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Interdependence between Exchange Rates: Evidence from Multivariate Fractional Cointegration

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ABSTRACT

In this paper, we examine the relationship between the Tunisian Dinar relative to US Dollar, the Euro and the Japanese Yen using multivariate fractional cointegration approach developed by Davidson (2002, 2006). The main advantage of this approach is to detect the long term relationship as well as the short term dynamics and to represent the interdependence between the variables. Moreover, we implement a causal analysis (Granger (1988)) to examine the links between the variables.

The empirical results provide evidence of fractional cointegration between the exchanges rates and show long-run causal links between the variables. In particular, we find a significant bidirectional causal links. Thus, we conclude to the existence of interdependence between the exchange rate series and we reject the hypothesis of weak informational efficiency in the Tunisian exchange market.

JEL Classification: C13, F31, G14

Keywords: *Dependence exchange markets; Weak exchange market efficiency hypothesis; Fractional cointegration model; Rank of cointegration; Causal analysis*

I. INTRODUCTION

The interdependence between the foreign exchange markets has been largely examined in the literature and numerous empirical studies have focused on investigating the relationship between the exchange rate series. The existence of such relationship is important and implies that the price of the current market is related to the price of the foreign market in the sense that we could be able to outperform the market using additional information of the others markets. Consequently, we reject the weak efficient market hypothesis which stipulates that current prices fully reflect past prices and information available.

Earlier studies consider that the exchange rate series are non stationary and thus employ the conventional cointegration tests developed by Engle and Granger (1987) and Johansen (1988) to check for a possible equilibrium relationship between the variables (Baillie and Bollerslev (1989), Diebold et al. (1994)).

Others studies show that the conventional cointegration tests may fail to find evidence of cointegration when the variables and particularly their equilibrium relationships exhibit characteristics that are more consistent with long memory (Cheung and Lai (1993), Andersson and Gredenhoff (1999), Gonzalo and Lee (2000), Smallwood and Norrbin (2002)). For that, they consider the fractional cointegration approach developed by Granger (1986). This approach consists on testing if the estimated residuals from the linear cointegration relationship are I(1) against distributed as a fractional process. Although this approach permits to test the presence of fractional cointegration, it supposes that the series are stationnary and is limited to univariate framework.

Recently, Davidson (2002) develops an alternative approach for testing fractional cointegration in multivariate framework. In particular, this approach considers that the processes are fractionally integrated and is powerful enough to distinguish between linear cointegration and fractional cointegration, and thus provide more robust results than conventional cointegration tests.

The aim of this paper is to examine the empirical relationship between the exchange rates using multivariate fractional cointegration approach (Davidson (2002)) and to analyse the causal links between the variables (Granger (1988)).

In order to determine the cointegrating rank, we apply in addition to the minimal algorithm of Davidson (1998a,b) the procedure of Nielson and Shimotsu (2007) based the exact local Whittle analysis of Shimotsu (2005).

This paper is structured as follows. Section II describes the fractional cointegration model as advanced by Davidson (2002) and describes the procedures of determining the cointegrating rank. Section III presents the data and reports the empirical results and Section IV concludes the paper.

II. FRACTIONAL COINTEGRATION

The concept of fractional cointegration is based on the basic principle that a set of k fractionally integrated variables $x_{jt} \sim I(d_j)$, $j = 1,\dots,k$, there exist r independent linear combinations that are integrated to lower orders.

This concept is introduced by Granger (1986) and developed by Davidson (2002, 2005) and Davidson et al. (2006). The general framework of the fractional vector error correction model (Granger (1986)) is given by:

$$[C(L) + \alpha\beta'((1-L)^{-b} - 1)](1-L)^{d}(X_t + \Phi D_t) = \varepsilon_t, \tag{1}$$

where $X_t(k \times 1)$ is a vector of observed variables, $D_t(s \times 1)$ is a vector of exogenous variables, typically dummies, $\Phi(k \times s)$ is a matrix of coefficients. $\varepsilon_t(k \times 1)$ is a vector of error terms $\varepsilon_t \sim$ i.i.d.$(0, \Sigma)$, $C(L)(k \times k)$ is a finite-order matrix polynomials in the lag operator with all roots outside the unit circle to represent short run effects. α and β are constant matrices of dimension $(k \times r)$, having rank r which represent, respectively, the error correction and the cointegration coefficients. d and b are nonnegative real term, with $d > 0$ and $0 < b \le d$. Setting $d = b = 1$ yields the Linear VECM (Johansen (1988)).

The equilibrium relationship is obtained if this condition is verified:

$$(1-L)^{d-b}\beta'(X_t + \Phi D_t) \sim I(0). \tag{2}$$

Davidson (2002) generalizes the model proposed by Granger (1986) as follows:

$$[C(L) + \alpha\beta'(K(L)^{-1} - I)]\Delta(L)(X_t + \Phi D_t) = \varepsilon_t, \tag{3}$$

where $\Delta(L)$ is the vector fractional differencing operator defined as:

$$\Delta(L) = \text{diag}\{(1-L)^{d_1}, \ldots, (1-L)^{d_k}\}, \tag{4}$$

where $d_1, \ldots, d_k$ are nonnegative real and represent the integration order of the k series, and

$$K(L) = \text{diag}\{(1-L)^{b_1}, \ldots, (1-L)^{b_k}\}, \tag{5}$$

where $0 \le b_i \le d_i$. These variables are set to be cointegrated if this condition is satisfied:

$$\beta'\Delta(L)K(L)^{-1}(X_t + \Phi D_t) \sim I(0). \tag{6}$$

It seems clear that to get this condition, the model requires some restrictions. First, if $\alpha = \beta = 0$ there is non cointegration. However, if $r > 0$, we must verify: $\beta' K(L)^{-1}(X_t + \Phi D_t) \sim I(0)$. Second, if $b_i > 0$ for one or more i, this implies cointegration. Third, if $d_i - b_i = a > 0$ for each i, this implies that $\beta'(X_t + \Phi D_t) \sim I(a)$, this is the case of fractional cointegration.

Davidson (2002) distinguishes between the cases of generalized and regular of fractional cointegration. The case of the generalized cointegration is defined as the case where the cointegrating variables are fractional differences of the observed series whereas in the regular cointegration, all the variables exhibit the same order of the fractional integration and the linear combinations of the variables are integrated to lower order. More precisely, the generalized fractional cointegration model (RFCM) is defined as:

$$[C(L)\Delta(L)-\alpha\beta(L)'L](X_t+\Phi D_t)=\varepsilon_t\,, \tag{7}$$

and the regular fractional cointegration model (GFCM) is given by:

$$[C(L)\Delta(L)-\alpha(L)\beta'L](X_t+\Phi D_t)=\varepsilon_t\,, \tag{8}$$

where $\alpha(L)$ and β (resp. α and $\beta(L)$) are matrices of dimension $k\times r$. The special lag polynomial matrices defined for these models take the respective forms $\alpha(L)=\{\alpha_{ji}(L)\}$ and $\beta(L)=\{\beta_{ji}(L)\}$ for all $1\le i\le k$ and $1\le j\le r$, and where:

$$\alpha_{ji}(L)=\alpha_{ji}(1-L)^{d_j-b_{ji}}\,, \tag{9}$$

$$\beta_{ji}(L)=\beta_{ji}(1-L)^{d_j-b_{ji}}\,. \tag{10}$$

Where $d_1,\dots,d_k$ are as in (4) and $b_{11},\dots,b_{kr}$ are additional parameters, not necessarily distinct.

To check the presence of cointegration against the alternative of non cointegration, Davidson (2002, 2005) considers that the conventional cointegration tests such as Dickey-Fuller tests are non valid since their null distributions are subject to the restriction $d_1=1$. To achieve this goal, Davidson (2002, 2005) proposes an alternative approach based on bootstrap tests. The main feature of this procedure is to draw bootstrap replications of the model (3) under H_0 such that $\alpha=\beta=0$ and so generate the null distributions of two regression-based test statistics, the F statistic for goodness of fit and the Durbin-Watson statistic. In order to determine the rank of cointegrating system, we consider the Minimal Algorithm proposed by Davidson (1998a,b) and the new method advanced by Nielson and Shimotsu (2007) which extends the procedure of Robinson and Yajima (2002) to contain stationary and non stationary fractionally integrated processes.

III. EMPIRICAL RESULTS

A. Data

The data consists of the daily exchange rates of the Tunisian Dinar (TND) relative to the American Dollar (USD), the Euro (EUR) and the Japanese Yen (JPY) between the 4 January 1999 and the 31 March 2008 thus totalling 2320 observations. The data are extracted from the Datastream and are transformed in logarithm form. Figure 1 plots the series in level and in first difference. At first sight, the series appear to have a non stationary behaviour in the sense that they do not converge towards their long term means while the series in first difference fluctuate around zero and seem to be stationary. Table 1 reports the descriptive statistics.

Figure 1
Exchange rates in level and in first difference

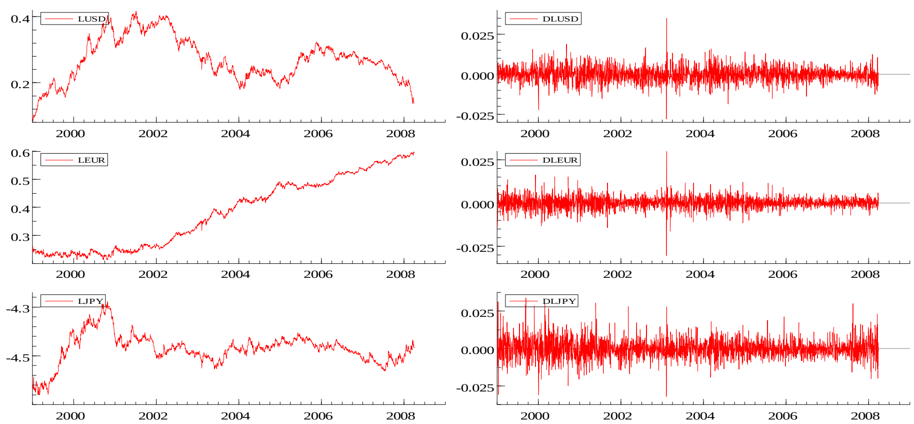


Table 1
Summary statistics of the exchange rate

	LUSD	LEUR	LJPY
Mean	0.271	0.380	-4.471
Median	0.269	0.383	-4.469
Maximum	0.417	0.597	-4.278
Minimum	0.086	0.213	-4.660
Std. Dev.	0.067	0.124	0.062
Skewness	0.085	0.131	-0.149
Kurtosis	2.481	1.506	4.448
Jarque-Bera	28.804	222.352	211.239
Probability	0.000	0.000	0.000
Observations	2320	2320	2320

B. Stationarity

Before modelling the relationship between the series, we examine the stationarity of the exchange rate series. For that, we apply the unit root tests of the Dickey-Fuller test (DF or ADF) (Dickey and Fuller (1979, 1981)), the Phillips-Perron test (PP) (Perron (1988)) and the KPSS test (Kwiatkowski et al. (1992)). Recall that the first two tests have as the null hypothesis the presence of a unit root whereas the null hypothesis of the KPSS test is the stationarity. The empirical Results of these three tests conducted on level of the exchange rate series and on the first difference are reported respectively in Table 2 and Table 3.

According to the three tests, we accept the null hypothesis of unit root for all series considered in level whatever the retained model is (with or without constant). However, for the series considered in first difference DLJPY while referring to the ADF and PP tests we reject the null hypothesis of non stationarity. This result is confirmed by the KPSS test. Thus, we conclude that the LJPY is I(1). For the DLUSD and the DLEUR, the null hypothesis of the ADF and PP test is rejected at 1% and 5% significance levels indicating that the series are stationary whereas the null hypothesis of the stationarity of the KPSS test is not accepted. Thus, the retained processes for these two series are neither I(0) nor I(1).

Table 2
Unit root test results for the series in level

		H_0: I(1)				H_0: I(0)
	Lags	τ	τ_μ	$Z(t_\alpha)$	$Z(t_{\alpha\bullet})$	$\hat{\eta}_\mu$
LUSD	1	0.124	-2.024	0.118	-2.011	12.376***
	4	0.083	-2.110	0.110	-2.021	4.973***
	8	0.051	-2.124	0.096	-2.142	2.776***
	15	0.063	-2.157	0.082	-2.211	1.576***
LEUR	1	0.638	2.992	0.514	2.827	60.789***
	4	0.779	3.416	0.641	3.104	44.237***
	8	0.781	3.343	0.702	3.242	24.611***
	15	0.796	3.470	0.734	3.404	13.878***
LJPY	1	0.753	-2.378	0.746	-2.372	10.876***
	4	0.690	-2.107	0.658	-2.354	5.645***
	8	0.658	-2.092	0.623	-2.349	3.812***
	15	0.619	-2.283	0.602	-2.347	2.241***

Note: τ (resp. $Z(t_\alpha)$) and τ_μ (resp. $Z(t_{\alpha\bullet})$) are the ADF (resp. PP) test statistics for the models without and with constant. $\hat{\eta}_\mu$ is the statistics of KPSS test where the residuals are issued from the regression with a constant. ***, ** and * denote significance at 1%, 5% and 10% level respectively.

Given that the conventional unit root tests has low power to discriminate between an I(1) process and a fractionally integrated I(d) process with ranging from 0 to $1/2$, we propose to estimate the fractional order of integration for each exchange rate series using Robinson's semi-parametric method and a Whittle-type pseudo-maximum likelihood estimator in addition to the exact local Whittle estimator and the feasible exact local Whittle of Shimotsu and Phillips (2005). These methods have the advantages to allow for non stationary processes. Indeed, differencing has the disadvantage that prior information is needed on the appropriate order of differencing. The estimation results are reported in Table 4 and show statistically significant values close to 1 for the integration orders of the logarithmic exchange rates. Thus, we conclude that these series are governed by an integrated process of order 1.

Table 3
Unit root test results for the series in first-differenced

		H_0: I(1)				H_0: I(0)
	Lags	τ	τ_μ	$Z(t_\alpha)$	$Z(t_{\alpha\bullet})$	$\hat{\eta}_\mu$
DLUSD	1	-33.303***	-33.311***	-47.896***	-47.899***	0.489**
	4	-20.437***	-20.446***	-47.892***	-47.894***	0.463**
	8	-14.973***	-14.989***	-47.900***	-47.903***	0.441*
	15	-11.841***	-11.861***	-47.926***	-47.924***	0.425*
DLEUR	1	-27.662***	-27.792***	-54.393***	-54.495***	0.523**
	4	-36.726***	-36.924***	-54.350***	-54.525***	0.497**
	8	-22.270***	-22.551***	-54.745***	-54.995***	0.470**
	15	-16.261***	-16.631***	-54.932***	-55.305***	0.389*
DLJPY	1	-18.087***	-18.641***	-48.784***	-48.635***	0.334
	4	-15.441***	-15.451***	-48.529***	-48.212***	0.295
	8	-13.861***	-13.991***	-48.349***	-48.251***	0.269
	15	-10.884***	-10.911***	-48.220***	-48.016***	0.258

Table 4
Estimates of the orders of integration in the levels of the logarithmic series

	LUSD	LEUR	LJPY	DLUSD	DLEUR	DLJPY
$\hat{d}_R$	-	-	-	0.003	0.004	0.002
Prob	-	-	-	[0.544]	[0.665]	[0.484]
$\hat{d}_W$	-	-	-	0.003	0.004	0.002
Prob	-	-	-	[0.732]	[0.689]	[0.577]
$\hat{d}_{ELW}$	1.042	0.921	0.996	0.003	0.004	0.002
Prob	[0]	[0]	[0]	[0.644]	[0.722]	[0.575]
$\hat{d}_{FELW}$	1.041	1.033	0.954	0.009	0.007	-0.012
Prob	[0]	[0]	[0]	[0.661]	[0.846]	[0.558]

C. Determining the Cointegrating Rank

To investigate the presence of one or more equilibrium relationships, we propose to determine the cointegration rank using the Minimal Algorithm of Davidson (1998a,b) and the procedure of Nielson and Shimotsu (2007).

The estimation results of the Minimal Algorithm are given in Table 5. The Wald and Philipps-Perron tests statistics suggest the existence of one cointegration relationship. We check this result using the procedure of Nielson and Shimotsu (2007). To apply this procedure, we analyze fractional integration of the data set by using the two-step Exact Local Whittle (ELW) estimator of Shimotsu (2005). This estimator is based on the modified ELW objective function and uses a tapered estimator. Compared to the ELW estimator, two-step ELW estimate has the principal advantage to allow for an unknown mean and polynomial time trend. The estimation results of the ELW are reported in Table 6. Given the values of $\hat{T}_0$, we easily accept the null of equality of the integration orders.

Table 5
Cointegration rank determination

	$\hat{\beta}$	Relation 1	Relation 2
LEUR	-5.689	-	-
LUSD	-13.339	-13.339^{***} (9.295)	-
LJPY	4.093	4.093^{***} (0.421)	-
Constante	24.347	24.347	-
Wald		4.129	
Phillips-Perron		-2.745	

Note: critical values for the chi-square test are adjusted according to the "rule of thumb" principle (Davidson (1998a)). Standard deviations are given in parentheses. ***, ** and * represent significance at 1%, 5% and 10% statistical levels respectively.

Table 6
Two step Feasible ELW estimates of fractional integration orders

Bandwidth	LUSD	LEUR	LJPY	$\bar{d}_*$
$m=[T^{0.6}]$	1.041 (0.105)	1.033 (0.105)	0.954 (0.105)	1.009
$m=[T^{0.5}]$	1.064 (0.153)	1.031 (0.153)	0.949 (0.153)	1.015

Note: standard errors are given in parenthesis, a nonzero mean was allowed in the estimation. When $m=[T^{0.6}]$, the values of the $\hat{T}_0$ statistic are 1.566 and 1.793 with $h(T)=1/\log(T)$ and $h(T)=1/\log(T)/\log(T)$, respectively. When $m=[T^{0.5}]$, the values of the $\hat{T}_0$ statistic are 1.108 and 1.348 with $h(T)=1/\log(T)$ and $h(T)=1/\log(T)/\log(T)$, respectively.

Now, we estimate the eigenvalues of $\hat{G}(\bar{d}_*)$ and $\hat{P}(\bar{d}_*)$ as advanced by Nielson and Shimotsu (2007). The obtained results are presented in Table 7. From these results, we expect that there will be evidence in favor of cointegrating relations. We propose in what follows to determine the cointegrating rank of the series using the model selection procedure with $\hat{P}(\bar{d}_*)$. The results which are displayed in Table 8 indicate clearly the possible presence cointegrating relations. We then propose to model the exchange rate series using the generalized fractional cointegration model.

Table 7
Estimated eigenvalues

Bandwidth	$\hat{\delta}_1$	$\hat{\delta}_2$	$\hat{\delta}_3$
Eigenvalues of $10000\times\hat{G}(\bar{d}_*)$			
$m_1=[T^{0.55}]$	0.069	0.025	0.003
$m_1=[T^{0.45}]$	0.068	0.024	0.002
Eigenvalues of $\hat{P}(\bar{d}_*)$			
$m_1=[T^{0.55}]$	1.810	0.842	0.348
$m_1=[T^{0.45}]$	1.921	0.822	0.257

Table 8
Estimated rank

$L(u)$	$v(n)=m_1^{-0.45}$	$v(n)=m_1^{-0.35}$	$v(n)=m_1^{-0.25}$	$v(n)=m_1^{-0.15}$	$v(n)=m_1^{-0.05}$
$m_1=[T^{0.55}], m=[T^{0.6}]$					
$L(0)$	-2.557	-2.322	-1.963	-1.414	-0.574
$L(1)$	-2.356	-1.201	-1.961	-1.594	-1.035
$L(2)$	-1.662	-1.584	-1.465	-1.282	-1.002
$\hat{r}$	0	0	0	1	1
$m_1=[T^{0.45}], m=[T^{0.5}]$					
$L(0)$	-2.369	-2.108	-1.739	-1.216	-0.477
$L(0)$	-2.323	-2.148	-1.902	-1.554	-1.061
$L(2)$	-1.711	-1.624	-1.501	-1.327	-1.081
$\hat{r}$	0	1	1	1	2

Note: the model selection procedure determines $\hat{r}$ as the arg min of $L(u)$ and the calculation of $L(u)$ allowed for a nonzero mean

D. Generalized Fractional Cointegration of the Exchange Rates

In this section, we model the exchange rate series by the generalized fractional cointegration model (GFCM) as defined by the equation (7). According to this model, the data dynamics are decomposed in a long term component and a short-term one. To choose the optimal lag length (in our case $p = 0$), we refer to Schwarz and Hannan-Quinn information criteria. The estimation results are summarized in Table 9. The estimated GFCM is given by:

$$(1-L)^{0.932} x_{1t} = -0.048\left((1-L)^{0.932-0.148} x_{1,t-1} - 7.535(1-L)^{0.881-0.176} x_{2,t-1} - 0.411(1-L)^{0.936-0.313} x_{3,t-1}\right) + \varepsilon_{1t},$$
$$(1-L)^{0.881} x_{2t} = -0.008\left((1-L)^{0.932-0.148} x_{1,t-1} - 7.535(1-L)^{0.881-0.176} x_{2,t-1} - 0.411(1-L)^{0.936-0.313} x_{3,t-1}\right) + \varepsilon_{2t},$$
$$(1-L)^{0.936} x_{3t} = -0.038\left((1-L)^{0.932-0.148} x_{1,t-1} - 7.535(1-L)^{0.881-0.176} x_{2,t-1} - 0.411(1-L)^{0.936-0.313} x_{3,t-1}\right) + \varepsilon_{3t},$$

where x_{1t}, x_{2t} and x_{3t} represent the LUSD, the LEUR and the LJPY exchange rate respectively.

Table 9
GFCM estimation results

	LUSD	LEUR	LJPY
d_i	0.932*** (22.800)	0.881*** (32.958)	0.936*** (21.636)
b_i	0.148*** (3.297)	0.176*** (4.785)	0.313*** (5.787)
α_i	-0.048 (-1.919)	-0.008** (-2.012)	0.038 (1.403)
β_i	- -	7.535 (1.049)	0.411 (1.243)
τ_1	3.073*	2.952*	4.122**
τ_2	57.064***	969.421***	1705.550***
Jarque-Bera	214284***	678134***	210679***
R^2	0.995	0.994	0.219
$Q(20)$	79.549***	63.733***	43.491***
$Q^2(20)$	256.548***	289.073***	348.029***

Schwarz: 18590.801

Hannan-Quinn: 18609.712

Log-likelihood: 18631.800

Note: t-statistics are given in parentheses. ***, ** and * denote significance at 1%, 5% and 10% statistical levels respectively. β_i represent the equilibrium relationship coefficient, α_i represent the short-term component, d_i represent the long memory parameter for the x_{it} variable and b_i represent a parameter satisfying the condition $0 < b_i \le d_i$.

According to these results, the values of the fractional integration parameters d_i lie between 1/2 and 1. We observe also that the estimated b_i is smaller than the estimated d_i verifying the condition $d_i - b_i > 0$. To validate the existence of a cointegration relationship between the exchange rates, we apply a Fisher test which is a test of coefficients restrictions whose null hypothesis is the absence of cointegration. Nevertheless, it should be stressed here that the statistical inference based on the asymptotic approximation is not applicable given that the asymptotic distribution under the null hypothesis depends on a nuisance parameter. To remain this issue, we rely on an alternative method initially proposed by Davidson (2002, 2005, 2006). This method based on bootstrapping allows correcting the distortions of the significance levels. The results of the validation test are reported in Table 10.

Table 10
Validation of the GFCM: p-values

	F statistics		
Bootstrap	Regular	Double	Fast-Double
$H_0 : \alpha_1 = \alpha_2 = \alpha_3 = 0$	0.195	0.147	0.158
$H_0 : \beta_1 = \beta_2 = \beta_3 = 0$	0.387	0.225	0.226
$H_0 : \alpha_i = \beta_i = 0$	0.486	0.413	0.394

The obtained results indicate that the hypotheses of nullity of model parameters are accepted at high levels for the all the series of study and suggest the absence of cointegration relationships between the series. Consequently, opting for a generalized fractional cointegration model seems to be inadequate. Since we reject the generalized fractional cointegration model, we check whether there is a regular fractional cointegration model in which the variables have the same order of integration.

E. Regular Fractional Cointegration of the Exchange Rates

In this section, we model the series by the regular fractional cointegration model (RFCM) as given by the equation 8. Recall that this model is characterized by the equality of the integration orders. This choice seems more appropriate as illustrated and demonstrated when analysing the rank of cointegration (Nielson and Shimotsu (2007)). The estimation results of the RFCM for the optimal lag length $(p = 1)$ are given in Table 11.

Table 11
Estimation results for the RFCM

	LUSD	LEUR	LJPY
d	0.898*** (41.895)	0.898*** (41.895)	0.898*** (41.895)
b	0.532*** (10.861)	0.532*** (10.861)	0.532*** (10.861)
α_i	-0.285*** (-6.337)	-0.059*** (-3.367)	0.521*** (3.023)
β_i	- -	1.469*** (6.257)	0.077*** (4.093)
δ_i	0.004*** (2.941)	0.002*** (4.898)	0.008*** (2.998)
c	0.086*** (31.246)	0.256*** (41.359)	-4.632*** (-87.833)
γ_{1i}	-0.436*** (-5.545)	0.203*** (4.664)	0.017*** (2.941)
γ_{2i}	-0.075*** (-2.872)	-0.261*** (-6.289)	-0.015*** (-6.364)
γ_{3i}	-0.078*** (-4.664)	-0.340*** (-6.481)	-0.022*** (-4.868)
τ_1	0.102	0.112	0.460
τ_2	4.819*	7.8191***	5.576**
Jarque-Bera	244.545***	1605.224***	460.729***
R^2	0.987	0.979	0.953
$Q(20)$	20.993	25.505	13.654
$Q^2(20)$	63.215***	127.695***	126.419***

Schwarz *:* 18595.121
Hannan-Quinn : 18629.524
Log-Likelihood*:* 18669.700

Note: t-statistics are given in parentheses. ***, ** and * denote significance at 1%, 5% and 10% statistical levels respectively.

Table 12
Validation of the RFCM: p-values

	F statistics		
Bootstrap	Regular	Double	Fast-Double
$H_0: \alpha_1 = \alpha_2 = \alpha_3 = 0$	0.070	0.052	0.011
$H_0: \beta_1 = \beta_2 = \beta_3 = 0$	0.002	0.001	0.000
$H_0: \alpha_i = \beta_i = 0$	0.005	0.003	0.001
$H_0: b > 0$	0.004	0.002	0.001
$H_0: d - b > 0$	0.005	0.001	0.001

According to the gotten results, the values of the fractional integration parameters d_i lie between 1/2 and 1. We see that the estimated values for b_i are strictly inferior to d_i verifying the condition $d_i - b_i > 0$. To validate the existence of a cointegration relationship between the series, we apply a Fisher test. This test leads us to reject the hypotheses of nullity of the parameters of the RFCM. The empirical results are reported in Table 12. Thus, we conclude that the exchange rates series move together to long-run stable equilibrium. Finding such cointegration relationships violates the weak form informational efficiency hypothesis in the Tunisian currency market. Equations of the estimated RFCM can be written as follows:

$$
\begin{aligned}
&(1-L)^{0.898} x_{1t} = -0.285(1-L)^{0.898-0.532}\left(x_{1,t-1} - 1.469x_{2,t-1} - 0.077x_{3,t-1}\right) \\
&+ (1+L)^{0.898}\left(-0.436x_{1,t-1} + 0.203x_{2,t-1} + 0.017x_{3,t-1} - (0.004t + 0.086)\right) + \varepsilon_{1t}, \\
&(1-L)^{0.898} x_{2t} = -0.059(1-L)^{0.898-0.532}\left(x_{1,t-1} - 1.469x_{2,t-1} - 0.077x_{3,t-1}\right) \\
&+ (1+L)^{0.898}\left(-0.075x_{1,t-1} - 0.261x_{2,t-1} - 0.015x_{3,t-1} - (0.002t + 0.256)\right) + \varepsilon_{2t}, \\
&(1-L)^{0.898} x_{3t} = 0.521(1-L)^{0.898-0.532}\left(x_{1,t-1} - 1.469x_{2,t-1} - 0.077x_{3,t-1}\right) \\
&+ (1+L)^{0.898}\left(-0.078x_{1,t-1} - 0.340x_{2,t-1} - 0.022x_{3,t-1} - (0.008t - 4.632)\right) + \varepsilon_{3t},
\end{aligned}
$$

We observe that the error correction term is negative and statistically significant for the LUSD and LEUR series. This reflects the presence of an adjustment mechanism towards the long term target allowing to compensate the disequilibrium between the exchange rate series in a way that they re-establish their common equilibrium tendency defined by the stable cointegration relationship. Nevertheless, for the LJPY series, the error correction coefficient is significant but takes a positive value. This does not match a long-run equilibrium mechanism.

We note that each exchange rate series depends negatively on its past values. This can be due to an internal adjustment mechanism within the currency market that allows to minimize the volatility risk. Furthermore, we remark that each exchange rate series depends on lagged values of the other series. This constitutes a violation of the weak form of the market efficiency hypothesis according to which the market immediately incorporates all available information and thus no available information can be used to make ahead forecasts of the series.

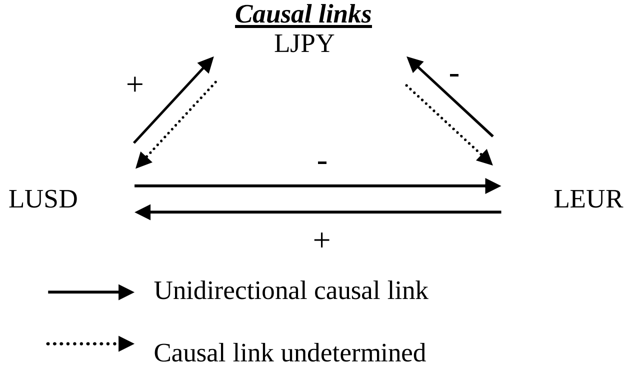
Examining the equation related to LUSD, we remark the LEUR and the LJPY positively influence the LUSD series. This positive influence is reflected via the error correction mechanism. Thus, we retain a unidirectional long-run causality link of positive sign running from LEUR to LUSD and from LJPY to LUSD. This implies that an appreciation of the LEUR and the LJPY exchange rates leads to an appreciation of the LUSD series. On the other hand, we observe an inverse causality relationship of negative sign running from LUSD to LEUR. Thus, we can state that anticipating an increase of the LEUR implies a depreciation of the LUSD. Analogously, an inverse causality link of negative sign exists from LUSD to LJPY. Nevertheless, according to the error correction model, the latter causality relationship becomes positive.

Looking at the equation allied to the LEUR, we see that the LEUR depends negatively on the LUSD variations. This negative relationship is also found in the error correction model implying the existence of a negative long-run relationship between the series. Besides, we remark that the LJPY affects negatively the LEUR series thus contradicting the error correction model according to which this influence is a positive sign. It seems then the effect of the LEUR depends on the period length hence justifying the use of the Granger causality over the different resolution scales. Besides, we observe an inverse causality link running from LEUR to LUSD of positive sign and a negative inverse causality relationship from LEUR to LJPY.

Moving to the equation associated to LJPY, we notice a long-run unidirectional causality relationship of positive sign running from LEUR to LJPY. Moreover, we see the existence of a negative causality link which is assumed to run from LUSD to LJPY. However, via the error correction mechanism, this effect of LUSD on LJPY is positive. Thus, we can not confirm the sign of the relationship over the long term.

The different causality links are reported on the following schematic:

Causal links



IV. CONCLUSION

The aim of this paper is to study the interdependence between the foreign exchange markets. For that, we have examined the relationship between the exchange rate series. The data consists in three exchange rate series of the Tunisian Dinar relative to the American Dollar, the Euro and the Japanese Yen. We adopt a multivariate fractional cointegration approach which presents several advantages. This approach allowed to detect the existence of one long term cointegration relationship as well as the short term relationships characterizing the error correction mechanism and implies a slower speed of return to the equilibrium compared with the conventional cointegration. A regular fractional vector error correction model is then estimated in order to investigate the long-run bidirectional causality links between the different exchange rate series.

The adopted multivariate approach involves to the rejection of the weak form informational efficiency hypothesis in the Tunisian exchange market. This implies that the prediction of the exchange rate series can be done exploiting information pertaining to other currency rates. Moreover, the existence of causal links implies that it is we can predict the exchange rate movements considering the fluctuations of the other variables.

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