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Dynamic versus Static Optimization of Hedge Fund Portfolios: The Relevance of Performance Measures

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ABSTRACT

This paper analyzes the relevance of a set of some performance measures for optimal portfolios including hedge funds. Four criteria are considered: the Sharpe Ratio, the Returns on VaR and on CVaR, and the Omega performance measure. The results are illustrated by an allocation on several indices: HFR (Global Hedge Fund Index), JPM Government Bond Index, S&P GSCI, MSCI World and the UBS Global Convertible. Both static and dynamic optimizations are considered. Due to the non-convexity of some of the criteria, we use the "threshold accepting algorithm" to solve numerically the optimization problems. The time period of the analysis is September 1997 to August 2007. Our results suggest that, for the dynamic optimization, the portfolio which maximizes the Omega measure has the more stable performances, in particular when compared to the Return-on-CVaR portfolio. As a by-product, we prove that all the optimal portfolios had to contain hedge funds for the time period 1997-2007.

JEL Classification: *C6, G11, G24, L10*

Keywords: *Hedge funds; CVaR; Tail risk; Omega measure*

I. INTRODUCTION

The seminal Markowitz's analysis is based on the first two moments of portfolio return and ignores the higher ones. By using an improved estimator of the covariance structure of hedge fund index returns, Amenc and Martellini (2002) prove that a portfolio including hedge funds can have a significantly smaller volatility (on an out-of-sample basis), while having almost the same mean return. This result suggests to incorporate hedge funds to get a mean-variance efficient portfolio. As illustrated by Cremers et al. (2005), hedge funds returns usually have higher means and lower standard deviations than standard assets, but they also have undesirable higher moment properties. For example, hedge funds returns can be negatively skewed. This is due to for instance to the non-linearity of their payoffs when they are generated by option-like strategies (see Goetzmann et al., 2002, 2003).

Many empirical studies have also proved that the mean-variance approach is no more valid when investors include hedge funds in their portfolios (Amenc and Martellini, 2002; Terhaar et al., 2003; McFall Lamm, 2003; Agarwal and Naik, 2004; Morton et al., 2006).

Empirical studies show that the assumption of normality in return distribution is not justified, in particular when dealing with hedge funds which have significant positive or negative skewness and high kurtosis (Fung and Hsieh, 1997; Ackerman et al., 1999; Brown et al., 1999; Caglayan and Edwards, 2001; Bacmann and Scholz, 2003; Agarwal and Naik, 2004).

One possible extension of the mean-variance analysis is the use of the expected utility theory. For a truncated utility at the order 4, we have to maximize a linear combination of the first four moments of the return. In this more general framework, the hedge fund portfolio is built according also to the skewness and kurtosis. This problem has been studied by Jurczenko and Maillet (2004), Malevergne and Sornette (2005), Anson et al. (2007)...Martellini and Ziemann (2007) extend the Black-Litterman Bayesian approach when building a portfolio of hedge funds with non-trivial preferences about higher moments. Their results show that active style allocation provides a significant value in a hedge fund portfolio if we take account of non-normality and parameter uncertainty in hedge fund return distributions.

Since the demise of Long Term Capital Management (LTCM), downside risk measures have been introduced to take more account of the tails of the distributions. In most cases, they are based on the Value-at-Risk (VaR) approach. As proved by Goetzman et al. (2003), Agarwal and Naik (2004), the VaR can be high, due to important leverage effect. VaR also is more appropriate than the first four moments to measure extreme risks. VaR optimization has been previously studied by Litterman (1997a, 1997b), Lucas and Klaasen, (1998). Additionally, Favre and Galeano (2002) introduce a new Value-at-Risk method to adjust the volatility risk with the skewness and the kurtosis of the distribution of returns. They minimize the modified VaR at a given confidence level. However, as pointed by Lo (2001), VaR has several shortcomings. For instance, it does not indicate the magnitude of the potential losses below itself. Additionally, it is not a convex risk measure (see Artzner et al., 1999) and its minimization is rather involved.

For these reasons, Acerbi and Tasche (2002) introduce the expected shortfall (ES). This measure, also called CVaR, measures the amount of losses below the VaR, is coherent and leads to portfolio optimization problems that can be solved more easily.

Indeed, Rockafellar and Uryasev (2000) prove that the CVaR minimization is equivalent to a convex optimization problem. Agarwal and Naik (2004) estimate ES from the HFR hedge fund indices. They show that downside risk is significantly underestimated by the mean-variance analysis, which suggests that mean-ES optimization must be introduced.

Given benchmark related investment objectives, Popova et al. (2007) study the optimal allocation to hedge funds, for criteria such as expected shortfall and semi-variance. Using a stochastic programming model based on Monte Carlo simulation, they prove that a 20% allocation to hedge funds is justified. Their optimal portfolios are more skew to the right relative to those of the optimal mean-variance portfolios. Thus, they have higher Sortino ratios.

Liang and Park (2007) examine the risk-return trade-off for the hedge funds. They compare semi-deviation, VaR, ES and Tail Risk (TR) at both the individual fund and the portfolio levels. They show that the cross-sectional variation in hedge fund return is well explained by the left-tail risk captured by ES and TR and not by the other risk measures. They prove that between January 1995 and December 2004, hedge funds with high ES outperform those with low ES (annual return difference of 7%).

As shown by Hübner (2007) for the information ratio and the alpha, the relevance of performance measures heavily depends on the kind of portfolios that is managed by the investor. In this paper, we propose also to examine the relevance of different performance measures when determining optimal portfolios including hedge funds, while taking account of portfolio constraints. In particular, we focus on the Sharpe, mean VaR and CVaR ratios. As a by-product, we also introduce the Omega performance measure, defined by Keating and Shadwick (2002) and Cascon et al. (2003). This ratio takes account of all the moments of the distribution. It penalizes the returns below a given level and emphasizes the returns beyond this threshold.

Section 2 recalls definitions and main properties of the performance measures that are used in the paper. Section 3 examines the static and dynamic optimization of the four performance measures, using VaR Cornish-Fisher estimates and the threshold accepting algorithm for portfolio optimization. The empirical analysis is based on a portfolio that includes five indices: HFR (Global Hedge Fund Index), JPM GBI, S&P GSCI TR, MSCI World and the UBS Global Convertible.

The results highlight the importance of the asset allocation model based on the maximization of the Omega performance measure, with or without specific constraints on portfolio weights. Indeed, this approach takes account of the characteristics of hedge fund portfolios. Some empirical illustrations are gathered in the Appendix.

II. RISK-ADJUSTED PERFORMANCE MEASURES AND OPTIMIZATION

To compare funds with different characteristics of returns and risks, several performance measures have been introduced (see Lhabitant (2004) for more details about the choice of performance measures). Among them, the Sharpe Ratio, the Returns on VaR and on CVaR, and the Omega performance measure.

For each criterion to be maximized, we consider that the investor chooses among n financial assets with returns $R = (R_1, \ldots, R_n)$. Let R_f be the return of the riskless asset. Denote $e = (1, \ldots, 1)$. Denote also by $w = (w_1, \ldots, w_n)$ the weighting vector satisfying :

$$w^t.e = \sum_{i=1}^{n} w_i = 1 \tag{1}$$

We assume that no short selling is allowed: $w_i \geq 0$. The portfolio return at maturity with cdf F and mean $\bar{R}$ is given by:

$$R^w = w^t.R = \sum_{i=1}^{n} w_i R_i \tag{2}$$

A. The Sharpe Ratio Maximization

It is the standard optimization problem, which is yet a benchmark. We have to solve:

$$\text{Max} \frac{w^t \bar{R} - R_f}{\sqrt{w^t \Sigma w}} \text{, with } w^t.e = 1 \text{ and } w \geq 0 \tag{3}$$

where Σ is the variance covariance matrix. The mean $\bar{R}$ is computed from data, observed along a given time period.

B. The Mean-VaR Optimization

The CVaR can be defined as the expected value of the portfolio loss below a given threshold, given the fact that this loss is higher that this level (see Acerbi, 2004). The Ro CVaR (or Mean CVaR) ratio is defined by:

$$\text{RoCVaR}(\alpha) = \frac{\bar{R} - R_f}{\text{CVaR}(\alpha)} \tag{42}$$

Then, the maximization problem is given by:

$$\text{Max} \frac{w^t \bar{R} - R_f}{\text{CVaR}} \text{, with } w^t.e = 1 \text{ and } w \geq 0 \tag{5}$$

Note that, if we assume that returns $(R_j^w)_{1 \leq j \leq m}$ are independent, the CVaR optimization problem is equivalent to: (notation: $\mathbf{1_A}$ is the indicator function of set A).

$$\text{Max} \frac{(w^t.\bar{R} - R_f)}{\left(\frac{\sum_{j=1}^{m} R_j^w 1_{R_j^w > VaR}}{\sum_{j=1}^{m} 1_{R_j^w > VaR}} \right)} \text{, with } w^t.e = 1 \text{ and } w \geq 0 \tag{6}$$

Another performance measure can take account of the characteristics of hedge funds returns: the Omega function, introduced by Keating and Shadwick (2002, 2003).

C. The Omega Optimization

The Omega function Ω(L) can potentially take account of the whole probability distribution of the returns. The Omega measure introduced by Keating and Shadwick (2002) is based on the stochastic dominance approach. The measure Ω(L) is equal to:

$$\Omega_F(L) = \frac{\int_L^b (1 - F(x))dx}{\int_a^L F(x)dx} \qquad (73)$$

where F(.) is the cdf of the random variable (for example equal to the portfolio return) with support in [a, b]. The level L is the threshold chosen by the investor: returns smaller than L are viewed as losses and those higher than L are gains. For a given threshold L, the investor would prefer the portfolio with the highest Omega measure. As shown by Kazemi, Schneeweis and Gupta (2003), the ratio Omega is equal to:

$$\Omega_{F_X}(L) = \frac{E_P\left[(X - L)^+\right]}{E_P\left[(L - X)^+\right]} \qquad (84)$$

This is the ratio of the expectations of gains above the given level L upon the expectation of losses below L. Therefore, $\Omega_{Fx}(L)$ can be interpreted as a ratio Call/Put defined on the same underlying asset X, with strike L and computed with respect to the historical probability P. The Put is the risk measure component. It allows the control of the losses below the threshold L. Kazemi et al. (2003) define the Sharpe Omega by:

$$\text{Sharpe}_\Omega(L) = \frac{E_P[X] - L}{E_P[(L - X)^+]} = \Omega_{F_X}(L) - 1 \qquad (95)$$

If $E_p(X) < L$, the Sharpe Omega measure is negative. Otherwise, it is positive. Suppose that X is the value at maturity T of a given asset S with a Lognormal distribution:

$$X = S_0 \exp[(\mu - \sigma^2 / 2)T + \sigma W_T] \qquad (106)$$

where W_T has a Gaussian distribution. Then $E_P(X)=S_0 \exp[\mu T]$ does not depend on the volatility σ. Thus, if $S_0 \exp[\mu T]<L$, then the Sharpe Omega ratio is an increasing function of the volatility σ, whereas, for $S_0 \exp[\mu T]>L$, it is a decreasing function of σ.

Due to the previous properties, the usual values of the level L are such that $L < \bar{X}$. Indeed, it is more convenient for a performance measure to be a decreasing function of the standard risk σ than the converse. The corresponding optimization problem is:

$$\text{Max}\,\Omega(L)[R^w] \text{ with } w^t.e = 1 \text{ and } w \geq 0 \qquad (117)$$

which is equivalent to:

$$\text{Max}\,\frac{E[(R^{w}-L)^{+}]}{E[(L-R^{w})^{+}]},\ \text{with } w^{t}.e=1 \text{ and } w\geq 0 \qquad (128)$$

Then, from the observation of independent returns $(R_j^{w})_{1\leq j\leq m}$, we have to solve:

$$\text{Max}\,\frac{\sum_{j=1}^{m}(R_j^{w}-L)1_{R_j^{w}>L}}{\sum_{j=1}^{m}(L-R_j^{w})1_{R_j^{w}<L}},\ \text{with } w^{t}.e=1 \text{ and } w\geq 0 \qquad (139)$$

Due to the non convexity of some of the previous objective functions, we use the threshold accepting algorithm to solve numerically the optimization problems. This algorithm has been introduced by Dueck and Scheuer (1990). It is a refined version of the standard local search procedure (for more details, see Winker, 2001). It is a local search procedure which accepts moves to neighbourhood solutions that improve the objective function value. Dueck and Scheuer (1990) prove that the Threshold Accepting Algorithm converges to the optimal portfolio solution when dealing with complex objective functions such as shortfall optimization case. Recently, Gilli and Kellezi (2001) have used this algorithm for CVaR and Omega portfolios optimization.

III. EMPIRICAL RESULTS

In what follows, we search for the optimal portfolio based on indices. The optimal allocation is determined among five main asset classes: Hedge funds, Equities, Bonds, Commodities and Convertibles. Two optimization methods are used:

1. A static optimization with only one period (the whole period of observations).
2. A dynamic optimization based on the back testing method.

A. The Data Analysis

The portfolio contains the following indices:

1. The HFRX Global Hedge Fund USD.

The index is built as follows: from Sep-97 to March-03, the index corresponds to the mean of performances of hedge funds, quoted by the Lipper basis (without survivorship bias, as in Liang, 2000); and, from April-03 to Aug-07, the index is the HFRX Global Hedge Fund USD.

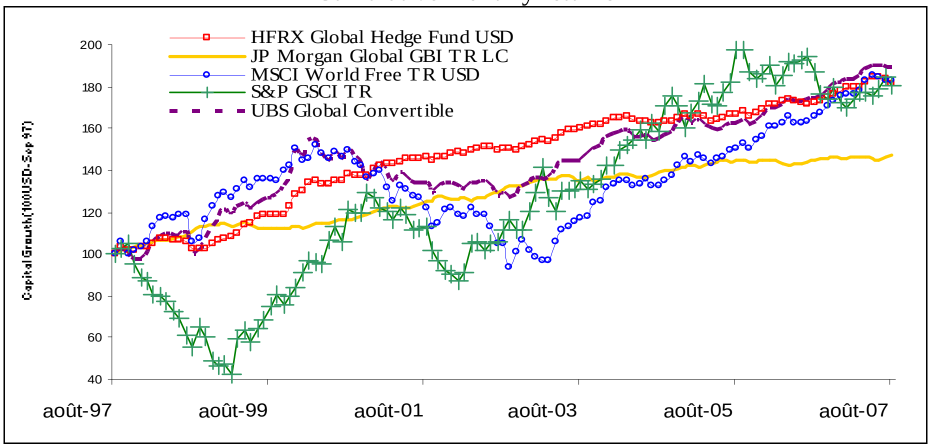
2. The UBS Global Convertible. The index represents convertibles.
3. The JPM Global GBI LC. The index represents the performance of bonds in local currency.
4. The MSCI World Free Equity. It is the international equity index.
5. The S&P GSCI. It is the commodities index.

The data on hedge funds are obtained from the commonly used Lipper TASS. The time period of the analysis lies between September 1997 and August 2007. The sample contains 121 net-of-fees monthly returns. Looking at realized returns for the given time period, the hedge funds and convertibles generally have better performance, as shown by Figure 1. The HFR index returns were smooth and steady despite the high correlation with stock markets (represented by MSCI World Free).

The index has shown real capacity to preserve capital through some crisis periods such as 1998 (LTCM collapse) or 2000 (Technology bubble). During the financial crisis 2000-2002, the hedge funds had clearly higher returns (positive returns while those of the equities were significantly negative).

Figure 1

Cumulative monthly returns



The S&P GSCI TR has the worst performance until 2002, while growing up and having the best performance from January 2005 to January 2007. The Global Hedge Fund index has regularly grown up. In particular, it has not fallen from 2000 to 2002, contrary to the MSCI World and has provided capital protection during this time period. Looking at cumulative returns, the UBS Convertible index has risen during two periods: from Sep-97 to Sep-00 and from Jan-03 to Sep-07, while it has fallen between these two periods, which can be explained by a falling stock market and spreads widening. Looking at correlations, for most of the cases, the equity index has the highest correlation with the other indices, in particular with the Convertible and Hedge indices. The Convertibles also have a high correlation with the hedge funds. The main statistical properties of the five indices are provided in Tables 1 and 2.

Table 1

Correlations between the indices

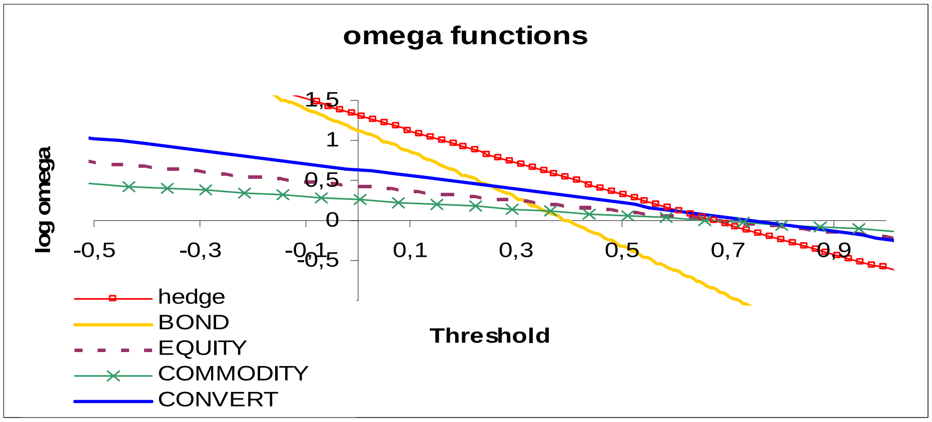
Indices		1	2	3	4	5
Hedge	1	1.00				
Bond	2	-0.12	1.00			
Equity	3	0.57	-0.27	1.00		
Commodity	4	0.20	0.03	0.01	1.00	
Convertible	5	0.73	-0.11	0.81	0.13	1.00

Table 2
Statistical characteristics of the indices (monthly)

	Hedge	Bond	Equity	Commodity	Convertible
Mean	0.68	0.40	0.69	0.67	0.74
Median	0.60	0.46	1.23	0.42	0.95
Maximum	5.69	2.19	9.07	16.89	11.21
Minimum	-3.53	-1.59	-13.32	-14.41	-9.34
Std. Dev	1.40	0.84	4.08	6.35	3.05
Skewness	0.42	-0.32	-0.64	0.10	-0.21
Kurtosis	4.25	2.64	3.76	2.67	4.24
VaR (p=5%)	1.62	1.25	8.42	10.33	4.52

The convertibles have the highest mean, but with a rather high standard deviation. The equity index has similar properties as the convertible index but with a more negative skewness. The hedge index has a weak standard deviation and a positive skewness. Its mean is similar to those of the equity and commodity indices. The highest VaR and CVaR at the level 5% have the same order of magnitude as the commodity index. Those of the convertibles are higher than those of the equity index. The Omega ratios of the five indices are displayed in Figure 2, as function of the threshold L.

Figure 2
Omega measures of the five indices



The hedge fund index dominates the other indices on the set of values $L \in [0\%;0.65\%]$. The bond index dominates the equity, commodity and convertible indices for thresholds $L \in [0\%;0.02\%]$ (which are usual value of L). Most of the time, the commodity index is dominated by the other ones. During the period 1997-2007, the commodities have not provided a true diversification for the four static optimal portfolios. This property is usually verified. This is due to the left tail of its distribution, as shown for example by its VaR at the level 5% which is equal to 10, 33.

B. The Static Framework

In what follows, we examine the static optimal allocations. Our base value for the Omega threshold L is equal to 0%. Table 3 indicates the optimal allocations (percentage) which correspond to each optimization criterion.

Table 3
Static optimal weights

Portfolio	Hedge	Bond	Equity	Commodity	Convertible
Sharpe	0.29	0.66	0.00	0.00	0.05
RoVaR	0.45	0.55	0.00	0.00	0.00
Omega	0.43	0.37	0.06	0.02	0.12
RoCVaR	0.41	0.02	0.54	0.03	0.00

Only the hedge fund index is a significant component of all optimal portfolios. This is due to the positive skewness of the hedge index (sk=0.42) while having a mean (E=0.68) similar to the other indices (except the convertibles). For example, for the equity index (sk=-0.64, E=0.69), the Sharpe Omega is weaker than for the hedge index. Indeed, the expectation of the losses $E[(-X)^+]$ is higher since the skewness is negative, contrary to the hedge index. The Omega portfolio is the most diversified: all its weights are different from zero. The MeanCvaR and Omega optimal portfolios contain much more hedge funds than convertibles. Indeed, despite its highest mean, the convertible index has high VaR and CVaR. However, the weights on equities and bonds are quite different for the MeanCvaR and Omega optimal portfolios. In fact, the Omega performance measure is more sensitive to the probability of a drawdown than the MeanCVaR criterion. This property is explained in what follows. Consider a given threshold *L* and a random return R. The EroCVaR (or Excess Mean CVaR) ratio is defined by:

$$\mathrm{ERoCVaR}(\alpha, L) = \frac{\bar{R} - L}{\mathrm{CVaR}(\alpha)} \tag{1410}$$

where $\bar{R}$ denotes the expectation of the rate of return $E_P[R]$. Since we have:

$$\begin{aligned} \mathrm{CVaR}(\alpha) &= E_p[-R \mid -R \geq \mathrm{VaR}(\alpha)] \\ &= E_p[L - R1_{-R \geq \mathrm{VaR}(\alpha)}] / E_p[-R \geq \mathrm{VaR}(\alpha)] \end{aligned} \tag{15}$$

we deduce that, for L=0, ERoCVaR(α,0) = RoCVaR(α) and also that

$$\mathrm{RoCVaR}(\alpha) = \frac{RP[-R \geq \mathrm{VAR}(\alpha)]}{E_p[-R1_{-R \geq \mathrm{VaR}(\alpha)}]}, \text{and}\, \mathrm{Sharpe}_{\Omega}(0) = \frac{\bar{R}}{E_P[-R1_{-R \geq 0}]}$$

Then, in that case, maximizing RoCVaR(α) does not penalize P[-R≥VaR(α)] as far as the ratio $\bar{R}/E_p[-R1_{R \geq VaR(\alpha)}]$ and the SharpeΩ ratio. For L=-VaR(α), we get:

$$RoCVaR(\alpha, L) = \frac{[\bar{R} + VaR(\alpha)]P[-R \geq VaR(\alpha)]}{E_p[-R1_{-R \geq VaR(\alpha)}]} \quad (1611)$$

and

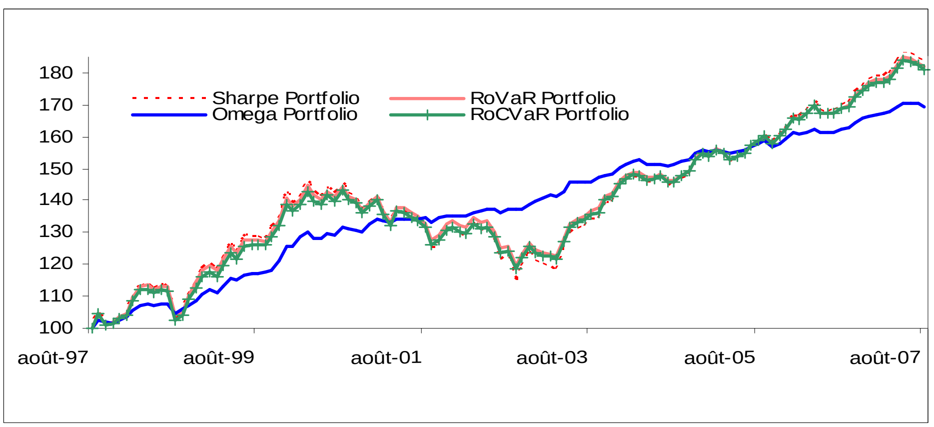
$$Sharpe\Omega(L) = \frac{\bar{R} + VaR(\alpha)}{E_p[-R1_{-R \geq VaR(\alpha)}] - VaR(\alpha)E_p[-R \geq VaR(\alpha)]}$$

Thus, both RoCVaR(α) and SharpeΩ maximizations do not penalize the probability P[-R≥VaR(α)] in the same manner. Note that both criteria do not penalize the probability to bear losses beyond VaR(α) as far as the ratio:

$$\frac{\bar{R} + VaR(\alpha)}{E_p[-R1_{-R \geq VaR(\alpha)}]}$$

Let us examine the cumulative returns of the four optimal portfolios. These returns are determined from the optimal allocations and the observed index returns on the given time period 1997-2007.

Figure 3
Cumulative returns of the four optimal portfolios



We note that the Omega optimal portfolio is the more stable one, while the other static portfolios have quite similar cumulative returns. This is in line with the purpose of such portfolio which is to limit the downside risk and potential losses for L=0%. The main characteristics of the returns are provided in Table 4. The Omega portfolio has the smaller mean but only this portfolio has a positive skewness. Additionally, it has the best kurtosis.

Table 4
Optimal portfolio characteristics

	Sharpe	RoVaR	Omega	RoCVaR
Mean	0.70	0.68	0.58	0.68
Median	1.11	1.04	0.50	1.01
Maximum	7.27	7.01	4.39	6.85
Minimum	-9.74	-8.87	-2.78	-8.77
Std. Deviation	0.97	2.64	1.10	2.58
Skewness	-0.60	-0.55	0.26	-0.58
Kurtosis	0.82	0.76	0.97	0.83
VaR (p= 5%)	7.42	6.16	1.69	3.20
CVaR	1.27	1.00	0.10	5.18
Sharpe	0.72	0.26	0.53	0.26

For the threshold L=0%, the Omega portfolio return is almost Gaussian. This is in particular due to the diversification effect, as illustrated in Table 5 and due to standard statistical property. As expected, the Omega optimal portfolio has the highest Omega value, for thresholds L below 0.6%. Note that this value corresponds (approximately) to the means of the four portfolios. The other portfolios have quite similar Omega performances for L smaller than 0.6%. The previous results show that the four optimal portfolios exhibit similar absolute performances (about 0.6% per month). Additionally, they suppose that parameters of interest are quite anticipated. In practice, this assumption is rather strong. In what follows, we propose a more realistic framework: the dynamic allocation, as used by fund managers.

C. The Dynamic Allocation

Now, we consider dynamic allocations. The data correspond to monthly observations from 30/09/1997 to 31/08/2007 (Currency: US Dollar).

1. Principle and Main Results

Dynamic allocations are based on two time periods: the analysis period and the managing period. The simulation is carried through two steps: First, the estimation of the optimal portfolio weights using the data in the analysis window and, second, the calculation of the optimal portfolio performances over the projection window on a monthly basis. Next Table 5 provides statistical characteristics of the four optimal portfolios according to various values of both the analysis window length and the projection window length.

Table 5

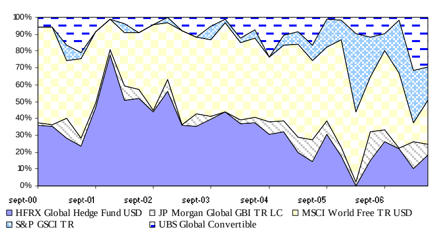
Statistical properties of the four optimal portfolios

	Omega th=0						Sharpe					
	Aw=36 Pw=6	Aw=24 Pw=6	Aw=12 Pw=6	Aw=36 Pw=3	Aw=24 Pw=3	Aw=12 Pw=3	Aw=36 Pw=6	Aw=24 Pw=6	Aw=12 Pw=6	Aw=36 Pw=3	Aw=24 Pw=3	Aw=12 Pw=3
Mean	0,54	0,59	0,69	0,55	0,60	0,72	0,46	0,56	0,74	0,47	0,54	0,70
Median	0,47	0,51	0,56	0,54	0,54	0,63	0,49	0,46	0,68	0,50	0,45	0,63
Max	3,28	3,46	5,11	2,59	4,68	5,32	2,29	2,79	6,82	2,29	2,79	6,84
Min	-2,45	-3,24	-2,86	-2,13	-2,70	-3,82	-1,79	-1,75	-2,34	-1,79	-1,75	-2,62
Std.Dev	1,09	1,22	1,31	0,93	1,37	1,67	0,67	0,87	1,32	0,58	0,88	1,38
Skweness	-0,22	-0,23	0,28	-0,22	0,23	0,67	-0,32	0,11	1,47	-0,32	0,21	1,39
Kurtosis	0,67	0,35	1,13	0,30	0,37	2,54	0,42	-0,06	5,41	0,52	0,11	4,96
Sharpe ratio	0,49	0,48	0,53	0,59	0,44	0,43	0,69	0,65	0,56	0,81	0,61	0,51
VaR 5%	1,14	1,55	0,69	0,94	1,62	0,86	0,74	1,28	0,88	0,17	0,07	1,30
CVaR	1,85	2,11	2,02	1,50	2,13	2,53	1,18	1,14	1,62	1,15	1,26	1,76
observations	84	96	108	84	96	108	84	96	108	84	96	108

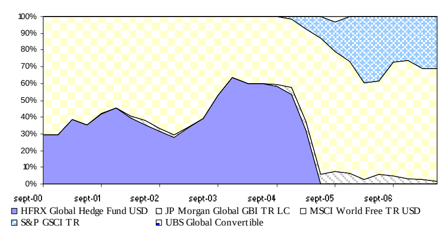
	VaR p=0,05						CVaR					
	Aw=36 Pw=6	Aw=24 Pw=6	Aw=12 Pw=6	Aw=36 Pw=3	Aw=24 Pw=3	Aw=12 Pw=3	Aw=36 Pw=6	Aw=24 Pw=6	Aw=12 Pw=6	Aw=36 Pw=3	Aw=24 Pw=3	Aw=12 Pw=3
Mean	0,46	0,56	0,71	0,47	0,56	0,66	0,38	0,75	0,61	0,47	0,60	0,86
Median	0,50	0,51	0,66	0,50	0,44	0,63	0,48	0,74	0,83	0,54	0,87	0,95
Max	2,27	2,56	5,84	2,28	2,57	6,18	6,04	6,90	5,32	2,35	12,72	6,44
Min	-1,74	-1,99	-2,84	-1,74	-1,99	-2,67	-8,76	-6,16	-7,75	-1,63	-10,71	-4,33
Std.Dev	0,73	0,87	1,29	0,72	0,88	1,33	2,82	2,26	2,33	0,70	3,28	2,22
Skweness	-0,31	0,02	0,69	-0,32	0,17	1,14	-0,73	-0,22	-0,75	-0,29	-0,26	-0,05
Kurtosis	0,61	0,05	2,94	0,72	0,18	4,23	1,26	1,23	1,27	0,84	2,77	-0,17
Sharpe ratio	0,63	0,64	0,55	0,65	0,64	0,50	0,14	0,33	0,26	0,68	0,18	0,39
VaR 5%	0,64	1,30	1,28	0,42	0,03	0,96	4,91	0,13	2,17	0,59	6,78	2,08
CVaR	1,18	1,23	1,98	1,13	1,18	1,92	6,69	4,67	5,08	1,08	7,56	3,54
observations	84	96	108	84	96	108	84	96	108	84	96	108

Figure 4

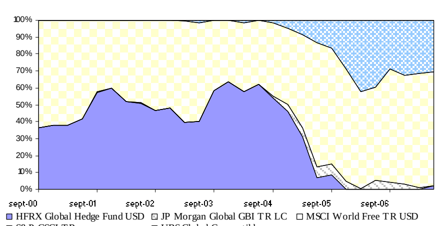
Optimal allocation for the four criteria (A_w=36; P_w=3)



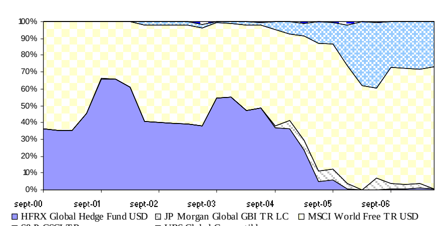
Omega optimal weights



Sharpe optimal weights



RoVaR optimal weights



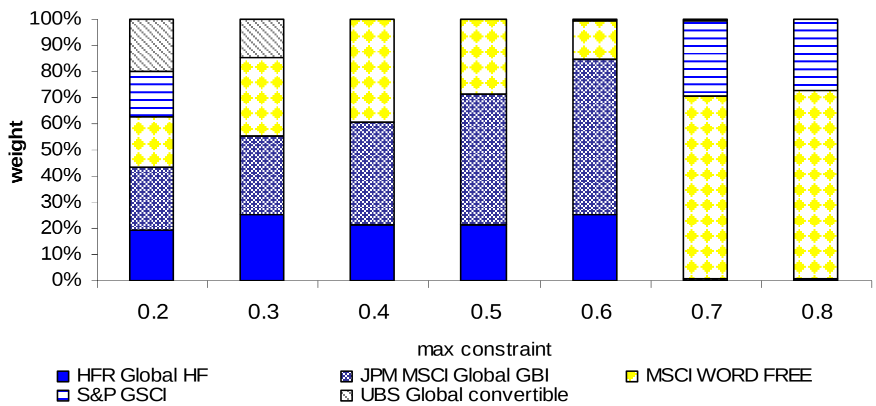
RoCVaR optimal weights

Figure 6 (see Appendix) provides the cumulative returns of the four optimal portfolios. It illustrates their comparison according to the choice of A_w and P_w. The first column corresponds to P_w=6 and the second one to P_w=3. As it can be seen, the performances of the four portfolios are more sensitive to the value A_w than to the value P_w. Indeed, for example, for A_w=36 and A_w=12, the Omega portfolio has better performances than the Mean-CVaR one whereas for A_w=24, it is the converse. Note that the Mean-CVaR portfolio is the most sensitive to the choice of A_w, while the Omega is the less sensitive. For the three other portfolios, the performances are relatively stable with respect to the choices of both A_w and P_w. Figure 4 describes the evolution of the four portfolio allocations. The parameter values are: A_w=36 P_w=3 , which are the most used in practice (36 months correspond to the average life time of hedge funds; 3 months is the standard rebalancing time). The Omega portfolio is relatively different from the three other ones. For example, it is the only one which contains convertibles and still contains hedge funds from 2006. The Sharpe, MeanVaR and MeanCVaR portfolios have similar weighting evolutions.

2. The Omega Portfolio

The main statistical characteristics of the Omega portfolio for L=0, A_w=6, and P_w=3 show that the Omega portfolio has the highest Sharpe ratio (equal to 0,59) and the smallest kurtosis. Figure 7 (see Appendix) shows how the optimal portfolio depends on the upper bound “a” which is imposed on weights. A small upper bound induces more diversification. Note for example that, for an upper bound about 30% to 40% on all weights, the Omega portfolio must include convertibles, whereas, for higher upper bound values, it does not contain any convertible. Figure 5 provides the optimal Omega allocations for May 2007, according to various upper bound constraints (A_w=6, P_w=3). For a=0.20, all the optimal weights are necessarily equal to 20%. The higher the upper bound, the smaller the diversification.

Figure 5
Omega portfolio weights according to the maximal constraint



In what follows, we examine the properties of the Omega optimal portfolio, according to the threshold L which is varying from 0% to 0.8% step by step equals to 0.05%. The choice of the upper and lower target L is justified by the characteristic of the index portfolio. In fact, the mean of the distribution for all assets in the index portfolio is 0.66%. From August 2004 to August 2007, the Omega portfolio associated to the threshold L=0.6% provides the best cumulative returns. Note that the Omega portfolio corresponding to the value L=0% has the more stable returns along the whole period (as for the static case). Table 6 illustrates the influence of the threshold on the four first moments and the extreme values.

Table 6

Characteristics of the Omega portfolios for different target thresholds L

	Threshold																
	0,00	0,05	0,10	0,15	0,20	0,25	0,30	0,35	0,40	0,45	0,50	0,55	0,60	0,65	0,70	0,75	0,80
Mean	0,55	0,52	0,57	0,47	0,58	0,52	0,52	0,56	0,58	0,36	0,52	0,62	0,65	0,38	0,54	0,84	0,55
Max	2,59	4,20	4,55	4,38	7,41	4,96	4,96	5,89	6,57	6,77	11,50	12,35	8,86	9,15	9,83	9,27	8,00
Min	-2,13	-4,31	-2,42	-4,57	-3,36	-3,35	-3,35	-4,54	-6,87	-6,62	-5,88	-6,70	-5,41	-8,07	-6,45	-7,67	-6,06
Std .Dev	0,93	1,33	1,47	1,62	2,05	1,76	1,76	2,10	2,20	2,35	2,68	3,14	2,63	2,90	2,90	2,89	2,78
Skewness	-0,22	-0,27	0,31	0,02	0,92	0,10	0,10	0,04	0,15	0,02	0,79	0,68	0,52	0,24	0,30	0,09	0,24
excess-Kurtosis	0,30	1,50	-0,25	0,49	1,61	0,04	0,04	0,48	1,30	0,45	2,77	2,02	0,43	0,67	0,61	0,91	0,31

The highest Sharpe ratio is reached for the threshold equal to 0%. However, the Sharpe ratio is not a decreasing function of the threshold (see for example the values for L=0.15 and L=0.25). Note also that none of the standard statistical characteristics is a monotonic function with respect to the threshold L. For values of the threshold above 0.10%, the skewness is positive since increasing the threshold corresponds to the search of higher right tail of the distribution.

IV. CONCLUSION

We have examined the relevance of four performance measures when they are used to determine optimal portfolios including hedge funds. Both CVaR and Omega measures are more appropriate, especially when the Cornish-Fisher expansion is introduced to calculate the CVaR. In the static optimization framework, the Omega provides more stable results whereas, for high volatility, the CVaR portfolio seems to perform better. In the dynamic case, corresponding to more actual portfolio management of hedge funds, the difference between the two methods is due to the different penalization of a given drawdown. We have also developed the analysis of the Omega performance measure when upper bounds on the weights are considered and when the associated threshold varies. As a by-product, we have shown that all the optimal portfolios had to contain hedge funds, for the time period 1997-2007. All these results are in line with those that can be deduced, for example when dealing only with purely hedge funds portfolios such as the commodities trading advisers (CTA).

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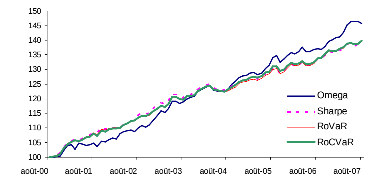
Appendix

Figure 6
Cumulative returns according to the analysis and projection windows



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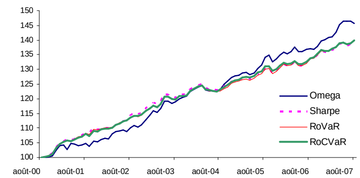

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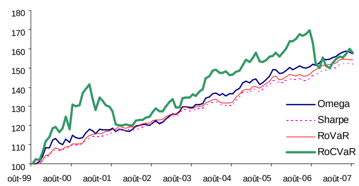

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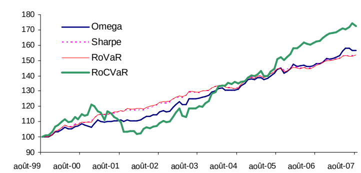
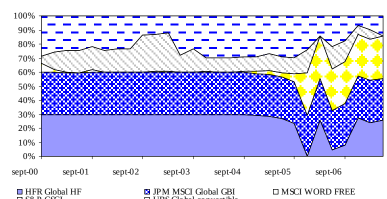
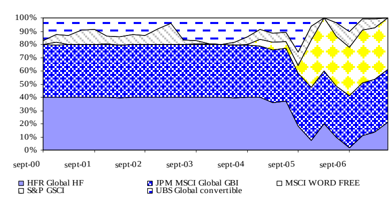


Aw=24 Pw=6

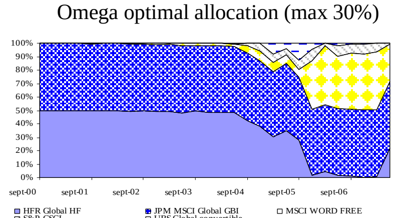
Figure 7
Omega optimal allocation according to the upper bound



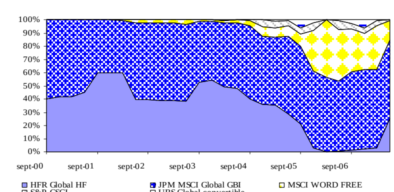
Omega optimal allocation (max 30%)



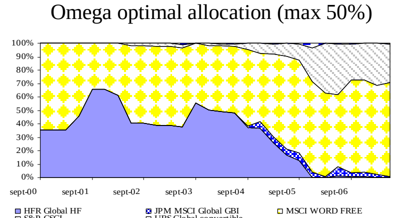
Omega optimal allocation (max 40%)



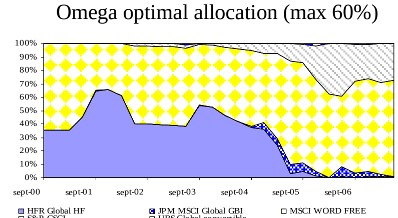
Omega optimal allocation (max 50%)



Omega optimal allocation (max 60%)



Omega optimal allocation (max 70%)



Omega optimal allocation (max 80%)