Leverage and Asset Allocation under Capital Market Distortion

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ABSTRACT

Within the structure of a simple general equilibrium model, corporate leverage, growth of assets and resource allocation, and their inter-relationship under condition of market distortion are examined. Market distortion here refers to plurality of interest rates, as noted in the Modigliani and Miller's classic work (1958) in the later section on the analysis of capital structure. In this work, the concept of leverage it construed in two different senses – *physical* and *financial* - conditioned only by the presence of market distortion, and after that distinction is spelled out, it is explained how the changes in debt, equity, prices of the firm products, and market distortion affect the growth (decline) of firms, component cost of capital assets, and valuation. It is shown that returns on debt capital in real terms for a firm are increased (decreased) by the relative increase in the product price of the firm, and by the appropriate measures of leverage. The magnification effects on the earnings of the firms are also sketched out by the *physical* leverage of firms, and the modification of such magnification is induced or even reversed by opposite *financial* leverage situation. Elasticity of substitution between assets, demand elasticity and supply-side elasticity also come into play, and asset allocation is further exposited with all the parametric changes within this two-firm general equilibrium model.

I.   INTRODUCTION

The criticism by Durand (1959) of the Modigliani-Miller (1958) results on capital structure has fostered two strands of thought in the literature. On the one hand, several pieces of research (e.g., Stiglitz (1969), Rubinstein (1973), Hamada (1969), Baron (1974), Fama and Miller (1972), and others) have proven that the Modigliani-Miller claims are valid even under more general conditions than the authors originally envisaged. However, in their original piece itself, Modigliani and Miller (1958) observe that if a firm's cost of borrowing is less than the investors' cost of borrowing, the value of the firm increases with increase in debt. Baumol and Malkiel (1967) argue that a firm is not leverage-indifferent if investors incur transaction costs in arbitrage activities. Later, Rubinstein (1973) also demonstrate that if security markets are partially segmented and if debt is traded in a separate market where traders are more risk-averse than investors in the firm's equity capital, the value of the firm and its debt level are inversely related. But a second line of research - by Kim (1978), Baxter (1967), Lee and Barker (1977), Scott (1977), Barnea, Haugen and Senbet (1981), Chen (1978), Chen and Kim (1979), and many others - has established more formally that unique level of optimum capital structure does indeed exist for a firm if market distortions caused by taxes, bankruptcy costs, agency problem, informational asymmetry, etc. are admitted of. In dynamic environment, sustained by equity accumulation and change in debt, Ghosh (1991) shows that optimum capital structure for a firm is unique even under frictionless and perfectly competitive capital market.

The existing literature has waxed eloquent on a related issue, - the issue involving various facets of allocation in both macroeconomic decisions at a firm level, and in macro-structures at economy-wide aggregate level, - sometimes in static framework, and at times in inter-temporal dynamic setting. Sharpe (1987), and Perold and Sharpe (1988, 1995) discuss strategies for asset allocation in an integrated framework. Grossman (1995), in his *Presidential Address*, examines dynamic asset allocation and efficiency of markets. Black and Litterman (1991) study the issue by combining investor views with market equilibrium. Scarf (1994), Hurwicz and Majumdar (1988), Craine (1988) explore allocation questions in dynamic environment through structured mathematical logic. Choi and Han (1991) attempt to ascertain wealth effect and macro uncertainty in allocation between stocks and bonds. Prakash, Dandapani and Karels (1988), and Anderson and Prakash (1990) examine allocation principle in capital budgeting context. The issue is brought out by Bolten and Besley (1991) under dynamic interaction of earnings and interest rates. Ghosh and Sherman (1993) employ a general equilibrium structure to analyze resource allocation in competitive capital market where this interaction has been played out in comparative static framework. Krasa and Yanelis (1994), Leibowitz et al (1994), Sarnat (1974), Scarf (1994), and Stapleton and Subrahmanyam (1977) analyze market imperfection in various paradigmic set-ups. Here, in this paper, an attempt is made to use a two-firm framework of general equilibrium with distortion in capital market to study how market imperfection caused by such distortion affects asset allocation, firm value, and component cost of capital under the assumption of different corporate leverage of the constituent firms.

It is shown, in Section II, that in imperfect capital market, a firm that is more levered in *physical* sense is not necessarily so levered in *financial* sense, and if that is the case, then the firm which is more debt (equity) financed may respond to a change in capital structure, cost of component capital, and price structure in the economy in perverse

fashion. Corporate growth, discussed also by Gup (1980), and Roll (1973) somewhat differently, is brought out here in our analytical framework that follows, and at some point later we discuss the differences in the conclusions arrived here and earlier in the existing literature. Specifically, we show that the earnings of the more levered firm rises faster than the rise in debt/equity ratio if the firm is more levered in physical sense, and the least levered firm's rises at the slowest rate (or even drops), which confirms the long-standing results in the existing literature. However, if the firm is only more levered in physical sense, and not in financial sense, the existing results are compromised or even reversed. The rate of change in distortion is also a modifier, and therefore the existing literature needs a more critical re-examination for further corroboration or reversal of the hitherto-accepted findings.

II.   THE ANALYTICAL STRUCTURE

Consider an economy with two types of capital - debt capital (D) and equity capital (E) - sustaining two firms, each producing a different output in condition of different capital market imperfection. Here, D represents the number of bonds (debt instruments), and E stands for the number of shares of common stock (equity instruments) outstanding in the economy. The structure of our economy is defined by the following sets of equations:

$$a_{D1}X_1 + a_{D2}X_2 = D, \qquad (1)$$

$$a_{E1}X_1 + a_{E2}X_2 = E, \qquad (2)$$

$$a_{D1}r_{D1} + a_{E1}r_{E1} = P_1, \qquad (3)$$

$$a_{D2}r_{D2} + a_{E2}r_{E2} = P_2, \qquad (4)$$

where

$$r_{Di} = A_i r_D, \qquad (5)$$

and

$$r_{Ei} = B_i r_E, \qquad (6)$$

Here a_{Di} and a_{Ei} measure, respectively, units of debt and equity capital needed to produce one unit of output of firm i, X_i represents the total net output of firm i (which can be construed as the earnings before interest in terms of product units)[1]. The symbols r_{Di}, and r_{Ei} stand, respectively, for return on debt instrument (that is, interest rate), and return on equity capital for firm i, and P_i the price of the product of firm i (i = 1, 2) and r_{Di} is the cost of debt in the absence of distortion in debt market. The ith firm's rate of interest (r_{Di}) and rate of return on equity (r_{Ei}) are assumed to be, respectively, proportional to their respective market counterparts r_D and r_E with constant of proportionality, A_i, and B_i. To simplify the analytical structure, we assume $B_i = 1$. Note that a_{Di} and a_{Ei} are not assumed constant, as

differentiations and later discussions will clearly spell that out, and so these should not be construed as fixed coefficients even though they may appear so at first sight, particularly to those not so familiar with literature using this approach to general equilibrium, developed by Amano (1964), Jones (1965, 1971), and many others since then.

To simplify the analysis, we assume that $B_i = 1$, and thus $r_{E1} = r_{E2}$ (=r_E, say), which means that in equity market rate of return across firms is identical. If we further assume that $A_2 = 1$, it is simply postulated that for the first firm cost of debt is higher (lower) if $A_1 > 1$ (< 1). Obviously, equations (1) and (2) describe the allocations of capital between firm 1and firm 2. Equations (3) and (4) - the dual to capital asset allocation - represent the price cost structure for the firms. More lucidly, equation (1), for instance, shows how many debt instruments are used to produce X_1 and X_2. Thus, $a_{D1}X_1$ and $a_{D2}X_2$ measure the number of bonds issued by firm 1, and firm 2, and the sum of these numbers is equal to the total number of debt instrument (D) existing in the economy, Equation (2) can be interpreted exactly in the same way for equity capital allocation between the firms. Equation (3), as already noted in Baxter (1967), in competitive condition should read as follows:

$$a_{D1}r_D + a_{E1}r_E = P_1,$$

If market is perfectly competitive, the rate of return on each capital is identical across firms. That is, r_D and r_E and the rates of return, respectively, on debt capital and equity, and these rates are will then be firm-independent. Then, in above equation, $a_{D1}r_D$ is the cost of debt and $a_{E1}r_E$ is the cost of equity per unit of net output of firm 1. Since perfectly competitive equilibrium is a zero-profit situation, unit cost is equal to unit price, and that means: $a_{D1}r_D + a_{E1}r_E = P_1$. Thus, it is the expression of zero profit cost allocation for firm 1. The relation (5), however, exhibits that r_{D1} is not equal to r_{D2}; it is a situation of plurality of interest rates, as in the work of Modigliani and Miller (1958). If $A_1 > 1$, firm 2 has higher cost for debt, and if $A_1 < 1$, firm 2 has lower cost of debt than firm 1. Thus, the parameter A_i measures the extent of interest rate differential.

In this structure of capital market distortion, let us examine the effects of changes in debt (D_i), equity (E_i), prices (P_i) of the firm products, and market distortion (A_i) - the parameters of the model - on the growth (decline) of firms, component cost of capital, and value. Here, let the asterisk as superscript denote the percentage change in a parameter or variable. That is, $D^*/dD/D$, $a^*_{D1} = da_{D1}/a_{D1}$ denote percentage change in debt (D), percentage change in a_{D1}, respectively, and so on. Total differentiation of equations (1) through (5) with some algebraic manipulations then results in the following expressions:

$$\alpha_{D1}X_1^* + \alpha_{D2}X_2^* = D^* - \left(\alpha_{D1}a_{D1}^* + \alpha_{D2}^*a_{D2}\right), \tag{7}$$

$$\alpha_{E1}X_1^* + \alpha_{E2}X_2^* = E^* - \left(\alpha_{E1}a_{E1}^* + \alpha_{E2}^*a_{E2}\right), \tag{8}$$

$$\beta_{D1}r_{D1}^* + \beta_{E1}r_E^* = P_1^* - \left(\beta_{D1}a_{D1}^* + \beta_{E1}a_{E1}^*\right), \tag{9}$$

$$\beta_{D2}r_{D2}^* + \beta_{E2}r_E^* = P_2^* - \left(\beta_{D2}a_{D2}^* + \beta_{E2}a_{E2}^*\right), \tag{10}$$

$$R_{Di}^* = A_I^* + r_D^*, \tag{11}$$

where $\alpha_{Di} \equiv a_{Di} Xi / D, \alpha_{Ei} \equiv a_{Ei} X_i / E$ are the distributive allocations of debt and equity capital to the i-th firm, and $\beta_{Di} \equiv a_{D1} r_{Di} / P_i$ and $\beta_{Ei} \equiv a_{Ei} r_E / P_i$ are the distributive shares of these capital assets to the value created in the i-th firm (i = 1, 2). Note a few interesting features now. The determinant of the coefficients of X_I^*'s in equations (7) and (8), given by

$$|\alpha| = \begin{vmatrix} \alpha_{D1} & \alpha_{D2} \\ \alpha_{E1} & \alpha_{E2} \end{vmatrix} = \left(\frac{\alpha_{E1} \alpha_{E2} X_1 X_2}{DE} \right) \left(\frac{\alpha_{D1}}{\alpha_{E1}} - \frac{\alpha_{D2}}{\alpha_{E2}} \right)$$

defines the relative leverage of the firms. If $|\alpha| > 0$, - (which is true *if and only if* $(a_{D1} / a_{E1}) > (a_{D2} / a_{E2)})$, - obviously the *first* firm has more debt equity ratio in numerical terms, and that means it is relatively more levered in *physical* sense; similarly, $|\alpha| < 0$ signifies that the *second* firm is more levered in *physical* sense. Substituting equation (11) into equations (9) and (10), then the determinant of the coefficients of r_{D1}^* and r_E^*, is:

$$|\beta| = \begin{vmatrix} \beta_{D1} & \beta_{E1} \\ \beta_{D2} & \beta_{E2} \end{vmatrix} = \left(\frac{r_D r_E a_{E1} a_{E2}}{P_1 P_2} \right) \left(A_1 \frac{a_{D1}}{a_{E1}} - A_2 \frac{a_{D2}}{a_{E2}} \right).$$

In the absence of distortion in debt market (where $A_i = 1$), the signs of $|\beta|$ and $|\alpha|$ are exactly the same (and in such a situation $|\alpha| |\beta| > 0$), which means the sign of $|\beta|$ also defines the relative leverage of the first and the second firm. However, if $A_1 > 1$, the sign of $|\alpha|$ may not *necessarily* be identical with that of $|\alpha| |\beta|$. Since $|\beta|$ is a stochastic matrix (that is, the sum of each row equals 1), one can easily find that: $|\beta| = \beta_{D1} - \beta_{D2} = \beta_{E2} - \beta_{E1}$, from which then it follows that if $|\beta| > 0$, the first firm is more levered in *financial* sense (and if the sign of $|\beta|$ is negative, the second firm is more levered in the same *financial* sense). This brings out the point that in the absence of distortion in the debt capital market (that is, for $A_i = 1$), a firm is uniquely levered, no matter if numerical debt equity ratio or distributive share of debt capital relative to equity capital (that is, value sense of leverage measure) is considered. As already noted, if $A_1 > 1$, and the first firm is less levered in *physical* sense, it may nonetheless be more levered in financial sense compared with the second firm. Let us next introduce a few more basic ingredients of general equilibrium structure. First, the cost-minimization condition on the usage of debt and equity for firm i (i = 1, 2), defined by:

$$-da_{Di} / da_{Ei} = r_E / r_{Di},$$

yields upon some algebraic manipulations:

$$\beta_{Di} a_{Di}^* + \beta_{Ei} a_E^* = 0. \tag{12}$$

Since

$$\sigma_i = -\frac{\left(a_{Di}/a_{Ei}\right)^*}{\left(r_{Di}/r_E\right)^*} = \frac{a_{Ei}^* - a_{Di}^*}{r_{Di}^* - r_E^*} = \frac{a_{Ei}^* - a_{Di}^*}{r_D^* + A_i^* - r_E^*} \tag{13}$$

is the elasticity of substitution between debt and equity capital in firm i, as shown in Ghosh and Sherman (1993), one may easily now find the following expressions by the solving the simultaneous equations (12) and (1 3) for i =1 first, and then for i = 2:

$$a_{D1}^* = -\beta_{E1}\sigma_1\left(r_D^* - r_E^*\right) - \beta_{E1}\sigma_1 A_1^*,$$
$$a_{Ei}^* = -\beta_{D1}\sigma_1\left(r_D^* - r_E^*\right) + \beta_{D1}\sigma_1 A_1^*,$$
$$a_{D2}^* = -\beta_{E2}\sigma_2\left(r_{D1}^* - r_E^*\right) + \beta_{E2}\sigma_2 A^*,$$
$$a_{E2}^* = -\beta_{D2}\sigma_2\left(r_{D1}^* - r_E^*\right) + \beta_{D2}\sigma_2 A^*.$$

The substitutions of these a_{ji}^* 's (j = D, E; i = 1, 2) into (7) and (8) give rise to the relations (14) through (17):

$$\alpha_{D1}X_1^* + \alpha_{D2}X_2^* = D^* + \lambda_D\left(r_D^* - r_E^*\right) + \left(\alpha_{D1}\beta_{E1}\sigma_1 A_1^* + \alpha_{D2}\beta_{E2}\sigma_2 A_2^*\right) \tag{14}$$

$$\alpha_{E1}X_1^* + \alpha_{E2}X_2^* = E^* + \lambda_E\left(r_D^* - r_E^*\right) + \left(\alpha_{E1}\beta_{D1}\sigma_1 A_1^* + \alpha_{E2}\beta_{D2}\sigma_2 A_2^*\right) \tag{15}$$

where $\lambda_D = \alpha_{D1}\beta_{E1}\sigma_1 + \alpha_{D2}\beta_{E2}\sigma_2 \geq 0$, and $\lambda_E = \alpha_{E1}\beta_{D1}\sigma_1 + \alpha_{E2}\beta_{D2}\sigma_2 \geq 0$, and

$$\beta_{D1}r_D^* + \beta_{E1}r_E^* = P_1^* - \beta_{D1}A_1^* \tag{16}$$

$$\beta_{D2}r_D^* + \beta_{E2}r_E^* = P_2^* - \beta_{D2}A_2^* \tag{17}$$

From (16) and (17) one can immediately derive that

$$r_D^* - r_E^* = \frac{1}{|\beta|}\left[\left(P_1^* - P_2^*\right) + \left(\beta_{D2}A_2^* - \beta_{D1}A_1^*\right)\right] \tag{18}$$

An algebraic manipulation further gives rise to the following relationships:

$$r_{D1}^* - r_E^* = \frac{1}{|\beta|}\left(P_1^* - P_2^*\right) + \frac{\beta_{D2}}{|\beta|}\left(A_2^* - A_1^*\right) \tag{19}$$

$$r_{D2}^* - r_E^* = \frac{1}{|\beta|}\left(P_1^* - P_2^*\right) + \frac{\beta_{D1}}{|\beta|}\left(A_2^* - A_1^*\right) \tag{20}$$

Here one can note that at constant distortions, - that is, if $A_i^* = 0$, a change in P_1 / P_2 (in percentage terms) causes a more than equi-proportionate change in r_{Di} / r_E ; but the changes may not be uni-directional. If $|\alpha| < 0$, and $|\beta| > 0$, then in spite of the fact that the first firm is less levered in physical sense, higher cost of debt for the first firm may induce a situation in which an increase in the relative price of the physically levered firm serves to lower the return to the debt-holders relative to the returns to equity-holders in both firms. Next, consider the effects of the change in distortions at constant P_i 's. As (18) and (19) show, an increased premium paid to debt-holders in the firm in which debt receives smaller distributive shares must raise cost of debt in both industries relative to the returns on equity capital. From (16) and (17) it is evident that an increase at A_i^* at constant P_i 's works like a decline in P_i 's in affecting the returns on debt and equity. It is instructive, however, to examine how the results just brought out will be influenced by the non-constancy of the P_i 's. From (16) and (17) one can derive the following expressions:

$$r_{D1}^* - P_1^* = \frac{\beta_{E1}}{|\beta|}\left(P_1^* - P_2^*\right) - \frac{\beta_{E1}\beta_{D2}}{|\beta|}\left(A_1^* - A_2^*\right) \tag{21}$$

$$r_{D1}^* - P_2^* = \frac{\beta_{E2}}{|\beta|}\left(P_1^* - P_2^*\right) - \frac{\beta_{E1}\beta_{D2}}{|\beta|}\left(A_1^* - A_2^*\right) \tag{22}$$

Equations (21) and (22) reveal that returns on debt capital in the first firm in real terms are increased (decreased) by the relative increase in the price of the product of the first firm $\left((P_1 / P_2)^* \equiv P_1^* - P_2^*\right)$ and/or decreased by the relative increase in the distortion in the first firm $\left((A_1 / A_2)^* \equiv A_1^* - A_2^*\right)$ if $/\beta/ > 0$. The question immediately is to ascertain the relative strength of these two divergent pulls. A close examination of (19) and (20) shows that since $0 < \beta_{Di} < 1$, impact of relative price change is stronger than relative change in distortion on the change in the real rate of return on debt capital, In more simple terms, one may state that if the price of the first firm relative to that of the second firm rises by 5 percent while the debt market distortion for the first firm relative to that of the second firm also rises by the same 5 percent, bondholders will benefit in real terms in both firms if the first firm is financially more levered. Now, from (15) and (16), we can easily show:

$$X_1^* - X_2^* = \frac{1}{|\alpha|}\left\{\left(D^* - E^*\right) + \left(\lambda_D + \lambda_E\right)\left(r_D^* - r_E^*\right) + M\right\} \tag{23}$$

where $M \equiv \left[\left(\alpha_{D1}\beta_{E1} + \alpha_{E1}\beta_{D1}\right)\sigma_1 A_1^*\right] + \left[\left(\alpha_{D2}\beta_{E2} + \alpha_{E2}\beta_{D2}\right)\sigma_2 A_2^*\right]$.

At constant returns to debt and equity capital ($r_{Di}^* = 0, r_E^* = 0$) and at the invariant levels of

distortion ($A_1^* = 0$). Obviously, if $D^* > E^*$ then $X_1^* > D^*$ and $X_2^* < E^*$ provided first firm is more numerically (physically) levered. If $D^* = E^*$ then $X_1^* = D^*$ and $X_2^* = E^*$ and if $D^* < E^*$, then $X_1^* < D^*E$, $X_1^* < D^*$ and $X_2^* < E^*$ provided first firm is more physically levered. Algebraically,

$$X_1^* > D^* > E^* > X_2^*$$
$$X_1^* = D^* = E^* = X_2^*$$
$$X_1^* < D^* < E^* < X_2^*$$

if $|\alpha| > 0$ (and the converse is true in the opposite *physical* leverage condition). Next, let us make use of (18) in (23) to get the expression defined by expression (24):

$$X_1^* - X_2^* = \frac{1}{|\alpha|}\left\{(D^* - E^*) + \frac{(\lambda_D + \lambda_E)}{|\alpha\| \,|}(P_1^* - P_2^*) + M\right\}$$

$$\frac{[(\alpha_{D1}\beta_{E1} + \alpha_{E1}\beta_{D1})\sigma_1\beta_{D2} + [(\alpha_{D2}\beta_{E2} + \alpha_{E2}\beta_{D2})\sigma_2\beta_{D1}]}{|\alpha\|\beta|}(A_1^* - A_2^2) \qquad (24)$$

A number of interesting observations can be made at this point. From (24), it is evident that at $P_i^* = 0$, and $A_i^* = 0$, - that is, at the constant prices and constant levels of distortion,

$$X_1^* > D^* > E^* > X_2^*$$
$$X_1^* = D^* = E^* = X_2^*$$
$$X_1^* < D^* < E^* < X_2^*$$

if $|\alpha| > 0$ (and the converse is true in the opposite physical leverage condition). All these mean that if debt capital increases relatively more that equity capital in the economy as a whole, the relatively levered firm (in *physical* sense) expands most and the less levered firm grows least (or may shrink). As a special case of this, one may conclude that if debt instruments are increased while equity instruments are held constant in the economy by some sort of pecking order policy, first firm will grow more than the rate of increase in debt, and the second firm will definitely shrink (provided the first firm is more levered in the *physical* sense, i.e., $|\alpha| > 0$). Technically, one finds the following scenario:

$$X_1^* > D^* > E^*(= 0) > X_2^*$$
.

Since $D^* > 0$, and $E^* = 0$, $E^* > X_2^*$ means $X_2^* < 0$. In the opposite condition, that is, in

the event of the economy-wide increase in equity instruments *alone*, less levered firm will expand more than the growth of equity, and the more levered firm will contract. In economic terms, growth in fixed income securities induce the physically levered firm to have more than proportional growth in EBIT, and that ultimately may increase earnings available to common stockholders of the physically more levered firm, and exactly the opposite fate will come to the stockholders of the other firm. Note that these results are predicated on the *physical* ranking of leverage. However, the results may be reversed in the presence of non-constancy of P_i 's and A_i 's. If leverage ranking of the firms are not the same in physical and financial senses, which means that $|\alpha|\,|\beta| < 0$, the unique price output response of the industries may not necessarily hold.

On a more involved inspection of this analytical structure one can here note that since

$$(\alpha_{D1}\beta_{E1} + \alpha_{E1}\beta_{D1})\sigma_1\beta_{D2} + (\alpha_{D2}\beta_{E2} + \alpha_{E2}\beta_{D2})\sigma_2\beta_{D1} > 0,$$

$|\alpha|\,|\beta| < 0$ signifies that if the distortion in the less levered firm relative to the other firm increases, the growth of the firm will be perverse as well. The implications of all of these on the earnings available to stockholders are therefore quite nebulous.

Thus far we have discussed only the supply-side response to various parametric changes. To close the general equilibrium model, it is necessary that one has to bring out the demand-side in the structural set-up. To do so, let the relative demand be specified in the following functional form: [2]

$$\frac{X_1}{X_2} = f\left(\frac{P_1}{P_2}\right), f(\bullet) < 0. \tag{25}$$

The functional relation (25) states that as P_1 / P_2 rises (falls), X_1 / X_2 falls (rises), as the normal demand structure should qualitatively suggest. A step further should lead one to the following expression of proportionate change:

$$X_1^* - X_2^* = -\xi_D\left(P_1^* - P_2^*\right), \tag{26}$$

where $\xi_D\ (> 0)$ measures the elasticity on the demand side (as Haugen and Wichern use the concept extensively). Equating this expression (26) to the expression (24), one gets the following:

$$\left(P_1^* - P_2^*\right) = \left(\frac{(\alpha_{D1}\beta_{E1} + \alpha_{E1}\beta_{D1})\sigma_1\beta_{D2} + (\alpha_{D2}\beta_{E2} + \alpha_{E2}\beta_{D2})\sigma_2\beta_{D1}}{\eta}\left(A_1^* - A_2^*\right)\right) \tag{27}$$

where $\eta = S_1\sigma_1 + S_2\sigma_2 + S_D\xi_D$, and $S_1 \equiv \alpha_{D1}\beta_{E1} + \alpha_{E1}\beta_{D1}, S_2 \equiv \alpha_{D2}\beta_{E2} + \alpha_{E2}\beta_{D2}$,

$S_D \equiv |\alpha\|\beta|$, and $\sum_i S_i = 1$. A little more algebraic manipulation can re-express (27) as follows:

$$\left(P_1^* - P_2^*\right) = \left(\frac{(\beta_{D2}S_1\sigma_1 + \beta_{D1}S_2\sigma_2)}{S_1\sigma_1 + S_2\sigma_2 + S_D\xi_D}\left(A_1^* - A_2^*\right)\right), \tag{28}$$

and this shows that under Anormal@ demand conditions, impact of distortion on price changes is less pronounced, and hence the relative change in expansion (or contraction) of the firms due to distortions is less significant provided the sign of $|\alpha\|\beta|$ is positive.

Next, putting equation (27) into equations (21) and (22), we obtain the following expressions of the relative returns on debt capital under distortion:

$$r_{D1}^* - P_1^* = \frac{\beta_{E1}}{\eta}\left[S_2\sigma_2 - \beta_{D2}|\alpha|\eta_D\right]\left(A_1^* - A_2^*\right) \tag{29}$$

$$r_{D1}^* - P_2^* = \frac{1}{\eta}\left[S_1\beta_{D2}\sigma_1 - S_2\sigma_2 - \beta_{D1}\beta_{D2}|\alpha|\xi_D\right]\left(A_1^* - A_2^*\right) \tag{30}$$

From these expressions (29) and (30) one can recognize that if the first firm is physically less levered, real returns on debt capital in this firm gets better if prices do not change and/or demand elasticity is very low and/or percentage change in market distortion for firm 1 is higher than that of the second firm. All these conclusions are again valid if both of these firms are uniquely levered. This validity will be disturbed by different physical and financial leverage rankings of the firms.

III.   CONCLUSION

In this paper we demonstrate that if the first firm is more levered in physical sense, and the debt capital is increased relative to equity capital in the economy, first firm expands most and the second firm least of all changes in percentage terms. A special case of this conclusion can be stated as follows: at constant returns to capital - debt and equity - and at constant distortion in the capital market, if debt alone increases (that is equity remains unchanged), relatively levered firm (in physical sense) expands and the less levered firm shrinks. If we go a step further we can conclude that at the constant level of debt and equity capitals with their respective returns remaining invariant as well, an increased premium paid to debt-holders in either industry will involve a substitution away from debt in that industry and affects the industry growth in the same way as the increase in debt and a decrease in equity. Also, the firm that is less levered in physical sense and yet pays a premium to the bondholders in that firm compared to the other firm, an increase in the price of the less levered firm product may not cause its output to grow, and thus the perversity can occur. This result is reminiscent of the result derived by Gup (1980), and yet a significantly different conclusion, as it involves leverage rankings of firms in two different ways in an integrated general equilibrium set-up as opposed to a partial equilibrium structure.

ENDNOTES

1. By standard normalization, - that is, by choosing the product units appropriately, one can make $P_i = 1$, and hence X_i can be called earnings before interest and taxes (EBIT) of firm i. Of course, here we do not have taxes in our paradigm. On a more critical analysis, it embraces the Cournot paradigm of production.
2. This type of demand function underlies the homothetic preference structure, often assumed in the general equilibrium literature.

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