138%, at a time when it was less than 0.03% when returns are measured in domestic currency (table 1). Equally, standard deviations of unhedged returns in the UK market exceed 0.95% when it was less than 0.029% in the first table. The impact of exchange rate on international investment return is clearly expressed in this table, since the depreciation of the Yen has contributed to the deterioration of returns realized on the Japanese market, when the depreciation of the EUR and GBP contributes to ameliorate returns realised on the European and English markets.

### B. Estimation of Hedged Bond Returns

Expected returns of hedged portfolios can be measured by the following equation:

$$\left[ \frac{dP_f}{P_f} \right]_{\text{couvert}} = \mu_f + \text{cov} \left( \frac{dP_f^*}{P_f^*}, \frac{dS}{S} \right) - r^f + r^d \quad (16)$$

Table 4 reports descriptive statistics of hedged portfolios. When considering the case of an American investor in an England portfolio formed of 3 and 5 year bonds, he realises an average return of 0.0374% from his investment in London and realises a negative return from his hedging position on the exchange market. Measured by the spread; rusd-rgbp, (-1.948%). Its net return in USD is about (-1.9106%). In this case also, the difference between this return and the return indicated in table 4 (-1.958) is always due to the covariance term between asset return and exchange rate returns.

**Table 4**  
Hedged returns summary statistics

	Means (%)	Std.Dev (%)	Skew.	Kurtos.	Auto-cor.
USA 3 – 5 years	0.02051	0.0194	7.9555	-1.1962	-0.312
USA 5 – 7 years	0.02307	0.0208	4.1009	0.4025	-0.109
USA 7 - 10 years	0.03614	0.0253	4.5097	0.2002	-0.205
USA 10 - 30 years	0.04510	0.0280	2.5186	-0.8057	-0.482
YEN 3 – 5 years	2.38508	0.4134	1.1846	0.5551	-0.794
YEN 5 – 7 years	2.38535	0.4131	1.1842	0.5558	-0.378
YENP 7 - 10 years	2.38669	0.4130	1.1851	0.5604	-0.278
YEN 10 - 30 years	2.38684	0.4144	1.1879	0.5708	-0.377
GBP 3 – 5 years	-1.95887	0.6551	1.1062	0.9423	-0.578
GBP 5 – 7 years	-1.95938	0.6544	1.1122	0.9549	-0.379
GBP 7 - 10 years	-1.95630	0.5562	1.1053	0.9371	-0.280
GBP 10 - 30 years	-1.95453	0.5356	1.1025	0.9459	-0.481
EUR 3 – 5 years	-0.46804	0.8701	0.7860	-0.7413	-0.388
EUR 5 – 7 years	-0.46755	0.8701	0.7854	-0.7442	-0.487
EUR 7 - 10 years	-0.45708	0.8703	0.7856	-0.7427	-0.380
EUR 10 - 30 years	-0.40803	0.8706	0.7857	-0.7390	-0.798

Comparison of unhedged returns and hedged returns of local portfolios in Tables 3 and 4 shows that the hedging strategy leads to a significant reduction of the hedged portfolio variances compared to those of unhedged portfolios. This reduction is about 70% and 80% in mean for all the portfolios. We remark that risk reduction is not

necessary coupled with an equivalent reduction in mean returns. Indeed, some means of return of hedged portfolios exceed those of unhedged portfolios. This can be due to the impact of the depreciation of exchange rate.

#### IV. ESTIMATION RESULTS OF FACTORS LOADING PER COUNTRY

We estimate the first four principal factors and the explained cumulative variance<sup>4</sup> for each country by applying principal component analysis to the covariance matrix of local currency excess returns.

Since the principal component analysis doesn't determine the exact number of factor loadings, we proceed in our methodology by estimating prices of risk factors separately in each country and then we compare results to those estimated through a cross-sectional analysis. Table 5 presents the first four principal factors obtained from the principal component analysis method applied to bond returns in each country.

The factor loadings values appear very small. This is due to the adjustment of the principal component analysis. The first factor affects bonds in all countries and can be regarded as a global factor, whereas the other factors seem to be more country-specific. For example, the second risk factor strongly affects bond in Japan, the third affects British bond and the fourth one affects bond returns in Germany.

**Table 5**  
Estimated factor loadings

The four first factors loading are estimated by the principal component analysis for each country.

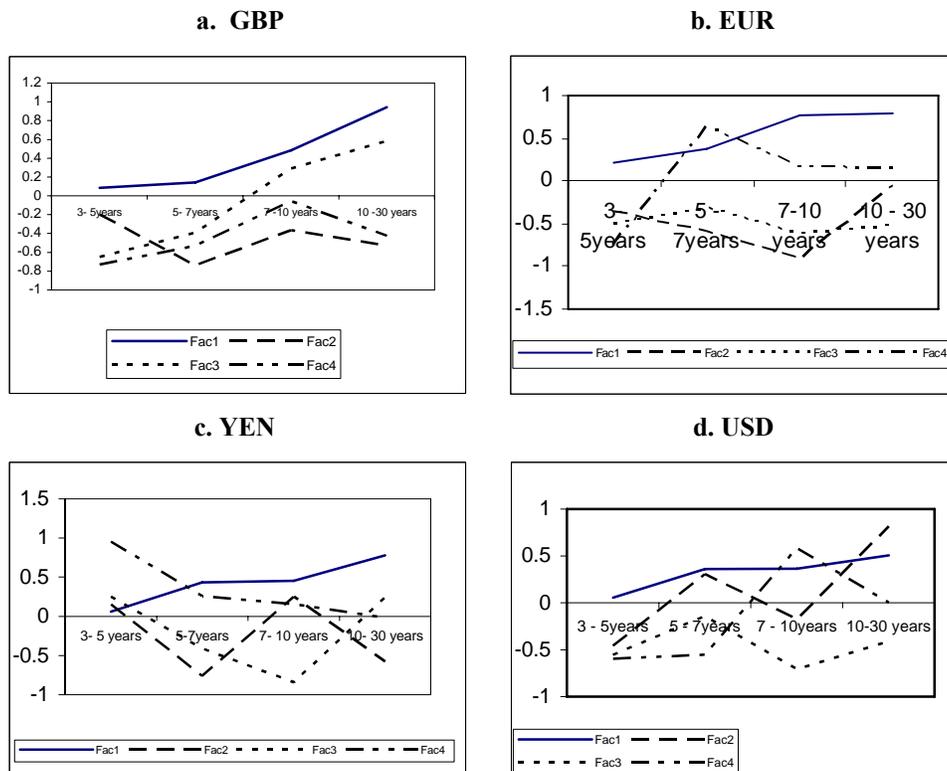
	1	2	3	4
USA 3 – 5 years	0.0549	-0.4569	-0.5576	-0.5953
USA 5 – 7 years	0.3608	0.3059	-0.1433	-0.5542
USA 7 - 10 years	0.3611	-0.1754	-0.7073	0.5818
USA 10 - 30 years	0.5046	0.8166	-0.4102	-0.0005
YEN 3 – 5 years	0,0574	0,1536	0,2528	0,9537
YEN 5 – 7 years	0,4320	-0,7606	-0,4109	0,2571
YENP 7 - 10 years	0,4527	0,2521	-0,8417	0,1550
YEN 10 - 30 years	0,7779	-0,5792	0,2431	-0,0177
GBP 3 – 5 years	0.0846	-0.1985	-0.6479	-0.7305
GBP 5 – 7 years	0.1417	-0.7386	-0.3907	-0.5308
GBP 7 - 10 years	0.4819	-0.3654	0.2923	-0.0578
GBP 10 - 30 years	0.9416	-0.5306	0.5849	-0.4258
EUR 3 – 5 years	0.2191	-0.3669	-0.5034	-0.7510
EUR 5 – 7 years	0.3736	-0.6049	-0.3261	0.6231
EUR 7 - 10 years	0.7728	-0.9276	-0.6107	0.1705
EUR 10 - 30 years	0.7840	-0.0706	-0.5169	0.1369

Estimations of the first four factors are presented in Figure 2. We note that variation of term structure level influence considerably long term bond returns. We interpret the first factor as a world level factor. This factor shifts the entire term structure in all countries in the same direction. The second factor can be interpreted as a slope factor, since movement of this factors exhibit the slope of the curve. when the third factor describe the curvature of yields curve, since it affect middle term bond returns in different sense of the short and long bond maturities.

**Figure 2 a-d**

The four first factor loadings estimated from principal component analysis

The principal component analysis is applied independently to the following currencies: GBP, EUR, USD et YEN.



Though risk factors express almost the same form across countries, the explained variances for each country differ for one country to another. Our results differ from those of Litterman and Scheikman (1991) who show that explained variance is lower for the first factor, and higher for the second and the third factors. This imply that for the last five years more movement of slope and curvature has occurred on the yield curve of Germany, Japan and UK than in the US market.

From Table 6 we deduce that the three first factors explain 99.99% of the variation of bond returns in the USA, 98.65% in Japan, 97.67% in the UK and 98.65% in Germany. The first factor appears very important for the four bond markets considered and explains 88% of variation in the USA, 76% in Japan, 67% in the UK and 79% in Germany.

**Table 6**  
Variance explained by principal components for each country

This Table reports explained variance in means for each compared with the total variance of bond returns.

	1	2	3	4
<b>Usd</b>	88.378	9.528	2.092	0.0007
<b>Eur</b>	79.161	14.201	5.129	1.507
<b>Yen</b>	76.829	14.486	7.337	1.347
<b>Gbp</b>	67.673	18.078	11.597	2.650

## V. EMPIRICAL RESULTS FOR A MULTI-COUNTRIES MODEL

### A. Risk Premium Estimation

The factor loadings estimated for each country will be used as inputs for the cross sectional estimation of currency-specific factors risk premium for each. From equation (14), the international asset pricing theory supports that hedged returns in a common currency can be expressed in terms of linear combination of principal compound and risk premium. In this section, we estimate risk premium of factors by regressing hedged returns on the principal factors estimated. In order to estimate specific prices of risk for each currency, expected returns of all bonds  $NM(=16)$  in the four currencies are measured, which give us  $NM^2$  (64) expected returns in our cross-sectional study. In a multi-country context equation (14) can be expressed in the following form:

$$R_e = [I \otimes B]\lambda \quad (17)$$

Where  $R_e$  is an  $NM^2$ -dimensional vector of expected hedged returns for all bonds in all countries.  $I$  is an identity matrix and  $\otimes$  is the Kronecker product.  $\lambda$  is an  $NK$ -dimensional vector containing specific risks premia for all factors and all currencies. Equation (20) can be reformulated in our case into:

$$\begin{bmatrix} R_e^{YEN} \\ R_e^{GBP} \\ R_e^{USD} \\ R_e^{EUR} \end{bmatrix}_\Delta = \begin{bmatrix} B & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & B & 0 \\ 0 & 0 & 0 & B \end{bmatrix} \begin{bmatrix} \lambda^{YEN} \\ \lambda^{GBP} \\ \lambda^{USD} \\ \lambda^{EUR} \end{bmatrix} \quad (18)$$

The parameters of this system can be estimated by the Generalised Moments Method developed by Hyeansen (1982)<sup>5</sup>. We estimate risk premium from one to seven risk factors. Table 7 presents specific risk premia for each currency. We remark that only six prices of risk are significant for all the countries.

This table shows that on the American market, prices of risk of the first factor and the third factor are significant. The second and the fifth factor prices of risk are significant only on the UK market. Whereas risk prices of the fourth and the fifth factors are significant for Germany and Japan, respectively.

**Table 7**  
Results of estimated prices of risk for the four-country model

This Table reports cross-section estimation of prices of risk of factors following equation (21). We suppose that each country has different prices of risk. Student-Statistics are indicated between parentheses for each prices of risk. \* indicates that prices of risks are significant at 95% level.

	$\lambda^{usd}$	$\lambda^{yen}$	$\lambda^{gbp}$	$\lambda^{eur}$
1	0.077* (5.245)	-0.062 (0.147)	0.012 (0.432)	-0.067 (1.325)
2	0.080 (0.146)	-0.059 (0.058)	0.009* (4.131)	-0.043 (0.980)
3	0.024* (2.034)	-0.025 (0.543)	0.068 (1.532)	-0.029 (0.654)
4	0.048 (0.120)	-0.052 (0.456)	0.047 (0.012)	-0.056* (2.654)
5	0.016 (0.149)	-0.043* (2.876)	-0.036* (-1.944)	0.027 (0.071)
6	0.063 (0.345)	-0.083 (0.855)	-0.083 (0.009)	0.054 (0.038)
7	0.045 (0.129)	-0.027 (0.243)	-0.110 (1.032)	0.079 (1.298)

Table 8 indicates that daily premium variations in international bond markets are explained by a common world factor. This result seems surprising when taking into account the complication process underlining the governmental bon market. This factor explains about 64.82% of the commune variation.

**Table 8**  
Explained variance for a multi-country model

This Table reports average explained variance for each factor compared to the whole variance of bond returns.

	<b>Returns of multi-factor model</b>
<b>1</b>	64.821 (%)
<b>2</b>	18.314 (%)
<b>3</b>	9.9640 (%)
<b>4</b>	0.4216 (%)
<b>5</b>	0.0125 (%)
<b>6</b>	0.0045 (%)
<b>Total</b>	<b>93.536 (%)</b>

## **B. Economic Interpretation**

While the principal component analysis shows that variation of premium results namely of a common factor, it appears insufficient to give any indication about underlying economic forces. This section examines this aspect in order to find a significant economic interpretation of the common factor. A factor is an abstract series that explains the common part variation of risk premium. In this sense it can be the world economic development, change in investors attitude face to risk.

Our methodology consists of analysing simple correlations between common factor series and variables reflecting world economic tendency. If it's impossible to precisely identify the significance of the common factor, this empirical investigation allows determination of the most important world tendency affecting bon risk premia. In particular, we propose examination of the explanatory power of the following variables: stock index return (S&P 500, FTSE and Nasdaq); long-term interest rates in the US and the slope of the yield curve in the US. We consider weekly data selected for the same period of study. Results are presented in Table 9.

The common factor is significantly correlated with many variables, which is explained by the high correlation existent between the world tendency variables, and the fact that common factor, in nature, associates all common determinants of risk premia in international bond markets. Globally, our study indicates positive correlation with American interest rate.

**Table 9**  
Correlation coefficients between common factor and economic variables

Returns are measured for 270 weekly observations over the period from 9/12/200 to 14/10/2005.

Correlation with economic variables	
<b>Stock Index</b>	
Nasdaq	0.214 (%)
FTSE	0.370 (%)
S&P 500	0.324 (%)
<b>United State interest rates</b>	
3-month EUR treasury returns	0.268 (%)
10-month EUR treasury returns	0.356 (%)
Yield curve	0.174 (%)

This positive relationship can be explained by the informative contain of the slope of the US yield curve, which is usually used as a common indicator to anticipate economic growth.

This is also corroborated by relatively narrow correlations with stock index<sup>6</sup>. For example, an increase of the return of S&P 500, is accompanied by an increase of the common factor, and in consequence an increase in bond returns.

### C. Prices of Risk Divergence across Country

Before testing price of risk conversion across-countries, we must, first of all, verify that price of risks is different across-countries. We apply the Wald to test the null hypothesis that risk premium are equivalent across-countries ( $\lambda_{usd} = \lambda_{yen} = \lambda_{gbp} = \lambda_{eur}$ ). Table 10 reports results of the Wald test. It's shown that price of risk of the first and the second factor are significantly different across-countries.

The first and the second factors show different price of risk across-countries. On one hand this difference must be reflected in the expected exchange rate, and on the other hand on the covariance between exchange rate and bond return in conformity to equations (14) and (15).

As a first step, in order to test the effect of price of risk differential on expected exchange rate, we regress the exchange return on risk prices spread and the interest rate differential across-countries. That is:

$$R_{Tchg} = \alpha + \beta(r_d - r_f) + \sum_{k=1}^K \delta_i(\lambda_{dk} - \lambda_{fk})dt \quad (19)$$

**Table 10**  
Wald hypothesis tests on currency-specific factor risk premia

This table reports p-value for Wald hypothesis tests. The null hypothesis testes is that factor risk premia are equal across countries:  $\lambda^{usd} = \lambda^{yen} = \lambda^{gbp} = \lambda^{eur}$ . Low p-values reject the null hypothesis.

	<b>P-Value</b>
<b>1</b>	0.04358*
<b>2</b>	0.00987*
<b>3</b>	0.23956
<b>4</b>	0.35512
<b>5</b>	0.78231
<b>6</b>	0.35056
<b>7</b>	0.89721

**Table 11**  
Estimation results of the regression of exchange rate returns on interest rates differential and prices of risk differentials

This table reports results of the test of expected variations of exchange rates that depends on interest rate differentials and prices of risk factors differential. \* indicates that  $\delta_i$  coefficient is significant at 10% level.

		<b>JPY/USD</b>	<b>GBP/USD</b>	<b>EUR/USD</b>
<b>One facteur</b>	$\delta_1$	0,013	0,014*	0,015
<b>Two factors</b>	$\delta_1$	0,051	0,052*	0,056*
	$\delta_2$	0,050	0,049	0,052*
<b>Three factors</b>	$\delta_1$	0,019	0,024	0,010
	$\delta_2$	0,064	0,055	0,059
	$\delta_3$	0,056	0,056	0,056
<b>Four factors</b>	$\delta_1$	0,013	0,014	0,015
	$\delta_2$	0,051	0,052	0,056
	$\delta_3$	0,050	0,049	0,052
	$\delta_4$	0,019	0,024	0,010

Table 11 reports results estimation of coefficients  $\delta_i$  for the first four factors. Though this table shows weak values of  $\delta_i$  coefficients in all countries, these coefficients appear significant for the case of two factors in EUR/USD and GBP/USD regression. However, the null hypothesis which support that exchange rate dynamic depends on risk prices differentials is rejected in the case of Japan.

The major remark to report from this table is that coefficients  $\delta_i$  is not significant for specifications with more than two factors for all countries. This is due to the fact that the first two factors are evaluated differently across-countries and must influence

the expected exchange rate. We conclude that prices differentials affect the expected exchange rate.

#### D. Role of Exchange Rate in Converting Price of Risks across Countries

With the aim of test if exchange rate has serve to convert prices of risk across countries, we test the null hypothesis which support that cross risk premium, estimated from a multi-countries model, equals the specific prices of risk spread to every currency.

Formally we test if  $\lambda_k^{\text{cross}} = \lambda_{dk} - \lambda_{fk}$ , where  $\lambda_k^{\text{cross}}$  is estimated from the following equation:

$$E\left[\frac{\Delta P_f}{P_f} \frac{\Delta S}{S}\right] = \sum_{k=1}^K b_{fk} \lambda_k^{\text{cross}} \quad (20)$$

We apply the Wald test to the product of expected bond returns and exchange return. Results of the test are presented in Table 12. Our results support the hypothesis that bond return covariance and exchange rate are identified by the spread of risk premium across-countries. At 10% level of significance, the null hypothesis is rejected only in the case of Yen/USD exchange rate.

**Table 12**

Wald hypothesis tests on expected cross products os asset returns and exchange rates

This table reports P-Values of the null hypothesis that:  $\lambda_k^{\text{cross}} = \lambda_{dk} - \lambda_{fk}$ , where  $\lambda_k^{\text{cross}}$  is the price of risk estimated through equation (23). \* indicates that the value is significant at level of 10%. Low values of p-values indicate rejection of the null hypothesis.

Nbr of factors	P-Value		
	JPY/USD	GBP/USD	EUR/USD
1	0 .013*	0 .314	0 .515
2	0 .051*	0 .452	0 .556
3	0 .050*	0 .249	0 .352
4	0 .219	0 .924	0 .210
5	0.025*	0.452	0.849
6	0.015*	0.547	0.955
7	0.000*	0.621*	0.125

Our cross-sectional analysis of bond returns supports the hypothesis that exchange rate serves to convert prices of risk as suggested by the International Asset Pricing Theory presented in section 2. Hence, expected variances of exchange rate can't be entirely explained by interest rates and risk price differentials in a cross-country model.

In this spirit, the rejection of equation (13) can be explained by incomplete market. In the case of incomplete market, exchange rate can be deduced from risk factors that are orthogonal to asset returns and can't be directly estimated through only market data.

Restriction of covariance must be maintained in the case of incomplete market. Its reject in the case of USD/Yen exchange rate indicates that Japanese market is not fully integrated with international markets. In the case of complete markets, assets with the same profile of risk must have the same expected returns after conversion to the commune currency.

Specific prices of risk are compatible with integrated markets as long as exchange risk recompenses spreads of these prices of risk. This is not the case of the Yen where differences in the first and the second factors are not adjusted by exchange rate. Limited integration of the Japanese market has been reported in many studies, i.e. French and Poterba (1991), Harvey (1991) and Campbell and Hamao (1992).

Two possible explanations can be advanced for this phenomenon. First, Japanese market was slightly liberalized. Hence, Japanese investors were faced with capital control that limited their investment opportunities in foreign markets. Second, the Japanese Yen was considered as a semi-floating currency, since the bank of Japan intervenes some times on the exchange market in order to imitate the appreciation of the Yen<sup>7</sup>. In this context, the bank of Japan has intervened in many reprises in 2001 to attenuate the appreciation of the Yen. These repeated interventions had maintained the Yen in an interval of 124 and 126 JPY per dollar at the middle of 2001.

## VI. CONCLUSION

The paper deduces conclusions about governmental bond returns through the exchange rate process. We have investigated loading factors that might account for the simultaneous increase in risk premia in many bond markets. Then we have tested two fundamental hypotheses of the international asset pricing theory. First, we test if hedged returns of international investment portfolio depend on the prices of risk of the domestic country of the investor or the price of risk where investment is realised. Second, we test the property of exchange rate to convert risk premium across-countries.

To test if factors of price of risk observed on bond market affect exchange process, we use a two-step approach. Firstly, we use the principal component analysis to estimate separately the principal factors in each bond market. Secondly, we estimated risk prices factors for all the countries considered. Specific factors of prices of risk to each currency are used to test if exchange rate convert expected exchange rate for all the currencies considered, only in the case of the Japanese market that is less integrated with international markets than American and European markets.

This paper involves three principal results. First, we show that international governmental bonds variation is due to a common loading factor. This one is, in mean, at the origin of the third of the whole daily variation. Idiosyncratic factors are in consequence dominating. Second, we confirm that prices of risk are different across countries. Finally, we have shown that exchange rates serve to convert prices of risk between countries and returns of hedged portfolios are determined by risk premium of

the domestic country of the investor and not by the prices of risk where investment has happened, i.e., an American investor can't apply German prices of risk to evaluate the performance of his investment in the German market.

#### ENDNOTES

1. Brown (1989), Litterman and Scheinkman (1991), Connor and Korajczyk (1993), etc.
2. This portfolio is formed by domestic and foreign assets.
3. German portfolio will be indicated by EUR since the exchange rate considered in the case of Germany is the Euro.
4. To estimate principal factors for each country, we precede Eisenberg (2003) approach used to extract principal factors from the share market of America, Japan, Germany and England.
5. This work is effectuated by Ogaki (1993).
6. A significant part of stock prices volatility is explained by variation of discount factor-degree of risk aversion (Cochrane (2001)).
7. Hiyao (2000).

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