Optimal International Diversification with Constraints

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ABSTRACT

We derive closed-form solutions to optimal international portfolio with constraints. The investor is assumed to trade simultaneously on several international financial markets (domestic and foreign). Typically, his investment strategy depends on the financial asset returns, on the domestic and foreign interest rates and also on exchange rates. The investor wants to maximize the expected utility of his portfolio value at maturity but he faces some specific constraints on his strategy. Such market frictions can potentially explain part of the well-known home bias. Using in particular the equilibrium of domestic and foreign interest rates with stochastic volatility introduced by Frachot (1995), we examine how the optimal international portfolio with constraints depends on these factors.

JEL Classifications: F3, G11, G15

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I.   INTRODUCTION

Many authors have well documented the gains from international diversification. Grubel (1968), Levy and Sarnat (1970), Lessard (1973) and Solnik (1974a) have shown how international diversification can provide lower return variances than those achievable by only the domestic market. Eun and Resnik (1988) have proved that international portfolio diversification outperform purely domestic portfolios (U.S.) even when taking exchange risk into account. However, as illustrated by Cooper and Kaplanis (1994) and French and Porterba (1991), individuals invest significantly much more in their respective domestic financial markets. This phenomenon is the so-called "home bias".

According to Stulz (1981), taxes on foreign asset investments can induce such bias. Solnik (1974b) and Stulz (1983), Adler and Dumas (1983) argue that home inflation risk can increase the demand for domestic assets. When investing in international markets, typically the investor must take account of all financial asset returns, of the domestic and foreign interest rates and also of exchange rates. Such problem has been previously studied for example by Solnik (1974a,b), Noetzlin and Solnik (1982), Adler and Dumas (1983) and related problems have been examined by Lessard (1983), Odier and Solnik (1993) or Campbell (1991). Uppal (1993) examines also the effect of consumption goods on the home bias.

As underlined in Coppeland and Weston (1988): "From the purely financial point of view, investors, whether they be firms or individuals, ought to consider the possibility of expanding their investments beyond the geographical limits of their own countries, if only because of the greater number and diversity of investment possibilities available." For example, Europeans are markedly under-diversified in international terms with regard to what would be an theoretical optimal portfolio or compared with the current practice of investors in other countries.

Additionally, some specific constraints may appear when determining the optimal portfolio. This is due to many "frictions": for example, information problems which imply additional costs, limited access to markets ...and exchange risk. Chiou (2009) examines for example the benefits of the optimal international diversification for a U.S.A. investor under various portfolio constraints. He suggests that introducing lower and upper weighting bounds may reduce but not completely eliminate the international diversification benefits (see also Bekaert and Urias, 1996; De Roon et al., 2001; Harvey 1995; Li et al., 2003; Pástor and Stambaugh, 2000; Wang, 1998) for no short selling conditions). Therefore, they emphasize that international diversification of investors must be examined according to specific investment constraints. This is precisely the purpose of the paper that is decomposed as follows.

Section II briefly recalls the main equilibrium relations between interest and exchange rates of Frachot (1995), for affine and lognormal models. In Section III, we solve[2] the international portfolio optimization problem with convex constraints, using results of Cvitanic and Karatzas (1992). In Section IV, we illustrate the general result for the two-country case.

II.   THE INTERNATIONAL FINANCIAL MARKET

A.   International Stock Markets

In this paper, asset prices, domestic and foreign interest rates as well as exchange rates are assumed to be exogenous: the investor is a price taker. The investor's portfolio is assumed to be denominated in domestic numeraire. Therefore, returns on foreign investments depend both on asset returns denominated in foreign currencies and on exchange rates. For example, from the point of view of an American investor, it is convenient to express foreign currencies as costing so many dollars.

Denote by S^0 the domestic stock and by S^j, $j \in \{1,...,n-1\}$ the foreign stocks, which are m_j-dimensional vectors. For simplicity of exposition and according to standard assumptions, their dynamics are supposed to be diffusions, given by:

$$\forall j \geq 0, \forall i \leq m_j, \, dS_t^{i,j} = S_t^{i,j}\left(m^{S^{i,j}}dt + [\sigma^{S^{i,j}}]dW_t\right),$$

where $(W)_t$ is a multidimensional Brownian motion that describes the different sources of uncertainty on the world market. We can assume for example that it is possible to introduce as many Brownian components as the number of assets under consideration. This leads to a complete market and avoid the choice of a particular risk-neutral measure. If other sources of uncertainty are identified, it is necessary to introduce more Brownian components. Then, using statistical estimation of pricing functional, a particular risk-neutral probability can be selected. Of course, this approach is based on a no-arbitrage assumption on the world market but, as usual, it is possible to consider that if a significant arbitrage exists, it will be "corrected" by the market. Moreover, we know that correlations exist between the equity markets of different countries. They can be for example computed using monthly returns on market indexes.

Introduce the domestic money market account (or domestic accumulation factor) b^0 defined by $b_t^0 = \exp[\int_0^t r_s^0 ds]$. Consider I_t^j, the domestic-denominated price of the foreign currency unit j and $B^j(t,T)$, the foreign zero-coupon bond of country j. Assume that the market price of risk satisfies sufficient regularity conditions to characterize the non arbitrage assumption by the existence of a risk-neutral probability measure Q^0 under which domestic zero-coupon bond prices discounted by the domestic spot rate are martingales. Under the non arbitrage assumption, all assets denominated in the domestic currency and discounted by the factor b^0 are martingales under a risk-neutral probability Q^0. That is: $\frac{S_t^{i,j}I_t^j}{b_t^0}$ and $\frac{B_t^jI_t^j}{b_t^0}$ are martingales under Q^0.

Denote by $\tilde{S}_t^{i,j}$ the domestic denominated stock $S_t^{i,j}I_t^j$ and by $\tilde{B}_t^j$ the domestic denominated zero-coupon bond $B_t^jI_t^j$. Recall that $(I_t^j)_t$ satisfies:

$$dI_t^j = I_t^j(m^{I^j}dt + [S^{I^j}]\P dW_t).$$

Therefore, the domestic-denominated stocks are given by:

$$d\tilde{S}_t^{i,j} = \tilde{S}_t^{i,j}(m^{\tilde{S}^{i,j}}dt + [S^{\tilde{S}^{i,j}}]\P dW_t),$$

with the following parameters, modified by the exchange rate:

$$m^{\tilde{S}^{i,j}} = m^{I^j} + m^{S^{i,j}} - 1/2S\ell^{I^j}S^{S^{i,j}}, \; S^{\tilde{S}^{i,j}} = S^{I^j} + S^{S^{i,j}}.$$

The domestic-denominated zero-coupon bonds are given by:

$$d\tilde{B}_t^j = \tilde{B}_t^j \overset{\text{æ}}{\underset{\text{è}}{\text{ç}}} m^{\tilde{B}^j}dt + [S^{\tilde{B}^j}]\P dW_t \overset{\text{ö}}{\underset{\text{ø}}{\div}},$$

with the following parameters, modified by the exchange rate:

$$m^{\tilde{B}^j} = m^{I^j} + m^{B^j} - 1/2S\ell^{I^j}S^{B^j}, \; S^{\tilde{B}^j} = S^{I^j} + S^{B^j}.$$

To illustrate the optimal portfolio problem, it is possible to consider a two-country economy. This allows to model the term structure of both domestic and foreign rates like Frachot (1995). He introduces a consistent framework for domestic and foreign term structures when arbitrage is precluded. In particular, stochastic volatilities are allowed with a shape of the term structure which is linear in terms of state variables. In fact, there are strong reasons to consider stochastic volatilities since there is empirical evidence that conditional variances depend on past values of interest rates and exchange rates. The next section is a brief review of the Amin and Jarrow (1991) context.

B.   Domestic term Structure

Under the assumption of no-arbitrage, domestic zero-coupon bond prices $(B^0(t,T))$ move over time in the following way:

$$dB^0(t,T) = r^0 B^0(t,T)dt + B^0(t,T)S^0(t,T)\P dW_t,$$

where $(W_t)_t$ is an m-dimensional standard Brownian motion associated to its natural filtration $(F_t)_t$, r^0 is the domestic spot rate and $S^0(t,T)$ is a q-dimensional continuous and adapted process such that $S^0(t,t) = 0$. Additionally, this volatility term satisfies some usual regularity conditions ensuring that the above stochastic differential equation has a unique strong solution. Defining the spot-forward rates $f^0(t,T)$ as

$$f^0(t,T) = -\partial_2 Ln B^0(t,T).$$

It can be proved that

$$f^0(t,T) = f^0(0,T) + \int_0^t S^0(s,T)\P_2 S^0(s,T)dW_s - \int_0^t \P_2 S^0(s,T)dW_s,$$

where $(f^0(0,t))$ is the initial forward curve, which is supposed to be given exogenously. In particular, the spot rate can be expressed as

$$r_t^0 = f^0(0,t) + \int_0^t S^0(s,T)\P_2 S^0(s,T)dW_s - \int_0^t \P_2 S^0(s,T)dW_s.$$

C.   Foreign Term Structure

If the probability measure Q^0 has been only defined in the domestic economy, there is no reason for the foreign zero-coupon bond price, discounted by the foreign spot rate, to be a martingale under Q^0. Define $(I_t)_t$ as the domestic-denominated price of a foreign currency unit and suppose that $(I_t)_t$ follows the stochastic diffusion equation

$$dI_t = m_t^I I_t dt + I_t[S_t^I]dW_t$$

under the historical probability P.

The domestic-denominated asset whose time- T value is $I_T e^{\int_t^T r_s^f ds}$, where r_t^f is the foreign spot rate, must satisfy:

$$I_t = E_{Q^0}[e^{-\int_t^T r_s^0 ds} I_T e^{\int_t^T r_s^f ds} | F_t]$$

This implies that $m_t^I = r_t^0 - r_t^f$ or, equivalently,

$$dI_t = \left(r_t^0 - r_t^f\right)m_t^I I_t dt + I_t[S_t^I]dW_t$$

Introduce the foreign zero-coupon bond $B^f(t,T)$. The domestic-denominated foreign zero-coupon bond whose price is $I_t B^f(t,T)$ is given by

$$I_t B^f(t,T) = E_{Q^0}\left[I_T e^{-\int_t^T r_s^0 ds} | F_t\right].$$

Particular cases can be studied. For example, recall the set of equations in this two-country economy under the non-arbitrage hypothesis (see Frachot, 1995):

$$\frac{dB^0(t,T)}{B^0(t,T)} = r_t^0 dt + [S^0(t,T)]dW_t$$

$$\frac{dB^f(t,T)}{B^f(t,T)} = r_t^f dt + [S^f(t,T)](dW_t - [S_t^I]dt)$$

$$\frac{d[I_t B^f(t,T)]}{I_t B^f(t,T)} = r_t^0 dt + (S^f(t,T) + S_t^I)dW_t$$

$$\frac{dI_t}{I_t} = (r_t^0 - r_t^f)I_t dt + I_t[S_t^I]dW_t$$

Consider the particular case of a one factor model where both zero-coupon prices and the exchange rate are assumed to be well-behaved and deterministic functions of a single state variable. By a change of variable, this factor can be identified to the domestic spot rate r_t^0. Recall the assumptions of Frachot (1995):

Assumptions:

-There exist three well-behaved and deterministic functions B^0, B^f and I such that:

$$B^0(t,T) = B^0(t,T,r_t^0), \ B^f(t,T) = B^f(t,T,r_t^0), \ I_t = I(t,r_t^0).$$

-Volatilities are homogeneous and separable:

$$S^0(t,T) = S^0(T-t)y^0(t,r_t^0),$$
$$S^f(t,T) = S^f(T-t)y^f(t,r_t^0),$$
$$S_t^I = S_t^I(t,r_t^0),$$

where S^0, S^f, S^I, y^0, and y^f are deterministic and well-behaved functions. These assumptions are sufficient to prove that zero-coupon prices are some exponential of a linear function of r_t^0 and that (r_t^0) and $\mathrm{Ln}(I_t)$ follow a square-root process, that has been introduced by Cox-Ingersoll-Ross (1985).

The zero-coupon prices and the exchange rate have the following simple form:

$$B^0(t,T) = \exp(-[A^0(T-t)r_t^0) + b^0(t,T)]),$$
$$B^f(t,T) = \exp(-[A^f(T-t)r_t^0) + b^f(t,T)]),$$
$$I_t = \exp(-[A^I r_t^0) + b_t^I]),$$

where A^0, A^f, b^0, b^f, and b^I are deterministic functions satisfying:

$$A^0(x) = \frac{1+k}{m} \frac{1-e^{-mx}}{1+e^{-mx}},$$

$$A^f(x) = -A^I + A^0(x) + A^I \frac{\P A^0(x)}{1 + \frac{a}{2} A^0(x)A^I}.$$

where m and k are convenient parameterizations for

$$m = \sqrt{l^2 + 2a}, \quad k = \frac{m-l}{m+l}.$$

Additionally, (r_t^0) and I_t satisfy:

$$dr_t^0 = (j - l\, r_t^0)dt + \sqrt{ar_t^0 + b}\, dW_t,$$

$$\frac{dI_t}{I_t} = (r_t^0 - r_t^f)dt - A^I \sqrt{ar_t^0 + b}\, dW_t.$$

III.   THE OPTIMAL PORTFOLIO

Consider now the optimization problem of an investor who maximizes his expected utility from terminal wealth. His portfolio is constrained to take particular values which are modeled by the following general condition: to be in a given closed, convex subset. This allows to model incompleteness of markets, no short-selling restrictions or specific bounds on portfolio strategies. The unconstrained version of this problem is well-known: recall for example the results Cox and Huang (1989). In the "constrained" case, such stochastic optimal control has been analyzed in Cvitanic and Karatzas (1992), by suitably embedding the constrained problem in an appropriate family of unconstrained ones and finding a member of this family for which the corresponding optimal policy obeys the constraint. Such an approach is very useful for example in the context of incomplete markets where "fictitious completion" can be introduced. The mathematical tools are those of continuous-time processes, convex analysis and duality theory (for detailed results, we refer to Cvitanic and Karatzas, 1992).

A.   Portfolio Value Dynamics

Consider the portfolio value process V associated to the strategy w of the investor which is the vector of proportions w_l of his wealth invested in the l-th asset. As usual, his decisions can only be based on the current information F_t, without anticipation of the future. More precisely, denote w^{B^j} and $w^{S^{i,j}}$ the respective weights invested on zero-coupon bonds and stocks. Recall that $(W_t)_t$ is an m-dimensional standard Brownian motion associated to its natural filtration $(F_t)_t$ Then, under the historical

probability P, the portfolio domestic currency denominated value V_t is given by:

$$dV_t = \sum_j w^{B^j} V_t \left(m^{\tilde{B}^j} dt + [S^{\tilde{B}^j}] dW_t \right) + \sum_{i,j} w^{S^{i,j}} V_t \left(m^{\tilde{S}^{i,j}} dt + [S^{\tilde{S}^{i,j}}] dW_t \right) + (1 - \sum_j w^{B^j} - \sum_{i,j} w^{S^{i,j}}) V_t r^0 dt$$

B.   The Investor's Utility and Portfolio

To describe the behavior of the investor, his utility function U is supposed as usual to be strictly increasing, strictly concave, of class C^1 and satisfies:

$$U'(0+) = \lim_{x \to 0,: x > 0} = +\infty, \ U'(+\infty) = \lim_{x \to +\infty} = 0.$$

Set $J = (U')^{-1}$ the (continuous, strictly decreasing) inverse of the marginal utility U'.

Proposition 1. Without specific portfolio constraints, the optimal solution is given by:

$$V_T^* = J(\lambda \, \eta_T),$$

where λ is the Lagrangian parameter corresponding to the budget constraint and η_T is the Radon-Nikodym density of the risk neutral probability.

C.   The Portfolio Constraints

Constraints on the strategies are described by a nonempty, closed, convex set K, model which covers most cases of practical interest. Recall some basic notions and definitions about convex optimization. For any nonempty, closed, convex set K, denote by

$$d(x) = d(x \mid K) = \sup_{p \hat{\in} K} (-w'x)$$

the support function of the convex set -K. The constrained optimization problem is defined by:

$$\text{Max } E[U(V_T)] \text{ under } w \, \hat{\in} \, K.$$

Following Cvitanic and Karatzas (1992), we deduce:

Proposition 2. Suppose that there exists a process λ such that the optimal solution w^λ of the unconstrained problem associated to the process λ is at any time in the convex K and $d(\lambda_t \mid K) + w_t^\lambda \lambda_t = 0$. Then, w^λ is optimal for the constrained optimization problem in the original market.

A special case is when all diffusion coefficients are deterministic: There exists a formal Hamilton-Jacobi-Bellman equation associated with the dual optimization problem. The optimal portfolio is computed in feedback form, in terms of the current wealth. For this, introduce the function $L(t,x)$:

$$L(t,y) = 1/y E\left[Y_T^{t,y} U'^{-1}(Y_T^{t,y})\right],$$

where $Y_t^{t,y} = y$ and

$$dY_s^{t,y} = -Y_s^{t,y}\left[\left(r_s^0 + d(\lambda_s \mid K)\right)ds + \left(q_s^0 + S_s^{-1}\lambda_s\right)' dW_s\right].$$

Denote by $G(t,.)$ the inverse of $L(t,.)$.

Corollary 3. The optimal portfolio is given in feedback form on the current level wealth by:

$$w_t^* = -[S_t S_t']^{-1}\left[\lambda_t + b_t - r_t^0 1\right]\frac{G(t,V_t^\lambda)}{V_t^\lambda \frac{\partial G(t,V_t^\lambda)}{\partial x}}.$$

Example. Consider the case of CRRA utility: $U(x) = \dfrac{x^a}{a}$ with $a \le 0$. In that case, we deduce:

$$w_t^* = \frac{1}{1-a}[S_t S_t']^{-1}\left[\lambda_t + b_t - r_t^0 1\right],$$

and

$$\lambda_t = \text{argmin}_{x \in \tilde{K}}\left[2d(x \mid K) + \| q_t^0 + S_t^{-1}x \|^2 + 2(1-a)d(x \mid K)\right].$$

Examine also the logarithmic case: $U(x) = \text{Log}(x)$. w^λ is optimal for the process λ solution of:

$$\lambda_t = \text{argmin}_{x \in \tilde{K}}\left[2d(x \mid K) + \| q_t^0 + S_t^{-1}x \|^2\right].$$

Thus the optimal portfolio w^* of the constrained problem satisfies:

$$w_t^* = [S_t S_t']^{-1}\left[\lambda_t + b_t - r_t^0 1\right].$$

As it can be seen, this solution is of the same kind as in the unconstrained problem (see Merton's solution), except that the process λ is added to the instantaneous conditional expectation ("drift" term) b. It appears like a modification of the instantaneous rates of returns of the assets, taking the constraints into account.

This process is completely determined by the specific set of constraints and, as it can be seen, depends on parameters of the market through the "risk premium" q^0 and the inverse of the volatility matrix S. Other some examples are provided in Mhiri (2011).

IV.   THE TWO-COUNTRY CASE

To illustrate the previous results, it is possible to introduce the term structure for domestic and foreign rates of Frachot (1995) exposed in Section II.

It is also necessary to specify the "volatility" matrix Σ of the instantaneous conditional variance-covariances of all assets. In what follows, we can determine the closed-form optimal portfolio and analyze its properties for two main cases.

A.   The Available Financial Assets

Consider an investor who trades on two bond indexes (domestic and foreign) and also on two stock indexes: one domestic S^1 and one foreign S^2.

Suppose that their dynamics are given by:

$$
\begin{cases}
dS^1_t & = & S^1_t[m^1 dt + S^{1,1}dW^1_t + + S^{1,2}dW^2_t], \\
dS^2_t & = & S^2_t[m^2 dt + S^{2,1}dW^1_t + S^{2,2}dW^2_t].
\end{cases}
$$

In particular, there exists a correlation between the indexes. Denote by r^f the foreign domestic rate. r_t and I_t satisfy:

$$
\begin{cases}
dr_t & = & (j - l\, r_t)dt + \sqrt{ar_t}\, dW^3_t, \\
\dfrac{dI_t}{I_t} & = & (r_t - r^f_t)dt + \sqrt{ar_t}\,(a^I dW^3_t + b^I dW^4_t).
\end{cases}
$$

B.   Optimization of the Portfolio Invested on the Four Basic Assets and Domestic Cash

Using all previous results, the optimal portfolio can be computed explicitly. For the numerical illustrations, we use the following parameter values:

$$r^f_t = 0.025; m_1 = 0.10; m_2 = 0.15; S^{1,1} = 0.2; S^{1,2} = S^{2,1} = 0.15; S^{2,2} = 0.2$$

and $a = 1; a^I = 1; b^I = 1$.

Next figure illustrates how the optimal weights depend on the domestic interest rate:

We can note that, without specific constraints on the weights, the optimal solutions are not realistic: they imply that the investor can use high financial leverages. Such results justify why we have to introduce specific constraints on investment strategies.
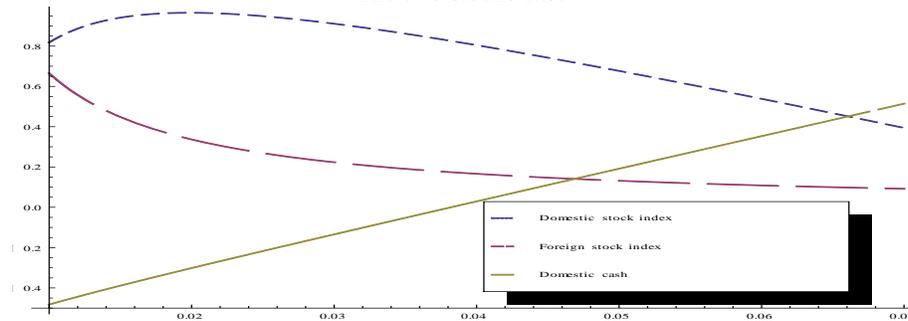
Figure 1
The four basic assets case
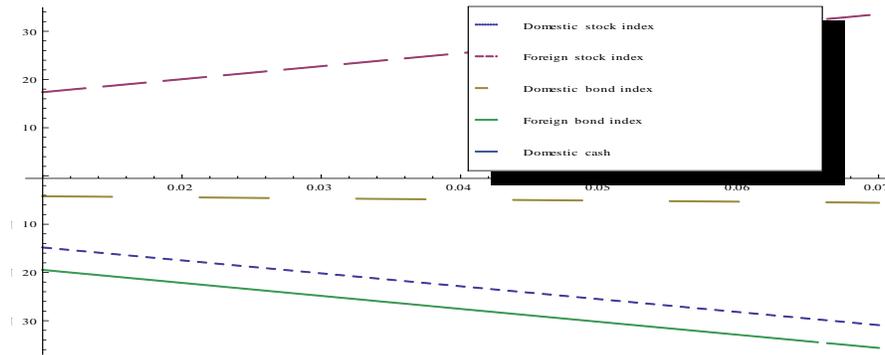
C.   Optimization of the Portfolio Invested only on Stocks and Domestic Cash

We complete the financial market by introducing two appropriate fictitious assets (see Mhiri, 2011). For this particular model, the domestic interest rate r_t is the key factor. We can examine how the optimal weights depend on it.

Figure 2
The two stocks case

For low values of the domestic interest rate, the investor uses a leverage on the monetary asset in order to increase his returns trough the two stocks. Despite a higher expected return, the foreign stock has not the highest weighting. The reason is that its volatility is higher due to the exchange.

V.   CONCLUSION

This paper provides a consistent framework to determine the optimal international portfolio for foreign and domestic investments when no arbitrage is allowed but when the investor faces specific constraints. The model is based on the non arbitrage assumption for the domestic market: all domestic-denominated assets discounted by a domestic

money market account are martingales under a risk-neutral probability. Following Frachot (1995), stochastic volatilities can be considered in the dynamics of both domestic and foreign rates and the foreign term structure can be endogenously determined. In practice, it is necessary to determine the "volatility" matrix and to estimate the risk premium associated to price functional, that is the risk-neutral probability of interest. Obviously, it depends on the dynamics of the exchange rates and so takes into account the change risk. Then, the optimal portfolio is deduced from the specific constraints by using an auxiliary unconstrained problem where all drifts are modified by the addition of a Lagrangian process. This process itself is the solution of a particular minimization problem which can be explicitly solved in some cases or relatively easily computed by numerical methods. Additionally, from the observations of the optimal portfolio, partial information about constraints can be deduced. This would be of particular interest to examine how the constraints of the investor can change according to variations of factors such as macroeconomic or financial variables or, within our framework, according to the level of specific costs such as transaction and information costs.

ENDNOTES

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2. All proofs are available upon request.

REFERENCES

Adler, M. and B. Dumas, 1983, "International Portfolio Choice and Corporation Finance: A Synthesis," *Journal of Finance* 38, 925-984.

Amin, K. and R. Jarrow, 1991, "Pricing Foreign Currency Options under Stochastic Interest Rates," *Journal of International Money and Finance* 10, 310-329.

Bekaert, G. and M. Urias, 1996, "Diversification, Integration and Emerging Market Closed-end funds," *Journal of Finance* 51, 835-869.

Campbell, R.H. 1991, "The World Price of Covariance Risk," *Journal of Finance* 46, 111-158.

Chiou, W.-J.P., 2009, "Benefits of International Diversification with Investment Constraints: An Over-time Perspective," *Journal of Multinational Financial Management*, 19, 93-110.

Coppeland, T.E. and J.F. Weston, 1988, "Financial Theory and Corporate Policy", Addison-Wesley.

Cooper, I. and E. Kaplanis, 1994, "What Explains the Home Bias in Portfolio Investment? " *Review of Financial Studies* 7, 45-60.

Cox, J.C., J.E. Ingersoll, and S.A. Ross, 1985, "A Theory of the Term Structure of Interest Rates," *Econometrica* 53, 385-407.

Cox, J.C., and C.F. Huang, 1989, "Optimal Consumption and Portfolio Policies When Asset Prices Follow a Diffusion Process," *Journal of Economic Theory* 49, 33-83.

Cvitanic, J. and I. Karatzas, 1992, "Convex Duality in Constrained Portfolio Optimization," *Annals of Applied Probability* 2, 767-818.

De Roon, F.A., T.E. Nijman, and B.J. Werker, 2001, "Testing for Meanvariance Spanning with Short Sales Constraints and Transaction Costs: the Case of Emerging

Markets," *Journal of Finance* 56, 721-742.

Eun, C. and B. Resnik, 1988, "Exchange Rate Uncertainty, Forward Contracts, and International Portfolio Selection," *Journal of Finance* 43, 197-215.

Frachot, A. 1995, "Factor Models of Domestic and Foreign Interest Rates with Stochastic Volatility," *Mathematical Finance* 5, 167-185.

French, K.R. and Porterba, J.M. (1991): "Investor diversification and international equity market"s. *American Economic Review* 81, 222-226.

Grubel, H., 1968, "Internationally Diversified Portfolios: Welfare Gains and Capital Flows," *American Economic Review* 58, 1299-1314.

Harvey, C., 1995, "Predictable Risk and Returns in Emerging Markets," *Review of Financial Studies* 8, 773-816.

Heath, D., R. Jarrow, and A. Morton, 1992, "Bond Pricing and the Term Structure of Interest Rates: a New Methodology for Contingent Claims Valuation," *Econometrica* 60, 77-106.

Lessard, D., 1973, "World, National and Industry Factors in Equity Returns," *Journal of Finance* 29, 379-391.

Lessard, D., 1983,"Principles of International Portfolio Selection," *International Finance Handbook,* J.Wiley.

Levy, H. and M. Sarnat, 1970, "International Diversification of Investment Portfolios," *American Economic Review* 60, 668-675.

Li, K., A. Sarkar, and Z. Wang, 2003,"Diversification Benefits of Emerging Markets Subject to Portfolio Constraints," *Journal of Empirical Finance* 10, 57-80.

Mhiri, M., 2011, "International Portfolio Choice : Diversification, Attitude towards Risk, and Barriers to Investment," PhD thesis, University of Cergy-Pontoise, France.

Noetzlin, B. and B. Solnik, 1982, "Optimal International Asset Allocation," *Journal of Portfolio Management* 9, 11-21.

Odier, P. and B. Solnik, 1993, "Lessons for International Asset Allocation," *Financial Analysts Journal*, 63-67.

Pástor, L. and R. Stambaugh, 2000, "Comparing Asset Pricing Models: An Investment Perspective," *Journal of Financial Economics* 56, 335-381.

Solnik, B., 1974a, "Why not Diversify Internationally rather than Domestically?" *Financial Analysts Journal* 30, 48-54.

Solnik, B., 1974b, "An Equilibrium Model of the International Capital Market," *Journal of Economic Theory* 8, 500-524.

Stulz, R.M., 1981, "On the Effects of Barriers to International Investment," *Journal of Finance* 36, 923-934.

Stulz, R.M., 1983, "The Demand for Foreign Bonds," *Journal of International Economics* 15, 225-238.

Uppal, R., 1993, "A General Equilibrium Model of International Portfolio Choice," *Journal of Finance* 49, 529-543.

Wang, Z., 1998, "Efficiency Loss and Constraints on Portfolio Holdings," *Journal of Financial Economics* 48, 359-375.