

An Empirical Comparison of Three Auction Strategies for Multiple Products

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ABSTRACT

This study investigates bidders' bidding strategies and the relative profitability of three auction strategies, one auction for the bundle, two simultaneous separate auctions, and two sequential separate auctions under a controlled environment. The results suggest that when there is high variation and no asymmetry among product values, the three selling mechanisms are equally profitable. When there is high asymmetry but no variation among product values, one auction for the bundle is more profitable than two separate auctions when there are two bidders, but less profitable than when there are ten bidders. Generally, selling products simultaneously or sequentially generates the same revenue.

JEL Classifications: M31, D01, D03, D44, C91

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I. INTRODUCTION

Auctioneers often have multiple complementary products to auction off. Complementarity is present when bidders' value for the bundle is higher than the sum of individual values for these products. Complementarity may be due to savings in transaction costs when one wins multiple products. For example, a winner of two eBay auctions run by the same auctioneer may save on shipping costs if the shipment can be combined. Complementarity may also be a consequence of extra utility from consuming the products together. A typical example is a set of antique furniture for which people usually are willing to pay more to have the complete set. An additional reason for people to pay more than the sum of the individual values is that it will take tremendous effort (if possible at all) to find all the individual pieces from different sellers.

Let us consider an auctioneer selling two different products using second price sealed-bid auctions (also called Vickery auctions)¹. An auctioneer, selling one unit each of two products, A and B, typically has three alternative selling strategies: (1) one auction for the bundle consisting of A and B, (2) two simultaneous separate auctions for A and B, and (3) two sequential separate auctions for A and B.

For the seller, each of the three strategies has its advantages and disadvantages. When complementarity is not present, separate auctions are efficient because winners are always the bidders with the highest values for the individual products. This efficiency minimizes the consumer surplus the winners have and therefore increases the revenues of separate auctions.

The following numerical example demonstrates the inefficiency of an auction for the bundle. Let's suppose there are three bidders bidding on the two products, A and B. Table 1(a) shows the values for A, B, and the bundle of A and B together, which is the sum of the values for A and B. In a Vickery auction, it is an incentive-compatible strategy for each bidder to bid her true value. As Table 1(b) shows, when A and B are auctioned off as a bundle, that price will be equal to the second highest value for and the bid for the bundle, which is \$140. When A and B are auctioned off separately, the individual prices will be \$80 and \$80, respectively, and the total revenue will then be \$160. Also note that two simultaneous and sequential separate auctions should generate the same revenue, since the outcome of one auction will not affect the bidders' strategies for the other auction.

However, when complementarity is present, bidders in separate auctions find themselves facing a so called exposure risk (Ausubel et al., 1997; Rothkopf et al., 1998; Bykowsky et al., 2000; Chakraborty, 2004; Popkowski Leszczyc and Häubl, 2010). This is the risk of winning only one of the products at a price higher than its individual value, or winning both products at a price higher than the value for the bundle, as bidders may bid above their value(s) in an attempt to win both auctions and receive the extra complementarity.

The exposure risk affects bidders differently in simultaneous and sequential auctions for two complements, say product A and product B. In two sequential Vickery auctions, bidders only face exposure risk in the first auction. In the second auction, the losing bidders' weakly dominant strategy is to bid the value for B, while the winner of A will bid an amount equal to the sum of the value for B and the complementarity. In the first auction, bidders may overbid to try to win A to increase their chances to win

Table 1

Illustration of the inefficiency of one auction for a bundle consisting of A and B when complementarity is not present

(a) Bidders' values for A, B, and the bundle of A and B

Bidder	Value for A	Value for B	Value for bundle
Bidder 1	\$100	\$40	\$140
Bidder 2	\$40	\$100	\$140
Bidder 3	\$80	\$80	\$160

(b) Comparison of a bundle auction and two separate auctions

Auction		Winner	Winner's Value	Price
One auction for the bundle		Bidder 3	\$160	\$140
Two separate auctions	Auction for B	Bidder 1	\$100	\$80
	Auction for A	Bidder 2	\$100	\$80

both auctions and thus receive the complementarity². Therefore, a bidder may win the first product by overpaying and then lose the second product. Take Bidder 1 in Table 1 as example. Let us assume that A and B are complements to all three bidders and the value for the bundle is \$40 more than the sum of the independent values for A and B, i.e., the value for complementarity is \$40 to all bidders. In the first of the two sequential auctions for A, Bidder 1 may bid \$130, \$30 higher than her value for A (\$100), to try to win A and have the opportunity of winning B and the complementarity in the second auction. However, if in the first auction for A, she is the highest bidder and the second highest bid is \$110 (say from Bidder 3 who also overbids in order to win A and thus the complementarity), Bidder 1 wins A by paying \$10 more than her value for A. However, if Bidder 1 fails to win B in the second auction (say Bidder 2 bids higher in the second auction), Bidder 1 ultimately wins A only and pays a price higher than her value for A.

In two simultaneous Vickery auctions, bidders run a potential exposure risk in both, as they may overbid in both auctions to increase their chances of winning the complementarity. They may then end up winning either A or B by paying higher-than-value price. For example, given that the complementarity is worth \$40, Bidder 1 in Table 1 may bid \$120 on A and \$60 on B in order to win both products and, therefore, gain complementarity. However, a possible outcome could also be that she wins only A and pays, say \$110, which is the second highest bid submitted by another bidder and is \$10 higher than her value for A. In certain instances, a bidder may even win both products by paying a total price that is higher than her value for the bundle.

As resulted, strategic bidders, in separate auctions, may bid less aggressively due to the exposure risk, reducing the profitability of separate auctions.

In contrast, in a bundle auction, bidders do not face this exposure risk and can bid more aggressively, adding the entire complementarity to their bid. This lack of exposure risk may translate to higher revenue. However, an auction for the bundle is generally inefficient because the winner is the bidder with the highest value for the bundle, and not necessarily the bidder with the highest values for the individual products. This inefficiency decreases the revenue of a bundle auction.

To sum up, when complementarity is present, in two separate auctions the efficiency increases the revenue while the exposure risk decreases the revenue. In an auction of the bundle, the inefficiency decreases the revenue but the lack of exposure risk increases the revenue. Therefore, when choosing a selling mechanism, an auctioneer of the two complements faces a tradeoff between the inefficiency of a bundle auction and the exposure risk problem in two separate auctions, and the optimality of each selling strategy depends on the net effect of these two mechanisms.

The primary objective of this empirical study is to compare the profitability of these three selling mechanisms under different conditions. The environment is characterized by (1) the number of bidders (N), (2) heterogeneity of bidders' individual values for the two products, and (3) complementarity of the two products (C). Another objective is to find out how the environment affects bidders' strategies for these three selling mechanisms. Specifically, I am interested in looking at how the environment affects bidders' perceived exposure risk and how the exposure risk affects bidders' overbidding in separate auctions. These issues are examined through laboratory experiments.

The remainder of this paper is presented as follows. The second section reviews the literature. The third and fourth sections offer the details and results of the experiment. The paper concludes with a discussion of the key findings in the fifth section, followed by an analysis of its limitations in the sixth section.

II. LITERATURE REVIEW

A. Bundling Literature in a Posted Price Context

Bundling, as a pervasive selling mechanism, is defined as "the sale of two or more separate products in one package" (Stremersch and Tellis, 2002), where "separate products" means products for which separate markets exist. It is a widely used marketing practice, to sell a wide variety of products, including seasonal tickets for sports events, high speed Internet and cable TV, air tickets, hotel and car rentals.

Research on bundling as a pricing mechanism was initiated by Stigler (1968). Since then bundling has received considerable attention by academics in the field of economics (Adams and Yellen, 1976; Schmalensee, 1984; McAfee, McMillan, and Whinston, 1989; Salinger, 1995) and marketing (Guiltinan, 1987; Gaeth et al., 1990; Yadav, 1994, 1995; Yadav and Monroe, 1993; Bakos and Brynjolfsson, 1999, 2000; Soman and Gourville, 2001; Stremersch and Tellis, 2002; Jedidi et al., 2003).

Bundling has been shown to increase sellers' profits by permitting more complete extraction of buyers' residual consumer surplus. This is because bundling can reduce the heterogeneity of buyers' reserve prices, by serving as a second-degree price discrimination mechanism (Adams and Yellen, 1976; Schmalensee, 1984). In their survey of the economics and marketing literatures on bundling, Stremersch and Tellis (2002) found that ambiguity exists concerning the concept of heterogeneity of reservation prices. They argued that the distribution of reservation prices consists of asymmetry and variation, and correlation alone is not sufficient to represent heterogeneity. Asymmetry refers to the difference among consumers' reservation prices for the separate products. For two separate products A and B, asymmetry occurs when one segment of buyers has a relatively higher reservation price for A, while the other segment has a higher reservation price for B. Variation means the difference among

consumers' reservation prices for the bundle of products. Asymmetry leads to negative correlation while variation leads to positive correlation. Stremersch and Tellis (2002) showed that these two dimensions affect the optimality of bundling in different ways, and, hence, it is important to incorporate both aspects of heterogeneity.

Besides heterogeneity of values, complementarity³ of multiple products has been shown to affect the profitability of bundling (Lewbel, 1985; Matutes and Regibeau, 1988, 1992; Telser, 1979; Guiltinan, 1987; Venkatesh and Kamakura, 2003). Venkatesh and Kamakura (2003) found that the optimality of different selling mechanisms (unbundled sales, pure bundling, and mixed bundling) is determined by the degree of complementarity. For example, when marginal cost is low, pure bundling is optimal for moderate-to-strong complements and mixed bundling is optimal for independently valued products and weak complements.

Examining the existing bundling literature, I identify heterogeneity of consumer's reservation prices (values) and the degree of complementarity as two key factors deciding the profitability of bundling. Therefore, I will incorporate both heterogeneity of values and complementarity in this study.

B. Auction Literature for Multiple Objects Auctions

Although most auction studies have focused on individual product auctions, auction of multiple products is a very active area of research (see Klemperer (2004) for a review). Prior economics studies have examined optimal auction design for multiple products (e.g., Maskin and Riley, 1984; Armstrong, 2000; Levin, 1997; Avery and Hendershott, 2000), simultaneous auctions (e.g., Wilson, 1979; Anton and Yao, 1992; Krishna and Rosenthal, 1996), sequential auctions (e.g., Bernhardt and Scoones, 1994; McAfee and Vincent, 1997; Jeitschko, 1999) and combinatorial auctions (see Milgram (2004) for a review). The multiple products can be either homogeneous (Wilson, 1979; Krishna and Rosenthal, 1996) or heterogenous (Palfrey, 1983; Chakraborty, 1999; Levin, 1997). The topic has also begun to receive attention from marketing researchers (Zeithammer, 2006; Cheema et al., 2005; Subramanian and Venkatesh, 2009; Popkowski Leszczyc and Häubl, 2010).

One track within the multiple product auction literature compares three typical selling mechanisms for multiple products in term of profitability based on the following analytical models:

(1) Bundle auction vs. Simultaneous auctions. Palfrey (1983) compared the profitability of one bundled Vickrey auction versus two simultaneous separate Vickrey auctions and showed that when there are only two bidders, the bundle auction is more profitable than separate auctions. Based on Palfrey (1983)'s framework, Chakraborty (1999) found that for two products whose values are independently distributed, there is a threshold for the number of bidders above which separate auctions will always be more profitable. So in general these two papers have concluded that without complementarity simultaneous separate auctions are more profitable than bundle auctions for more than two bidders. Both studies assumed that bidders' values for the component products are independently distributed and there is no complementarity.

(2) Bundle auction vs. Sequential auctions. Subramanian and Venkatesh (2009) examined the profitability of one auction for the bundle versus two sequential auctions for two complementary products. They concluded that when complementarity is small

and there are more than four bidders, separate auctions are more profitable. However, when complementarity is moderate or large, a bundle auction is always more profitable. Although their conclusions are in part based on the assumption that the individual values for the two products are independently distributed.

(3) Simultaneous auctions vs. Sequential auctions. Krishna & Rosenthal (1996) argued that for two complements, simultaneous and sequential auctions are approximately equally profitable. However, their results are limited due to the very strict assumptions made⁴. Hausch (1986) compared simultaneous and sequential auctions for two affiliated value identical products. He identified two opposing effects in sequential auctions: (i) when bids are announced between auctions, they may convey information about the values for products to be sold later on, which increase the revenues (an information effect); (ii) bidders who are aware of the information effect tend to bid lower in the first auctions and therefore reduce revenue (a deception effect). The optimality of sequential auctions depends on the net effect of these two effects. Feng and Chatterjee (2008) looked at a seller who has multiple identical products to sell to N bidders who arrive sequentially and only want one unit of the product. They indicated that the ratio of the number of items to the number of bidders decides whether sequential auctions are more profitable or not. When the ratio of the number of bidders to the number of items for sale is below a threshold value, sequential auctions have higher expected revenue than simultaneous auctions.

While auctioning off multiple products with complementarity is of significant managerial importance, a close examination of the literature reveals several significant gaps. First, there is no analytical model that compares all three mechanisms and shows under what conditions sellers should choose to sell products in a bundle or sell them in separate auctions (either simultaneously or sequentially). Second, very few empirical studies have tested the above-mentioned analytical models and their theoretical predictions on bidders' bidding strategies and auction revenues. Popkowski Leszczyc and Häubl (2010) empirically compared the seller revenue of the bundle auction relative to the revenue from separate auctions of the components based on evidence from eBay. The primary conclusion then is that although bundle auctions tend to be less profitable for noncomplementary products, these auctions are on average 50% more profitable than separate auctions when there is complementarity between the component products. However, the authors observed only the outcome (i.e., revenue) of the auctions, not the values and bids of all bidders for both the bundles and the separate component items. Therefore, it is not fully clear how all bidders' bids are influenced by the auction environments.

This study contributes to the auction literature by empirically investigating how bidders bid under different conditions, which are defined by the number of bidders, the complementarity and heterogeneity of bidders' values, for each of the three auction mechanisms, and comparing the revenues of all the three typical selling mechanisms under different conditions.

III. RESEARCH DESIGN

A. The Auctions

A revenue maximizing auctioneer has one unit of two products A and B, which can be either identical or different, to sell to N bidders ($N \geq 2$). These two products are to be auctioned by one of the following three auction mechanisms:

1. One Vickrey auction for the bundle consisting of products A and B. Each bidder submits just one bid (b_{bu}) for the bundle. The bidder with the highest b_{bu} wins, and the price the winner pays equals the second highest bid.

2. Two simultaneous separate Vickrey auctions. Each bidder submits two bids (b_A, b_B) respectively for products A and B. In each of the two auctions, the bidder with the highest bid wins, and the price the winner pays equals the second highest bid. The winners are announced simultaneously; hence, when placing a bid on one product, they are unaware of the outcome of the other auction.

3. Two sequential separate Vickrey auctions, with the first auction for product A followed by a second auction for product B⁵. Each bidder first places a bid (b_A) for product A, followed by a bid for product B, which is conditional on the outcome of the auction for product A ($b_{B|winA}$ or $b_{B|loseA}$). In both auctions, the bidder with the highest bid in each auction wins and pays a price equal to the second highest bid in that auction.

The following assumptions are made in this paper:

1. The number of bidders (N) is the same in the two separate auctions and in the bundle auction. The number of bidders is common knowledge to all bidders and to the seller.

2. Complementarity (C) for products A and B is the same for all bidders, regardless of their individual values for A and B (V_A and V_B). C is common knowledge to all bidders and the seller⁶.

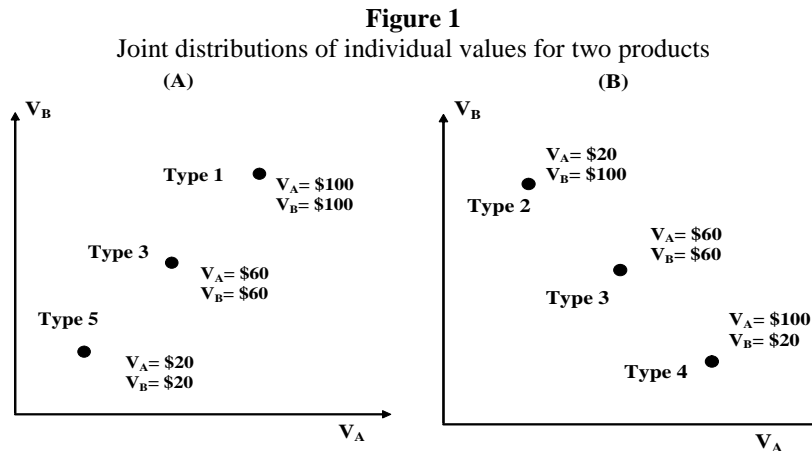
3. A bidder's value for the bundle of A and B (V_{bu}) equals the sum of her individual values for A and B (V_A and V_B) and the complementarity (C).

4. Each bidder's V_A and V_B are privately known and are realizations of the same distribution that is common knowledge to all bidders and the seller.

B. Experiment Design

In this study there are two distributions of bidders' values (shown in Figure 1), in each there are three bidder segments (types) with three different combinations of V_A and V_B . Each bidder has one third chance of being chosen by nature to be of one of the three potential types. In the first distribution (see Figure 1(a)), V_A and V_B of the three types Type 1, Type 3, and Type 5 are respectively (\$100, \$100), (\$60, \$60), and (\$20, \$20). In the second distribution (see Figure 1(b)), V_A and V_B of the three types Type 2, Type 3, and Type 4 are (\$20, \$100), (\$60, \$60), and (\$100, \$20) respectively. In this first distribution, there is only variation and no asymmetry in bidders' individual values, and, hence, the two values are perfectly positively correlated. There is only asymmetry and no variation in the second distribution, implying that the two values are perfectly negatively correlated. Therefore a main advantage of adapting these two distributions is that I can look at the effect of each of the two dimensions of heterogeneity in bidders' individual values while controlling the other.

I set the number of bidders at either 2 or 10⁷. Given that Palfrey (1983), Chakraborty (1999), and Subramanian and Venkatesh (2009) identified two, three and four as the threshold number of bidders to decide the relative profitability of bundle auction, I believe 2 bidders is low and 10 is high. I choose C =\$20 and C =\$50 as low and high levels of complementarity.



Therefore I obtain eight ($2 \times 2 \times 2$) different combinations (scenarios) of the heterogeneity of bidder's two values (distributions 1 and 2 in Figure 1), the number of bidders N (2, 10) and the level of complementarity C (\$20, \$50), as shown in Table 2.

For each combination for the number of bidders N , complementarity C and distribution of V_A and V_B , bidders come up with their bids in each of the three auction mechanisms. The seller, with the knowledge of the number of bidders, the distribution of bidders' types and all the type contingent bids, calculates and compares the expected revenues of the three selling mechanisms. The objective of this study is to find out under what conditions which of the three selling mechanisms is most profitable.

The bidders were 68 undergraduate business students at a North American university. Participants were provided with detailed instructions and shown an example of a Vickrey auction. The instructions included the following:

1. They would attend a series of auctions and bid on two hypothetical products, products A and B. Each participant would be provided with a value for each of the two products.

2. The values for these products were drawn from one of the two distributions demonstrated in Figure 1. In each auction, each participant was told the specific distribution from which her and her rivals' values for products A and B were drawn.

3. In each auction, the winner of an auction would obtain an amount equal to the difference between her value for the product and the amount of the second highest bid. Each bidder had 100 "e-dollars" in her account (Each e-dollar equaled one cent). All gains (losses) from the auctions in this study would be added to (or subtracted from) the subjects' accounts.

4. Whenever a bidder won both A and B, she would get an extra bonus, which represented the complementarity between the two products up for auction.

5. The bidders were told the number of opponents they would compete against in each auction.

To help bidders understand the concept of Vickrey auction, I conducted one practice run of a Vickrey auction for a hypothetical product. The outcome of the auction was revealed. Next all participants completed a short quiz about Vickrey auction and the correct answers were announced. Finally, bidders entered the real experimental auctions.

Table 2
Eight combinations (scenarios) of individual values, number of bidders and complementarity

Scenario No.	1	2	3	4	5	6	7	8
# of Bidders	2	2	10	10	2	2	10	10
Complementarity	\$20	\$50	\$20	\$50	\$20	\$50	\$20	\$50
Heterogeneity	High				No			
Variation Asymmetry	No				High			
Type 1 (VA=\$100, VB=\$100)	Included							
Type 2 (VA=\$20, VB=\$100)					Included			
Type 3 (VA=\$60, VB=\$60)	Included				Included			
Type 4 (VA=\$100, VB=\$20)					Included			
Type 5 (VA=\$20, VB=\$20)	Included							

Each bidder was required to bid in all eight scenarios for each of the three auction mechanisms. Thus, the experiment employed a four-factor (auction mechanism, N, C and distribution of values), twenty four-level (scenario) within-subject design.

Auctions using the same mechanisms were always put in the same block. So there are three blocks (mechanisms) with eight scenarios in each, and there are six possible orders for the three blocks. The order of blocks (mechanisms) was randomized, as well as the order of the eight scenarios within each block.

In each of the eight scenarios, subjects were told the number of opponents they competed against, the distribution from which their opponents' values were drawn, and the amount of complementarity for the two products. In each scenario there were three (pairs of) auctions, in each (pair of) auction a subject was given a pair of values, V_A and V_B (one out of the three in the given distribution) and was required to bid on each auction. Thus in a scenario defined by N, C and a distribution of V_A and V_B , I have each bidder's bids for each of the three pairs of values. Participants were told that only one of the three auctions would actually be conducted. For example, in a scenario where $N=2$, $C=\$20$ and V_A and V_B are drawn from the second distribution, a bidder participated in three auctions in which her values are respectively (\$20, \$100), (\$60, \$60) and (\$100, \$20). The bidder was told to place bid in each of the three auction based on these values, while her opponent's values could be any of the three pairs with equal chance. Only one auction was executed to determine the bidder's profit (for this scenario).

In an auction for the bundle, each subject was asked to place one bid. In two simultaneous separate auctions, each subject placed one bid on A and one on B. In two sequential separate auctions, a bidder was required to submit one bid for the first product A, and submit two bids for B; one if she were to win A ($b_{B|WinA}$) and one if she

were to lose A ($b_{B|LoseA}$). Bidders did not know the outcome of the first auction when they bid in the second auction.

To ensure that bidders understood the rules of each selling mechanism before the real auctions in each block (selling mechanism), two practice rounds were run and outcomes were shown for demonstration purposes. This was followed by a short quiz with several questions about the selling mechanism. These quizzes served as filters for each of the selling mechanism (experimental blocks). In the following data analysis for each mechanism, I only include the bids from the subjects who correctly answered all questions on the quiz about this mechanism⁸. One typical session lasted about 75 minutes.

IV. RESULTS

The average bids are summarized in Tables 3 to 5 for the three selling mechanisms.

A. One Bundle Auction

In all auctions of the bundle, bidders' bids are approximately equal to their corresponding values for the bundle (V_A+V_B+C), regardless of N and heterogeneity of values. The bids on the bundle increase as C increases, and an increase in N has little impact on bids⁹.

B. Two Simultaneous Auctions

When only variation is present and V_A and V_B are perfectly positively correlated (scenarios 1 to 4), comparison of all types of bidders' bids in scenarios 1 and 2 and comparison of bids in scenario 3 and 4 reveal that people increase their two bids to the same extent when C increases from \$20 to \$50, regardless of N.

In scenarios 5 to 8, where only asymmetry is present, Type 2 and Type 4 bidders do not change their bids according to N, while Type 3 bidders bid less aggressively due to the exposure risk. A comparison of scenarios 5 and 6 shows that, when N=2, Type 3 bidders increase the sum of their two bids by \$21.63 (paired $t=5.12$, $df=54$, $p=.00$) when C increases from \$20 to \$50. However, when N=10, Type 3 bidders actually decrease the sum of their two bids by \$2.39 (paired $t=-0.283$, $df=53$, $p=.778$) when C increases from \$20 to \$50.

These results indicate that Type 3 bidders' bids are more sensitive to exposure risk than Type 2 and Type 4 bidders'. This is because Type 2 and Type 4 bidders have a high value for one product and therefore have a good chance to win this product without adding any complementarity to the bid, while Type 3 bidders have median values for both products so they have to add complementarity to both products to win them both.

Table 3
Average bids in one auction for the bundle

(a) High variation, no asymmetry				
Scenario No.	1	2	3	4
# of Bidders	2	2	10	10
Complementarity	\$20	\$50	\$20	\$50
Type 1 ($V_A=\$100, V_B=\100)	$b_{bu}=\$212.76$	$b_{bu}=\$240.95$	$b_{bu}=\$219.56$	$b_{bu}=\$243.36$
Type 3 ($V_A=\$60, V_B=\60)	$b_{bu}=\$138.22$	$b_{bu}=\$171.46$	$b_{bu}=\$139.85$	$b_{bu}=\$168.59$
Type 5 ($V_A=\$20, V_B=\20)	$b_{bu}=\$62.71$	$b_{bu}=\$95.07$	$b_{bu}=\$72.61$	$b_{bu}=\$96.68$
(b) No variation, high asymmetry				
Scenario No.	5	6	7	8
# of Bidders	2	2	10	10
Complementarity	\$20	\$50	\$20	\$50
Type 2 ($V_A=\$20, V_B=\100)	$b_{bu}=\$139.00$	$b_{bu}=\$168.64$	$b_{bu}=\$140.95$	$b_{bu}=\$170.22$
Type 3 ($V_A=\$60, V_B=\60)	$b_{bu}=\$139.80$	$b_{bu}=\$169.29$	$b_{bu}=\$140.58$	$b_{bu}=\$169.17$
Type 4 ($V_A=\$100, V_B=\20)	$b_{bu}=\$139.54$	$b_{bu}=\$168.80$	$b_{bu}=\$141.19$	$b_{bu}=\$170.46$

C. Two Sequential Auctions

In the second of the two sequential auctions, all types of bidders' $b_{B|loseA}$ were very close to their corresponding V_B and $b_{B|winA}$ to (V_B+C) , regardless of N and the distribution of their opponents' values, fully consistent with the predictions of Subramanian and Venkatesh (2009).

When only variation exists in bidders' V_A and V_B (scenarios 1 to 4), in the first auctions, all types of bidders' b_A are affected by C but not by N . When only asymmetry exists in bidders' V_A and V_B , N has little impact on Type 2 bidders' b_A . When C increases from \$20 to \$50, Type 2 bidders increase their b_A as much when $N=2$ (scenarios 5 and 6) as when $N=10$ (scenarios 7 and 8)¹⁰. N has a significant impact on Type 3 and Type 4 bidders' b_A . A comparison of scenarios 5 and 6 shows that when $N=2$, Type 3 bidders increase their b_A by \$26.93 when C increases from 20 to 50. However, when $N=10$, Type 3 bidders only increase their b_A by \$7.75 when C increases from \$20 to \$50¹¹. Type 4 bidders increase their b_A by \$24.87 from scenario 5 to 6 (when $N=2$) and only \$0.58 from scenario 7 to 8¹² (when $N=10$).

Table 4
Average bids in two simultaneous separate auctions

(a) High variation, no asymmetry				
Scenario No.	1	2	3	4
# of Bidders	2	2	10	10
Complementarity	\$20	\$50	\$20	\$50
Type 1 ($V_A=\$100, V_B=\100)	$b_A=\$115.52$ $b_B=\$114.75$	$b_A=\$131.29$ $b_B=\$129.34$	$b_A=\$115.49$ $b_B=\$116.09$	$b_A=\$130.34$ $b_B=\$126.98$
Type 3 ($V_A=\$60, V_B=\60)	$b_A=\$73.33$ $b_B=\$71.38$	$b_A=\$86.49$ $b_B=\$84.42$	$b_A=\$73.60$ $b_B=\$72.96$	$b_A=\$84.26$ $b_B=\$82.34$
Type 5 ($V_A=\$20, V_B=\20)	$b_A=\$34.77$ $b_B=\$33.21$	$b_A=\$46.60$ $b_B=\$44.90$	$b_A=\$33.09$ $b_B=\$31.57$	$b_A=\$44.29$ $b_B=\$42.21$

(b) No variation, high asymmetry				
Scenario No.	5	6	7	8
# of Bidders	2	2	10	10
Complementarity	\$20	\$50	\$20	\$50
Type 2 ($V_A=\$20, V_B=\100)	$b_A=\$40.19$ $b_B=\$103.90$	$b_A=\$62.96$ $b_B=\$116.06$	$b_A=\$37.73$ $b_B=\$106.94$	$b_A=\$62.49$ $b_B=\$109.32$
Type 3 ($V_A=\$60, V_B=\60)	$b_A=\$71.75$ $b_B=\$70.88$	$b_A=\$85.28$ $b_B=\$78.98$	$b_A=\$69.67$ $b_B=\$69.96$	$b_A=\$68.79$ $b_B=\$68.45$
Type 4 ($V_A=\$100, V_B=\20)	$b_A=\$105.62$ $b_B=\$39.73$	$b_A=\$116.74$ $b_B=\$64.34$	$b_A=\$108.21$ $b_B=\$37.83$	$b_A=\$109.57$ $b_B=\$61.19$

Table 5
Average bids in two sequential separate auctions

(a) High variation, no asymmetry				
Scenario No.	1	2	3	4
# of Bidders	2	2	10	10
Complementarity	\$20	\$50	\$20	\$50
Type 1 ($V_A=\$100, V_B=\100)	$b_A=\$122.13$ $b_{B WinA}=\$120.49$ $b_{B LoseA}=\$102.27$	$b_A=\$142.89$ $b_{B WinA}=\$139.25$ $b_{B LoseA}=\$103.73$	$b_A=\$120.65$ $b_{B WinA}=\$123.58$ $b_{B LoseA}=\$101.55$	$b_A=\$142.18$ $b_{B WinA}=\$140.69$ $b_{B LoseA}=\$101.02$
Type 3 ($V_A=\$60, V_B=\60)	$b_A=\$75.51$ $b_{B WinA}=\$79.69$ $b_{B LoseA}=\$62.44$	$b_A=\$98.69$ $b_{B WinA}=\$98.56$ $b_{B LoseA}=\$63.53$	$b_A=\$74.91$ $b_{B WinA}=\$76.65$ $b_{B LoseA}=\$60.76$	$b_A=\$97.55$ $b_{B WinA}=\$98.49$ $b_{B LoseA}=\$63.00$
Type 5 ($V_A=\$20, V_B=\20)	$b_A=\$35.58$ $b_{B WinA}=\$38.78$ $b_{B LoseA}=\$22.82$	$b_A=\$59.00$ $b_{B WinA}=\$60.74$ $b_{B LoseA}=\$23.98$	$b_A=\$32.98$ $b_{B WinA}=\$40.98$ $b_{B LoseA}=\$23.49$	$b_A=\$58.24$ $b_{B WinA}=\$60.17$ $b_{B LoseA}=\$23.50$

Table 5 (continued)

(b) No variation, high asymmetry					
Scenario No.	5	6	7	8	
# of Bidders	2	2	10	10	
Comple-mentarity	\$20	\$50	\$20	\$50	
Type 2 ($V_A=\$20$, $V_B=\$100$)	$b_A=\$39.24$ $b_{B WinA}=\$113.81$ $b_{B LoseA}=\$97.64$	$b_A=\$58.56$ $b_{B WinA}=\$135.20$ $b_{B LoseA}=\$100.00$	$b_A=\$ 37.13$ $b_{B WinA}=\$116.87$ $b_{B LoseA}=\$95.85$	$b_A=\$ 56.35$ $b_{B WinA}=\$135.42$ $b_{B LoseA}=\$100.00$	
Type 3 ($V_A=\$60$, $V_B=\$60$)	$b_A=\$77.11$ $b_{B WinA}=\$78.31$ $b_{B LoseA}=\$61.24$	$b_A=\$104.04$ $b_{B WinA}=\$99.98$ $b_{B LoseA}=\$63.64$	$b_A=\$ 70.05$ $b_{B WinA}=\$79.05$ $b_{B LoseA}=\$59.73$	$b_A=\$77.80$ $b_{B WinA}=\$101.40$ $b_{B LoseA}=\$63.36$	
Type 4 ($V_A=\$100$, $V_B=\$20$)	$b_A=\$119.15$ $b_{B WinA}=\$51.87$ $b_{B LoseA}=\$23.73$	$b_A=\$144.02$ $b_{B WinA}=\$69.42$ $b_{B LoseA}=\$24.18$	$b_A=\$ 110.51$ $b_{B WinA}=\$50.25$ $b_{B LoseA}=\$23.73$	$b_A=\$ 111.09$ $b_{B WinA}=\$66.05$ $b_{B LoseA}=\$24.67$	

Table 6

Observed revenues for the eight scenarios

(a) High variation, no asymmetry					
Scenario No.	1	2	3	4	
# of Bidders	2	2	10	10	
Complementarity	\$20	\$50	\$20	\$50	
Hetero- geneity	Variation Asymmetry	High No	High No	High No	High No
Bundle auction		\$101.51 (52.73)	\$130.88 (56.52)	\$213.78 (23.85)	\$246.68 (26.11)
Simultaneous Auctions		\$105.80 (56.20)	\$128.66 (61.48)	\$224.54 (26.37)	\$252.19 (32.36)
Sequential Auctions		\$100.55 (56.12)	\$125.56 (58.20)	\$218.31 (27.15)	\$246.86 (29.88)

(b) No variation, high asymmetry					
Scenario No.	5	6	7	8	
# of Bidders	2	2	10	10	
Complementarity	\$20	\$50	\$20	\$50	
Hetero- geneity	Variation Asymmetry	No High	No High	No High	No High
Bundle auction		\$135.50 (12.67)	\$161.54 (20.68)	\$148.88 (8.02)	\$181.30 (10.37)
Simultaneous Auctions		\$109.20 (26.62)	\$142.92 (25.00)	\$214.24 (16.04)	\$211.58 (15.48)
Sequential Auctions		\$114.82 (26.34)	\$146.77 (27.36)	\$205.24 (16.26)	\$211.04 (14.08)

Note: Standard deviations are in the parentheses.

D. Comparison of Revenues

Table 6 summarizes the mean of the revenue for each selling mechanisms in each of the eight scenarios. Mean revenue in each scenario was calculated by bootstrapping. In each iteration, I randomly choose N ($N=2$ or 10) bidders from all the bidders who attended the auctions in this scenario. For each bidder chosen, I randomly choose one of the three types and the type contingent bids. Based on these N (pairs of) bids, the revenue is decided according to the rule of each selling mechanism.

Based on the results provided in Table 6, I summarize how N , C and heterogeneity affect revenues of the three selling mechanisms as follows:

1. Number of bidders. For all of the three mechanisms for given C and heterogeneity of individual values, larger N leads to higher revenues. In separate auctions, on the one hand, a larger N leads to less aggressive bidding (due to increased exposure risk), but, on the other hand, it results in a higher likelihood of having bidders with higher product values. The net effect of N on revenues is positive in separate auctions.

2. Complementarity. Generally, a higher C leads to higher revenues. There are two exceptions. A comparison of scenario 7 and 8 ($N=10$) shows that when C increases from 20 to 50, the revenue of two sequential auctions increases only by \$5.8 dollars ($z=.27$, $p=.3936$, one tailed), while the revenue of two simultaneous auctions actually decreases by \$2.66 dollars ($z = -.12$, $p = .452$, one tailed). In these two cases, bidders added little C to their bids due to the increased exposure risk.

3. Heterogeneity of individual values. For all three mechanisms, the effect of heterogeneity of individual values on revenues depends on N . When there are two bidders, asymmetry of values generates higher revenues. When there are ten bidders, variation of values leads to higher revenues.

The optimality of the three mechanisms depends on the combination of the three factors discussed above. When V_A and V_B are perfectly positively correlated and only variation is present (scenarios 1, 2, 3 and 4), the three selling mechanisms are approximately equally profitable with the biggest difference being 10.7 dollars ($Z=.28$, $p=.7795$), which occurred in Scenario 3 ($N=10$, $C=20$), accounting for only 5.07% of the expected revenue in Scenario 3. In scenarios 5 and 6 where only asymmetry is present and $N=2$, selling two complements in a bundle is more profitable. When $N=10$, the two separate auction mechanisms generate approximately the same revenues, and both are more profitable than bundle auctions. A comparison of scenarios 7 and 8 where $N=10$ shows that when C increases from 20 to 50, the revenue of auction for the bundle increases on average by \$32.4 ($z=2.47$, $p=.0068$, one tailed), the revenue of two sequential auctions increases only by \$5.8 dollars ($z=.27$, $p=.3936$, one tailed), while the revenue of two simultaneous auctions actually decreases by \$2.66 dollars ($z=-.12$, $p=.4522$, one tailed).

V. SUMMARY

The primary research question in this study was “which of the three selling mechanisms is most profitable, when selling two complementary products, namely, A and B?” Based on the empirical evidence, I present the following findings.

None of the three mechanisms strictly dominates the others. Superiority in profitability of each of them depends on the heterogeneity of individual values, number of bidders and the magnitude of complementarity. The relative profitability of these three selling mechanisms depends on the net effect of the inefficiency in bundling and the exposure risk in separate auctions.

When there is high variation in product values, which are positively correlated, the three selling mechanisms are equally profitable, regardless of number of bidders and complementarity. This is due to both the absence of exposure risk in separate auctions and inefficiency in bundle auctions.

When high asymmetry exists and product values are negatively correlated, one bundle auction is more profitable than two separate auctions (simultaneous or sequential) when there are two bidders and less profitable when there are ten bidders. Simultaneous auctions and sequential auctions are approximately equally profitable, although the nature of exposure risk is different in these two separate selling strategies.

VI. LIMITATIONS AND DIRECTIONS FOR FUTURE RESEARCH

Although this study generates solid evidence for analyzing bidders' bids and comparing the profitability of the three selling mechanisms, the generalizability of the results is somewhat limited due to the use of undergraduate students in a lab experiment environment and the use of hypothetical products. However, this study does offer a precise methodology that may be replicated in other populations having greater diversity in more real settings. Future research is also needed to address how product characteristics, a larger number of products (3 or more), and bidders' risk attitude will affect bidders' bids and seller revenue.

ENDNOTES

1. In the remainder of this paper, I assume that sellers use a Vickrey auction and that bidders will bid their value. A Vickrey auction is an auction where bidders submit written bids without knowing the bids of the others in the auction. The highest bidder wins, but the price she pays is equal to the second highest bid. A major reason for using a Vickrey auction is that without complementarity bidders' weakly dominant strategy is to bid value for the product, regardless of their risk attitude or the number of competing bidders. Most previous research has also used Vickrey auctions (e.g., Palfrey, 1983; Krishna and Rosenthal, 1996; Chakraborty, 1999; Subramanian and Venkatesh, 2009).
2. Subramanian and Venkatesh (2009) showed that it is an optimum strategy for bidders to bid above their value for the first product in a sequential auction to increase the chance of winning both products.
3. Oxenfeldt (1966) identified eight important sources of complementary of demand: One-Stop Shopping; Impulse buying; Broader Assortment; Related Use; Enhanced Value; Prestige Builder; Image Effects; Quality Supplements relationships. Guiltinan (1987) categorized complementary into three types: saving purchasing time and effort; enhancing satisfaction with other products; enhancing image of the seller so all products are valued more highly.

4. They assume that there are only two kinds of bidders; local and global bidders. A global bidder has equal values for these multiple products and for her the value for the bundle exceed the sum of the individual values, while a local bidder wants only one of the products and received no complementarity for winning both products. It is not clear if their conclusions apply when global bidders have unequal values for the individual products.
5. Values for A and B have same individual distribution, so the order of individual auctions does not matter.
6. Here I examine the heterogeneity of individual values V_A and V_B , not complementarity. In many cases, while people may have different individual values for two products, they have similar value for the complementarity (see the FCC auctions and eBay auctions examples mentioned before). This assumption was also made by Krishna and Rosenthal (1996).
7. Previous studies show monotonicity in the effect of number of bidders on the relative profitability of bundle vs. separate auctions. For example, Palfrey (1983), Chakraborty (1999), and Subramanian and Venkatesh (2009) all showed there exists a threshold for number of bidders. When the number of bidders is greater than this threshold, separate auctions are more profitable. The first essay confirms this finding. Therefore we consider two levels of number of bidders in this study.
8. Out of 68 subjects, respectively 59, 55 and 55 subjects correctly answered all questions about the auctions for the bundle, the simultaneous auctions and the sequential auctions. Four subjects failed all three quizzes and four subjects failed two quizzes.
9. The only exception is the Type 5 bidder's bid in scenario 3 ($t=2.877$, $df=58$, $p=.005$).
10. \$19.32 vs. \$19.22, paired $t=0.034$, $df=54$, $p=.937$.
11. \$26.93 vs. \$7.75, paired $t=6.008$, $df=54$, $p=.000$.
12. \$24.87 vs. \$0.58, paired $t=6.808$, $df=54$, $p=.000$.

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