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Is the Growth Potential of Stock Prices Underestimated? A Real Option Approach

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ABSTRACT

Practitioners consider that the intrinsic value of equity is given by a discounted cash flow (DCF) valuation method. The Modigliani and Miller theory (1963) reveals that the cost of debt beyond the risk free rate has no impact on the weighted average cost of capital (WACC) and that the net debt which is included in the WACC computation is related to the amount which is in the firm's accounts. However, the net debt, which is deducted from the enterprise value to get the equity value, should be based on its economic value. The reference to the options literature (mainly Black & Scholes (1973), Merton (1973), Dixit and Pindyck (1994)) enables to propose a new breakdown of enterprise value between equity and net debt economic values. For healthy listed companies, the additional value embedded in the option model is not meaningful as evidenced by empirical tests. But, the main parameters of the model and one of its outputs, namely the recovery rate given default provides an explanation of the growth potential of the stock.

JEL Classifications: *G12, G13, G32*

Keywords: *growth; real option; DCF; WACC*

I. INTRODUCTION: LIMITS OF THE DISCOUNTED CASH FLOW (DCF) VALUATION METHOD

From a DCF point of view, the value of the firm (EV) corresponds to the present value of the future free cash flows (FCF), the discount rate used being the weighted average cost of capital or WACC (K):

$$EV = \sum_{t=1}^{+\infty} \frac{FCF_t}{(1+K)^t}$$

with

$$K = k\frac{E}{E+D} + i.(1-\tau)\frac{D}{E+D}$$

where E is the equity value and D is the net debt. The embedded WACC in the firm value calculation is based on the equity value which is looked for in the DCF approach. For that reason, practitioners include a loop in their DCF model.

Assuming a perpetuity growth rate *g* of FCF from year 1 onwards and a WACC equal to K:

$$EV = \frac{FCF.(1+g)}{K-g}$$

and

$$K = k\frac{E}{EV} + i.(1-\tau)\frac{D}{EV}$$

The reference to the Modigliani and Miller's adjusted cost of capital enables to get rid of the loop. Indeed, as

$$K = \rho.(1-\frac{D.\tau}{EV})$$

where ρ is the cost of capital of the unleveraged firm with the same sector risk. In other words, thanks to the Capital Asset Pricing Model, CAPM,

$$\rho = r + \beta^*.\left[E(R_M) - r\right]$$

where r is the risk free rate. Then

$$EV = \frac{FCF.(1+g)}{\rho.(1-\frac{D.\tau}{EV}) - g}$$

and

$$EV = \frac{FCF.(1+g)+D.\tau.\rho}{\rho-g}$$

$D.\tau$ is the tax shield which is justified by the tax deductibility of interests which are due on the assumed perpetual financial debt. Indeed, in the Modigliani and Miller's theory, $D.\tau$ results from the simplification of $\frac{i.D.\tau}{i}$ where $i.D.\tau$ is the tax saving on interests and i the corresponding capitalization rate. In that case, D is obviously the outstanding debt which can be found in the last available financial statements.

When practitioners deduct D from the EV to obtain the equity value, a closed form of *E* can be obtained:

$$E = \frac{FCF.(1+g)+D.\left[g-\rho.(1-\tau)\right]}{\rho-g}$$

The Modigliani and Miller's theory evidences that the spread on the risky debt has no impact on the WACC and therefore on the equity value: the cost of debt does not appear in the two previous formulas and any increase of the spread of the debt corresponds to an increase of the risk which is borne by the bondholders and banks. It is therefore consistent with a decrease of the risk which is borne by the shareholders. The Table below uses a simple example to show the risk transfer between stakeholders and the unchanged WACC:

Table 1

The unchanged WACC according to Modigliani and Miller's theory

FCF	100	100
Perpetuity growth = g	3.00%	3.00%
Risk free rate = r	2.00%	2.00%
Market risk premium	7.00%	7.00%
Unleveraged beta = β*	0.90	0.90
Cost of debt		
Pretax cost of debt	3.40%	5.50%
Post tax @ 36.1%	2.17%	3.51%
Beta of the debt	0.20	0.50
Leveraged beta = β	1.20	1.07
Cost of equity = k	10.38%	9.49%
WACC =K	7.11%	7.11%
Ρ	8.30%	8.30%
Adjusted cost of capital	7.11%	7.11%
EV	2,509	2,509
Debt	1,000	1,000
Equity	1,509	1,509

For a FCF which is equal to 100, a risk free rate of 2%, a market risk premium of 7% and an unleveraged beta of 0.9, 2 assumptions regarding the pretax cost of debt are taken into account: 3.40%, based on a debt's beta of 0.20 and 5.50% based on debt's beta of 0.50. The corresponding leveraged betas, based on the Hamada's formula, are respectively 1.20 and 1.07 and the implied costs of equity are respectively 10.38% and 9.49%. Then both WACC and adjusted costs of capital are 7.11%. Then the enterprise value is the same in both cases: 2509.

A. Discount Rates

The WACC calculation is a bit subjective as a lot of assumptions have to be taken into account: the market risk premium depends on a assumption regarding the perpetuity growth rate of the listed firms' dividends; when the firm is listed, the cost of equity can either include its beta (which is different according to the data provider) or a leveraged beta based on the industry's unleveraged beta, which depends on the peers which have been included in the sample; the weighting coefficients can correspond to a target – or normative - financial structure or be based on an iterative calculation. In that case, E is the outcome of the DCF valuation approach. In other terms, each valuator can justify a specific ad-hoc discount rate.

B. Net Debt

The equity value (E) is the difference between the EV and the net debt (D). The net debt is the difference between the financial debt on the one hand, cash and cash equivalents on the other hand. The maturity of the debt is not taken into account. Then if the EV is 100 and the financial debt is 60, assuming no cash and cash equivalents, the equity value will be 40, whether the debt matures tomorrow or in 2023. The reason is that practitioners generally take the book value of the debt (which corresponds to the face value under the assumption of no repayment premium) instead of taking the economic value of the debt into account whereas the financial theory relies on economic values of funds provided by the firm's stakeholders. If the debt matures tomorrow, its economic value is its face value but, if it matures in 2023, its economic value is the present value of the bondholders expected cash flows, the discount rate reflecting the bankruptcy risk of the firm. In other terms, the firm's bankruptcy risk is not embedded in the debt which is deducted from the EV and included in the weighting coefficients of the WACC.

C. Free Cash Flow Computation

The DCF approach is based on FCF which are implicitly looked upon as deterministic ones. They are discounted which enables to reduce the weight of the remote ones in the EV. As the cost of equity, which is embedded in the WACC, is based on the CAPM, the reference to the firm's leveraged beta enables to refer to its systematic risk. Moreover, a sensitivity analysis is generally provided by practitioners in order to underline the uncertainty regarding the achievement of the underlying business plans. However, the full risk, i.e., the total volatility (σ) of the FCF is not taken into account.

II. REVIEW OF LITERATURE

A. Black and Scholes

The financial literature dedicated to equity valuation based on option starts with Black and Scholes' seminal article (1973). This article presents a company which is financed by common stock and bonds and whose only asset is shares of common stock in another company. The bonds are zero-coupon and have a maturity of 10 years. The company plans to sell all the stocks it holds at the end of 10 years, pay off the bondholders if possible and pay any remaining money to the stockholders as a liquidating dividend.

Under these conditions, the stockholders have the equivalent of an option on the company's assets which has been provided to them by the bondholders. At the end of 10 years, the equity value, w(x,t), is the value of the assets, x, less the face value of the bonds or zero, whichever is greater. Then, the economic value of the bonds is x – w(x,t). If the company holds business assets instead of financial assets, and if, at the end of the 10-year period, it issues new common stock to pay off the bondholders (and pay any money that is left to the old stockholders to retire their stocks), the economic value of bonds remains x – w(x,t) where x is the enterprise value. Black & Scholes underline that an increase in the company's debt, keeping the enterprise value constant, increases the probability of default and thus reduces the market value of the bonds. It hurts the existing bond holders and helps the existing stockholders. Then the bond price falls and the stock price increases. In this sense, changes in the capital structure of a firm may affect the price of a common stock, when these changes become certain, not when the actual changes take place.

B. Merton

Merton (1973) also considers the equity value as a call premium on the company's assets in the background of the pricing of corporate liabilities. For that purpose, the dynamics for the enterprise value, through time, is described by a diffusion-type stochastic process with the following stochastic differential equation:

$$dV = (\alpha V - C).dt + \sigma V.dz$$

where α is the instantaneous expected rate of return on the firm per unit time, C is the total payouts by the firm per unit time to either shareholders or liabilities-holders (eg.: dividends or interest payments) if positive and the cash received by the firm from new financing if negative, σ^2 is the instantaneous variance of the return on the firm per unit time, dz is a standard Wiener process. Moreover, F is the economic value of debt and D is the par value of the debt, i.e., the amount the firm has promised to pay to the bondholders on a specified calendar date.

In the event the payment of D is not met, the bondholders take over the company and the shareholders receive nothing. If there are no coupons, the PDE applied to D is:

$$\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 F}{\partial V^2} + r.V.\frac{\partial F}{\partial V} - r.F + \frac{\partial F}{\partial t} = 0$$

Let F(V, τ) be the economic value of debt when the length of time until maturity is τ and then F(V,0) = min(V,D). Let f(V, τ) be the economic value of equity when the length of time until maturity is τ and then f(V,0) = max(0;V-D) and: f(V, τ)= V. Φ (d_1) – De^{-rt}. Φ (d_2). As F = V – f:

$$\begin{aligned} F &= V - [V.\ \Phi(d_1) - De^{-rt}.\ \Phi(d_2)] \\ &= V.\ [1 - \Phi(d_1)] + De^{-rt}.\ \Phi(d_2) \\ &= V.\ \Phi(-d_1) + De^{-rt}.\ \Phi(d_2). \end{aligned}$$

$$F = D.e^{-r\tau}\left[\Phi(d_2) + \frac{V}{D.e^{-r\tau}}.\Phi(-d_1)\right]$$

Let

$$d = \frac{D.e^{-r\tau}}{V}$$

or

$$\frac{V}{D.e^{-r\tau}} = \frac{1}{d}$$

Then

$$F = D.e^{-r\tau}\left[\Phi(d_2) + \frac{1}{d}.\Phi(-d_1)\right]$$

This formula enables to express the spread on the risky debt. In that context, let R be the yield to maturity.

Then:

$$F = D.e^{-R\tau}$$

or

$$\frac{F}{D} = e^{-R\tau}$$

and

$$R = -\frac{1}{\tau}.\ln\frac{F}{D}\ .$$

Therefore,

$$R = -\frac{1}{\tau}\ln.\left\{e^{-r\tau}\left[\Phi(d_2) + \frac{1}{d}.\Phi(-d_1)\right]\right\}$$

$$R = -\frac{1}{\tau}\ln e^{-r\tau} - \frac{1}{\tau}\ln\left[\Phi(d_2) + \frac{1}{d}.\Phi(-d_1)\right].$$

$$R = r - \frac{1}{\tau}\ln\left[\Phi\left(d_2\right) + \frac{1}{d}.\Phi\left(-d_1\right)\right].$$

Finally,

$$R - r = \text{spread} = -\frac{1}{\tau}\ln\left[\Phi\left(d_2\right) + \frac{1}{d}.\Phi\left(-d_1\right)\right].$$

Merton (1973)'s pricing of corporate debt does not include any enhancement of the enterprise value by the tax shield which is generated by the tax deductibility of the financial expenses on debt. Such a principle was pioneered by Modigliani and Miller (1963) who established that the enterprise value of the leveraged firm is equal to that of the unleveraged one increased by a tax shield. In that context, the maximisation of the enterprise value can result from the maximisation of the level of corporate debt. But, as reminded by Brennan and Schwartz (1978) such a conclusion leads to the inconsistency between the premise that management has to maximise the wealth of shareholders and the empirical observation that most firms do not maximise their indebtness. This discrepancy is justified by Modigliani and Miller themselves who remind that retained earnings is a cheaper source of financing than debt and insist on the need for preserving flexibility. Another explanation regarding the limitation of the firm's leverage can be found in the bankruptcy costs which weigh on the enterprise value, as highlighted by Kraus and Litzenberger (1973).

C. Brennan and Schwartz

Brennan and Schwartz (1978) focus on the optimal capital structure taking corporate tax and bankruptcy costs into account. They assume that the enterprise value of the unleveraged firm, U, follows a GBM:

$$\frac{dU}{U} = \mu.dt + \sigma.dz$$

where dz is a standard Wiener process. The enterprise value of the leveraged firm, *V*, is a function of the enterprise value of the unleveraged firm (both firms having the same assets) on the one hand, of the time *t* until maturity of the debt on the other hand. In other terms, V=V(U,t). Then, the PDE is:

$$\frac{1}{2}\sigma^2 U^2 \frac{\partial^2 V}{\partial U^2} + r.U.\frac{\partial V}{\partial U} - r.U + \frac{\partial V}{\partial t} = 0\,.$$

On the maturity T of the debt:

$$V(U,T) = U \text{ for } U \geq D$$

$$V(U,T) = U - C(U) \text{ for } U < D$$

where C(U) corresponds to the bankruptcy costs if the firms files for bankruptcy. Moreover, assuming that t^- and t^+ denote respectively the instants before and after the dividend payments, d: $V(U,t^-) = V(U\text{-}d,t^+) + d$.

If the coupon payment, *iD*, is included and if τ is the corporate tax rate:

$$V(U,t^-) = V(U,t^+) - i.(1-\tau).D + i.D$$

where $i.(1-\tau).D$ corresponds to the required capital increase to restore the enterprise value of the leveraged firm after the coupon payment. The development of the brackets and the simplification by i.D provides:

$$V(U,t^-) = V(U,t^+) + i.D.\ \tau\ .$$

If the dividend and the coupon are paid on the same day:

$$V(U,t^-) = V(U\text{-}d,t^+) + d + i.D.\ \tau\ .$$

Finally, taking bankruptcy costs, C(*U*), into account:

$$V(U,t^-) = V(U\text{-}d,t^+) + d + i.D.\ \tau\ .\ \text{for}\ U \geq D$$

$$V(U,t^-) = U\text{-}\ C(U)\ \text{for}\ U < D$$

These two last formulas correspond to boundary conditions to solve the abovementioned PDE. But such an equation does not have a closed form solution. This is why Brennan & Schwartz utilize numerical techniques to determine an optimal leverage.

D. Leland

Leland (1994) solves the Brennan and Schwartz (1978)'s PDE assuming that the firm has run into debt to perpetuity. Such an assumed time independent environment is fully justified when the debt is rolled over by a new one. F being the economic value of the claim, as in Merton (1973)'s seminal paper, V the enterprise value and C the coupon paid, per instant of time when the firm is solvent:

$$\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 F}{\partial V^2} + r.V.\frac{\partial F}{\partial V} - r.F + \frac{\partial V}{\partial t} + C = 0$$

Then, if the claim has no time dependency:

$$\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 F}{\partial V^2} + r.V.\frac{\partial F}{\partial V} - r.F + C = 0$$

As the derivation only relates to V, the PDE can be written more simply:

$$\frac{1}{2}\sigma^2 V^2 F''(V) + r.V.F'(V) - r.F(V) + C = 0$$

The solving of such a PDE requires taking firstly the homogeneous equation corresponding equation (ie without C) into account:

$$\frac{1}{2}\sigma^2 V^2 F''(V) + r.V.F'(V) - r.F(V) = 0$$

This reminds Dixit and Pindyck (1994)'s PDE - which is:

$$\frac{1}{2}.\sigma^2 V^2.F''(V) + (r-\delta).V.F'(V) \text{ - r.F(V) = 0] -for } \delta = 0.$$

In that case, the solutions of the characteristic equations are β_1 and β_2 is that:

$$\beta_1 = \frac{-\left(r - \frac{1}{2}\sigma^2\right) + \sqrt{(r - \frac{1}{2}.\sigma^2)^2 + 2.r.\sigma^2}}{\sigma^2} = \frac{-\left(r - \frac{1}{2}\sigma^2\right) + \sqrt{r^2 - r\sigma^2 + \frac{\sigma^4}{4} + 2.r.\sigma^2}}{\sigma^2}$$

Then

$$\beta_1 = \frac{-\left(r - \frac{1}{2}\sigma^2\right) + \sqrt{(r + \frac{\sigma^2}{2})^2}}{\sigma^2} = \frac{-\left(r - \frac{1}{2}\sigma^2\right) + r + \frac{\sigma^2}{2}}{\sigma^2} = 1$$

$$\beta_2 = \frac{-\left(r - \frac{1}{2}\sigma^2\right) - \sqrt{(r - \frac{1}{2}.\sigma^2)^2 + 2.r.\sigma^2}}{\sigma^2} = \frac{-\left(r - \frac{1}{2}\sigma^2\right) - \sqrt{r^2 - r\sigma^2 + \frac{\sigma^4}{4} + 2.r.\sigma^2}}{\sigma^2}$$

Then

$$\beta_2 = \frac{-2r}{\sigma^2}$$

The solution of the homogeneous equation is therefore:

$$F(V) = A_1.V^1 + A_2.V^{-X}$$

where $X = \frac{-2r}{\sigma^2}$.

Taking C into account, the general solution of the PDE is:

$$F(V) = A_0 + A_1.V^1 + A_2.V^{-X}$$

where the constants of which (ie, A_0, A_1 and A_2) are determined by boundary conditions.

Let α be the fraction of value which will be lost to bankruptcy costs, leaving debtholders with value (1- α).V_B and stockholders with nothing, V_B being the level of enterprise value at which bankruptcy is declared. Then, the value of debt, D(V) is such that:

$$\text{If } V=V_B\text{, then } D(V) = (1-\alpha).V_B \qquad (1)$$

$$\text{If } V \to +\infty\text{, then } D(V) = \frac{C}{r} \qquad (2)$$

But, if $V \to +\infty$, V^{-X}=0; then the condition (1) requires that $A_0 = \frac{C}{r}$ and $A_1 = 0$.
Moreover taking the condition (2) into account:

$$\frac{C}{r} + A_2.V_B^{-X} = (1-\alpha).V_B$$

Then

$$A_2 = \frac{(1-\alpha).V_B - \frac{C}{r}}{V_B^{-X}}$$

and

$$D(V) = \frac{C}{r} + \left[(1-\alpha).V_B - \frac{C}{r}\right].\left[\frac{V}{V_B}\right]^{-X}$$

Regarding the bankruptcy costs, BC:

$$\text{If } V=V_B\text{, then } BC(V) = \alpha.V_B \qquad (3)$$

$$\text{If } V \to +\infty\text{, then } BC(V) = 0 \qquad (4)$$

As, if $V \to +\infty$, V^{-X}=0; then the condition (4) requires that $A_0 = 0$ and $A_1 = 0$.
Moreover taking the condition (3) into account:

$$A_2.V_B^{-X} = \alpha.V_B\,.$$

Then

$$A_2. = \alpha.\frac{V_B}{V_B^{-X}}$$

and

$$BC(V) = \alpha.V_B.\left(\frac{V}{V_B}\right)^{-X}$$

Regarding the tax benefits, TB:

$$\text{If } V=V_B\text{, then } TB(V) = 0 \tag{5}$$

$$\text{If } V \to +\infty\text{, then } TB(V) = \frac{\tau C}{r} \tag{6}$$

As, if $V \to +\infty$, V^{-X}=0; then the condition (6) requires that $A_0 = \frac{\tau.C}{r}$ and $A_1 = 0$. Moreover taking the condition (5) into account:

$$\frac{\tau.C}{r} + A_2.V_B^{-X} = 0\,.$$

Then

$$A_2 = -\frac{\frac{\tau.C}{r}}{V_B^{-X}}$$

and

$$TB(V) = \frac{\tau.C}{r} - \left[\frac{\tau.C}{r}\right].\left[\frac{V}{V_B}\right]^{-X}$$

Finally, the enterprise value EV, taking bankruptcy costs and tax benefits into account, is:

$$EV = V + TB(V) - BC(V) = V + \frac{\tau.C}{r}\left[1 - \left(\frac{V}{V_B}\right)^{-X}\right] - \alpha.V_B.\left(\frac{V}{V_B}\right)^{-X}$$

And the equity value,

$$E(V) = EV - D(V)$$

Then

$$E(V) = V + \frac{\tau.C}{r}\left[1-\left(\frac{V}{V_B}\right)^{-X}\right] - \alpha.V_B.\left(\frac{V}{V_B}\right)^{-X} - \frac{C}{r} - \left[(1-\alpha).V_B - \frac{C}{r}\right].\left[\frac{V}{V_B}\right]^{-X}$$

Finally,

$$E(V) = V - (1-\tau).\frac{C}{r} + \left[(1-\tau).\frac{C}{r} - V_B\right].\left[\frac{V}{V_B}\right]^{-X}$$

Furthermore, the expression of EV evidences that the asset value is maximized by setting V_B as low as possible, assuming it is not imposed by a covenant. The value of V_B which enables to maximize the equity value, E(V), is such that:

$$\frac{dE(V)}{dV} = 0 \text{ for } V = V_B.$$

$$\frac{dE(V)}{dV} = 1 - X.\left[(1-\tau).\frac{C}{r} - V_B\right].\left[\frac{V}{V_B}\right]^{-X-1}.\frac{1}{V_B} = 0$$

For $V = V_B$:

$$1 - \frac{2r}{\sigma^2}.\left[(1-\tau).\frac{C}{r} - V_B\right].\frac{1}{V_B} = 0$$

$$1 - \frac{2r}{\sigma^2}.\left[(1-\tau).\frac{C}{r} - V_B\right].\frac{1}{V_B} = 0 \Leftrightarrow \frac{2r}{\sigma^2}.\left[(1-\tau).\frac{C}{r.V_B} - 1\right] = 1 \Leftrightarrow (1-\tau).\frac{C}{r.V_B} = 1 + \frac{\sigma^2}{2.r}$$

and

$$V_B = \frac{(1-\tau).C}{r + \frac{\sigma^2}{2}}$$

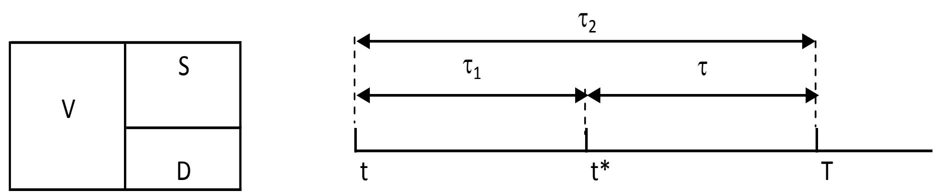
Therefore, V_B is independent from V and α. Moreover, if r, σ or τ increases, V_B decreases; if C increases, V_B increases too.

E. Geske

Geske (1977) proposed a valuation formula of corporate liabilities which includes n-1 individual coupon payments due before the principal plus interest must be repaid. Such a valuation is based on the generalisation of a compound options pricing model.

Geske (1979) used a compound options pricing model in which the stock can be viewed as an option on the value of the firm. In this setting, a call on the common stock is an option on an option. Let V be the value of the firm (or its enterprise value), S the value of stock, D the face value of debt and K the strike price of the call on equity. Let t^* be the expiration date of the call on equity and T the maturity date of the debt. The following graphs illustrate Greske's principles which drive to the compound option's premium

Table 2
Stock as an option on the value of the firm



At the intermediate date t^*, the call holder exercises his option on the stock if the call is in the money, ie if $S_{t^*}>K$. Otherwise, if $S_{t^*}=K$ (or if $S_{t^*}<K$), the call on the stock remains unexercised. As the value of the stock, S, depends on the value of the firm's assets, V, such a situation happens when the enterprise value V is equal to (or lower than) V^*. Then, V^* is the value of V such that $S_\tau - K = 0$. In other terms, the call holder pays K on $t=t^*$ if, at this date, $V>V^*$ in order to keep the possibility to pay M on t=T in order to get the firm's assets. In that case:

$$C = V.N(a_1, b_1, \rho) - D.e^{-r\tau_2}.N(a_2, b_2, \rho) - K.e^{-r\tau_1}.\Phi(a_2)$$

where

$$a_1 = \frac{\ln(\frac{V}{V^*}) + (r + \frac{\sigma_V^2}{2}).\tau_1}{\sigma_V.\sqrt{\tau_1}}, \quad a_2 = a_1 - \sigma_V.\sqrt{\tau_1}, \quad \rho = \sqrt{\frac{\tau_1}{\tau_2}}$$

$$b_1 = \frac{\ln(\frac{V}{D}) + (r + \frac{\sigma_V^2}{2}).\tau_2}{\sigma_V.\sqrt{\tau_2}}$$

and

$$b_2 = b_1 - \sigma_V.\sqrt{\tau_2}$$

where N(.) and Φ are respectively the bivariate and univariate cumulative normal distribution functions.

F. Charitou and Trigeorgis

Charitou and Trigeorgis (2004) transpose Geske's compound option pricing model when a coupon interest I comes due at an intermediate date t^*, while the debt's maturity is T. If, on $t=t^*$, V is lower than V^* such that $E(V^*, \tau_1)-I=0$, the stockholders voluntarily default on the interest payment, I. In that case, based on Geske's notations, K is replaced by I and C is replaced by E which corresponds to the equity value. Indeed, the stockholders have the option to pay I, at the intermediate date t^*, to keep the possibility to pay M on t=T in order to get the firm's assets. In that case:

$$E = V.N(a_1, b_1, \rho) - D.e^{-r\tau_2}.N(a_2, b_2, \rho) - I.e^{-r\tau_1}.\Phi(a_2)$$

and the probability of default on the intermediate date T is $P[E<I] = P[V<V^*] = \Phi(-a_2)$ where a_1, a_2, b_1, b_2 and ρ have the values which have been defined by Geske (1979).

III. EMPIRICAL STUDY

A. Database

A sample of 40 firms belonging to the French CAC 40 has been set up. For each firm, 4 data as at 22/02/2013 have been extracted from the Facset financial data base: the market capitalization, the brokers' consensus on EV, the brokers' consensus on the target price and the volatility of the shares which corresponds to the standard deviation of the return over one year. For most brokers, the enterprise value is the output of a DCF valuation. Each daily volatility has then been multiplied by $\sqrt{252}$ in order to turn it into a yearly one. The French risk free rate, paid on 10-year T-Bonds, is 2.20% which corresponds to 2.18% in continuous time. Moreover, the financial last available financial debt has been found in each firm's balance sheet as at 31/12/2011.

The financial institutions (Axa, Crédit Agricole, BNP Paribas and Société Générale) can be valued by brokers taking a cash flow approach into account. But in that case, the cash flow corresponds to the excess equity which could be paid out to the stockholders taking solvency constraints into account. Then the cash flow is a theoretical dividend (hence the "dividend discount model" which is given to such a valuation approach) and the sum of the present value of the forecasted free cash flows enables to get directly the equity value, the discount rate being the cost of equity. For insurance companies, like Axa, the target Solvency 1 ratio, which is consistent with the company's targeted rating, is taken into account. For banks, the target common equity tier 1 ratio, which is required by the Basel 3 regulation, is taken into account. Anyway, for insurance companies as for banks, no enterprise value is embedded in the valuation.

When the firm has a negative net debt, the DCF approach changes from a broker to another: some calculate the present value of free cash flows based on the cost of equity, others calculate algebraically the WACC with a negative net debt. In order to get rid of such possible discrepancies from a methodology point of view, the 5 firms with a negative net debt as at 31/12/2011 - Cap Gemini, EADS, L'Oreal, STMicroelectronics, Technip – have been excluded. Moreover, Renault, whose weight of the consumer credit business in the accounts is significant, has also been excluded from the sample. At the end of the day, the empirical study is based on 30 firms.

Table 3
Firms characteristics

Name	Market Cap	Target equity value	EV Consensus	Equity's volatility
				σ_E
Accor S.A.	6,414	6,599	7,074	29%
Air Liquide S.A.	28,942	31,507	34,666	20%
Alstom S.A.	10,170	10,846	12,777	36%
ArcelorMittal SA	19,185	23,427	33,957	39%
Bouygues S.A.	6,517	7,362	11,541	32%
Carrefour S.A.	14,894	15,209	20,306	33%
Compagnie St-Gobain	16,720	18,169	24,221	34%
Danone S.A.	33,932	32,517	39,300	21%
Electricite de France S.A.	27,030	29,788	67,684	28%
Essilor International S.A.	16,261	16,381	16,356	20%
France Telecom	19,922	26,652	51,448	27%
GDF Suez S.A.	35,444	41,789	79,613	25%
Gemalto N.V.	6,001	6,197	5,567	28%
Lafarge S.A.	14,114	14,891	24,324	33%
LeGrand S.A.	9,229	8,398	10,157	23%
LVMH	67,241	77,707	70,663	26%
Michelin	12,124	14,427	13,998	31%
Pernod Ricard S.A.	25,969	25,025	33,425	19%
PPR S.A.	21,635	20,855	22,942	25%
Publicis Groupe S.A.	10,601	10,151	10,372	19%
SAFRAN S.A.	14,260	15,000	15,092	24%
Sanofi S.A.	96,611	103,893	103,138	22%
Schneider Electric S.A.	32,727	31,138	35,684	34%
Solvay S.A.	9,292	9,454	12,156	33%
Total S.A.	90,047	105,145	108,524	21%
Unibail-Rodamco SE	16,464	16,812	28,495	20%
Vallourec S.A.	5,291	5,000	6,653	41%
Veolia Environnement S.A.	4,774	5,472	17,728	39%
Vinci S.A.	20,640	25,363	33,963	28%
Vivendi	20,449	24,258	34,146	30%

From a Black & Scholes-Merton's perspective, the brokers' consensus on enterprise value corresponds to the price of the underlying asset and the strike price to the amount of the debt in the accounts. Beyond that, the application of the option pricing models requires 2 other parameters: the time to expiration and the underlying asset's volatility. In the Black & Scholes and Merton's seminal papers, the debt is a zero coupon. Then, the option's time to expiration corresponds to the residual maturity of the bond. For most firms, the debt is made of bonds with coupons and financial borrowings from banks. From a theoretical point of view, a compound option with several maturities should be taken into account. However, in order to apply the Black-Scholes-Merton's pricing model, an average residual maturity of each company's debt has been calculated as a proxy of the time to expiration τ.

The underlying asset's volatility corresponds, from a corporate finance point of view, to the enterprise value's volatility. But the assets are rarely listed, except for holding companies which are not represented in the CAC 40 index. If they are not listed, their value (ie spot EV) and their volatility have to be estimated.

The EV and its volatility is based on the methodology which is proposed by Hull, Nelken and White (2004) and commonly used by Moody's rating agency. Based on Ito's lemma:

$$dF(x,t) = [\frac{\partial F}{\partial t} + a(x,t)\frac{\partial F}{\partial x} + \frac{1}{2}b^2(x,t)\frac{\partial^2 F}{\partial x^2}].dt + b(x,t).\frac{\partial F}{\partial x}.dz$$

Then with F = E (for equity), x=V (for enterprise value), a(x,t) = m.V and b(x,t) = $\sigma_V V$

$$dE(V,t) = [\frac{\partial E}{\partial t} + m.V\frac{\partial E}{\partial V} + \frac{1}{2}\sigma_V^2 V^2 \frac{\partial^2 E}{\partial V^2}].dt + \sigma_V V.\frac{\partial E}{\partial V}.dz$$

In that, case:

$$\sigma_V V.\frac{\partial E}{\partial V}.dz = \sigma_E E..dz$$

and

$$\sigma_V V.\frac{\partial E}{\partial V} = \sigma_E E$$

Finally,

$$\sigma_E E = \sigma_V V.\Phi(d_1)$$

Moreover, thanks to the Merton's formula:

$$E = V.\ \Phi(d_1) - De^{-rt}.\ \Phi(d_2).$$

The values of V and σ_V can be obtained thanks to Excel's solver applied to the following nonlinear system:

$$V.\ \Phi\left[\frac{\ln\left(\frac{V}{D}\right)+\left(r+\frac{\sigma_V^2}{2}\right).\tau}{\sigma_V\sqrt{\tau}}\right] - De^{-rt}.\quad \Phi\left[\frac{\ln\left(\frac{V}{D}\right)+\left(r-\frac{\sigma_V^2}{2}\right).\tau}{\sigma_V\sqrt{\tau}}\right] = E$$

$$\frac{\sigma_V V.\Phi\left[\frac{\ln\left(\frac{V}{D}\right)+\left(r+\frac{\sigma_V^2}{2}\right).\tau}{\sigma_V\sqrt{\tau}}\right]}{E} = \sigma_E$$

Only the obtained value of σ_V is taken into account, as the enterprise value is based on the DCF approach.

The following table provides the detailed calculation of Merton's economic values of debt and equity based on the previous example assumptions regarding the EV (2509) and risk free rate (2%). The assets' volatility is supposed to be 30% and the average maturity of the financial debt is 5 years.

Table 4
Merton approach based on DCF

Check EV (no loop on K)	2,509
D	1,000
σ of assets	30%
T	5.00
R	2.00%
d1	1.86
d2	1.18
F(d1)	0.97
F(d2)	0.88
F(-d1)	0.03
Economic value of debt	878
Equity	1,631

The table below evidences that the DCF's equity value (1509) is obtained only if the time to expiration, i.e., the average residual maturity of the financial debt is nil. Otherwise, the longer the maturity is, the higher the time premium and therefore the equity value is. Moreover, the higher the volatility is, the more important the likelihood of an increase in the share price is which implies a higher equity value. Such calculations have been made for the 30 firms belonging to the sample[12].

Table 5
Sensitivity of equity value

t	Volatility of assets					
1631	0%	10%	20%	30%	40%	50%
0	1,509	1,509	1,509	1,509	1,509	1,509
5	1,604	1,604	1,606	1,631	1,684	1,754
10	1,690	1,690	1,703	1,764	1,856	1,958
15	1,768	1,768	1,793	1,876	1,986	2,098
20	1,838	1,839	1,872	1,968	2,087	2,199
25	1,902	1,903	1,942	2,046	2,165	2,272

B. Empirical Models

The empirical study is focused on the growth potential of the stock price of healthy listed firms which belong to the French CAC 40. Such a growth potential can be based on brokers' target prices which can be compared to the listed prices of stocks. In that case, the target price is the enterprise value, which corresponds to the present value of future free cash flows, as determined by brokers, reduced by the net debt that can be found in the accounts. But such a net debt, which is based on its face value without taking its maturity into account and therefore the probability of bankruptcy, may be overestimated. The growth potential of the stock price may increase, should the equity value be based on Black & Scholes-Merton in order to include the bankruptcy risk which depends on the debt's face value but also on its maturity and the assets' volatility. The Black & Scholes-Merton approach provides a new breakdown of the DCF enterprise value (V) between equity and net debt economic values. In that case: Equity value = Brokers' EV – [V. Φ ($-d_1$) + De^{-rt}. Φ (d_2)-cash and equivalents] where:

$$d_1 = \frac{\ln(\frac{V}{D}) + (r + \frac{\sigma_V^2}{2}).\tau}{\sigma_V.\sqrt{\tau}}$$

and

$$d_2 = d_1 - \sigma_V.\sqrt{\tau}$$

The comparison between both growth potentials may be explained by the corresponding leverage ratios. For that reason, the net debt to EV is calculated based on the net debt which is in the accounts on the one hand, on the economic value of the net debt which is given by the Black and Scholes-Merton's model on the other hand. These ratios are respectively noted D/EV and B/EV[1]. An alternative to Merton's debt economic value, *B*, is the following breakdown:

$$B = V.\ \Phi\ (-d_1) + De^{-rt}.\ \Phi\ (d_2) - De^{-rt} + De^{-rt} = De^{-rt} + V.\ \Phi\ (-d_1) + De^{-rt}.[\ \Phi\ (d_2) - 1]$$

$$B = De^{-rt} + V.\ \Phi\ (-d_1) - De^{-rt}.\ \Phi\ (-d_2)$$

and

$$B = D.e^{-r\tau} - \varphi(-d_2).\left[D.e^{-r\tau} - \frac{\varphi(-d_1)}{\varphi(-d_2)}.V\right]$$

where $\frac{\varphi(-d_1)}{\varphi(-d_2)}.EV$ is the amount of debt which will be recovered by the bondholders should the firm file for bankruptcy. Then $\frac{\varphi(-d_1)}{\varphi(-d_2)}$ is the recovery rate and $D.e^{-r\tau} - \frac{\varphi(-d_1)}{\varphi(-d_2)}.V$ is the expected discounted loss which will be borne by the bondholders given the assumed default of the firm. As $\varphi(-d_2)$ is the probability of bankruptcy, $\varphi(-d_2).\left[D.e^{-r\tau} - \frac{\varphi(-d_1)}{\varphi(-d_2)}.V\right]$ is the expected discounted shortfall. Finally, as used by Moody's KMV and the risk departments of banks in the background of risk weighted assets calculations:

Value of debt = par value of debt – probability of default x expected discounted LGD, where LGD means "Loss Given Default". Gemalto and Legrand, which have a probability of default of 0%, are excluded of the sample for this part of the analysis that is therefore limited to 28 firms.

The 3 main parameters of the economic value of the net debt seem to be its maturity (τ), the recovery rate given default ($\frac{\varphi(-d_1)}{\varphi(-d_2)}$), which includes the probability of default and the weight of its face value which be expressed as a percentage of the enterprise value (D/EV). In that context, a multiple regression is tested in order to explain the growth potential based on the Black & Scholes Merton's equity value[12].

C. Empirical Results

1. Equality test of assets' and equities' volatilities

The means of the stocks and assets volatilities are respectively 28% and 22%. The significance of the 6% discrepancy can be tested using the data provided in the following tables. The table is dedicated to the equality test of variances.

If the variances are equal, the ratio of the standard variances obeys a Fisher-Snedecor's distribution:

$$\frac{S_x^2}{S_Y^2} \to F(n_p - 1; n_Q - 1)$$

where n_P=30 and n_Q=30. Hence:

$$T = \frac{S_x^2}{S_Y^2} \to F(29; 29)$$

Table 6
Equality test of variances (F-test)

	σ_E	σ_V
Mean	28.0%	22.2%
Variance	0.4%	0.3%
Observations	30	30
Degrees of freedom	29	29
F	1.41	
P(F<=f) unilateral	0.18	
Critical value for F (unilateral)	1.86	

The Fischer-Snedecor's table provides: P[T>1.86] = 5%. In other words, if the variances are equal, T has a 5% probability to be higher than 1.86. By experimentation, $t_0^* = 1.41 < 1.86$. Hence, with a 5% error risk, the variances of the volatilities of the stocks on the one hand, of the assets on the other hand, are equal. Then a Student's test enables to know whether the stocks' and assets' volatilities are significantly different. The table below is dedicated to such a test:

Table 7
Equality test of means: 2 observations with equal variances

	σ_E	σ_V
Mean	28.0%	22.2%
Variance	0.4%	0.3%
Observations	30	30
Weighted variance	0.4%	
Hypothetical means difference	0	
Degrees of freedom	58	
Stat t	3.76	
P(T<=t) bilateral	0.00	
Critical value for F (bilateral)	2.00	

If the means are equal, the following ratio obeys a Student's distribution:

$$T=\frac{\bar{X}-\bar{Y}}{\sqrt{\frac{(n_P-1)S_X^2+(n_Q-1)S_Y^2}{n_P+n_Q-2}}\cdot\sqrt{\frac{1}{n_P}+\frac{1}{n_Q}}}$$

where n_P=30 and n_Q=30. Hence, $T \rightarrow S(58)$. The Student's table provides: P[-2.00<T<2.00]=95%. In other words, if the means are equal, T has a 95% probability to be in a [-2.00; 2.00] range. By experimentation, $t_0^* = 3.76 > 2.00$. Then, with a 5% error risk, the means of the volatilities of the stocks on the one hand, of the assets on the other hand, are different. Such a conclusion justifies the determination of the assets' volatilities to use the Black & Scholes-Merton's approach of equity valuation.

2. Equality test of stock prices' potential growth based on brokers' and Black and Scholes-Merton's approach

The means of the growth potential based on brokers' target prices and Black & Scholes-Merton's approach of equity valuation are respectively 7.5% and 13.7%. The significance of the 6.2% discrepancy can be tested thanks to data provided in the following tables. The table bellows is dedicated to the equality test of variances.

Table 8
Equality test of variances (F-test)

	g brokers	g vs BS' E
Mean	7.5%	13.7%
Variance	1.1%	5.1%
Observations	30	30
Degrees of freedom	29	29
F	0.20	
P(F<=f) unilateral	0.00	
Critical value for F (unilateral)	1.86	

As in the former equality test of variances, if the variances of growth potentials are equal, T= $\frac{S_x^2}{S_Y^2}$ has a 5% probability to be higher than 1.86.

By experimentation, $t_0^* = 0.20 < 1.86$. Hence, with a 5% error risk, the variances of the potential growth based on brokers on the one hand, on the Black and Scholes-Merton's approach on the other hand, are equal. Then a Student's test enables to know whether the average growth potentials are significantly different. The table below is dedicated to such a test.

Table 9
Equality test of means: 2 observations with equal variances

	g brokers	g vs BS' E
Mean	7.5%	13.7%
Variance	1.1%	5.1%
Observations	30	30
Weighted variance	3.1%	
Hypothetical means difference	0	
Degrees of freedom	58	
Stat t	-1.37	
Critical value for F (bilateral)	2.00	

If the means are equal, the following ratio obeys a Student's distribution: T= $\rightarrow S(58)$ as in the former equality test of means. The Student's table provides: P[-2.00 < T < 2.00]=95%. By experimentation, t_0^* = -1.37. Then t_0^* is obviously in the [-2.00; 2.00] range. Hence, with a 5% error risk, the means of the potential growth based on brokers on the one hand, on the Black & Scholes-Merton's approach on the other hand, are equal. The difference between the brokers' and the Black & Scholes-Merton's approaches corresponds to the net debt's amount which is deducted from the DCF's enterprise value. The reason of such a result is that the firms belonging to the CAC 40 index are healthy ones. Then, their probability of default is very low, narrowing 0%. In that case, $\Phi(-d_2) = 0$ which means that $\Phi(d_2) = 1$ which happens when $d_2=+\infty$.Then $d1=+\infty$ too which implies that $\Phi(d_1) = 1$. Based on the Black & Scholes-Merton formula that: $E=S - D.e^{-r\tau}$. As τ is relatively low, E is not far from S – D.

The explanation of the equality of growth potentials can be completed by a statistical test of equality of leverage ratios which correspond to net debt / enterprise value.

3. Equality test of leverage ratios based on the net debts in the firms' accounts and on recalculated net debts including Black and Scholes-Merton's approach

The means of the leverage ratios based on brokers' target prices and Black & Scholes-Merton's approach of equity valuation are respectively 25% and 18%. The significance of the 7% discrepancy can be tested thanks to data provided in the following tables. The table bellows is dedicated to the equality test of variances.

As in the former equality tests of variances, if variances of the leverage ratios are equal, T= $\frac{S_X^2}{S_Y^2}$ has a 5% probability to be higher than 1.86. By experimentation, t_0^* = 2.08 > 1.86. Hence, with a 5% error risk, the variances of the potential growth based on brokers on the one hand, on the Black & Scholes-Merton approach on the other hand, are different. Then an Aspin Welch's test enables to know whether the average leverage ratios are significantly different. Table 11 is dedicated to such a test.

Table 10

Equality test of variances (F-test)

	D/EV	B/EV
Mean	25.0%	18.5%
Variance	3.9%	1.9%
Observations	30	30
Degrees of freedom	29	29
F	2.08	0
P(F<=f) unilateral	0.03	0
Critical value for F (unilateral)	1.86	

Table 11
Equality test of means: 2 observations with different variances

	D/EV	B/EV
Mean	25.0%	18.5%
Variance	3.9%	1.9%
Observations	30	30
Hypothetical means difference	0	
Degrees of freedom	52	
Stat t	1.47	
Critical value for F (bilateral)	2.01	

If the means are equal, the following ratio obeys a Student's distribution:

$$T= \frac{\bar{X}-\bar{Y}}{\sqrt{\frac{S_X^2}{n_P}+\frac{S_Y^2}{n_Q}}} \to S(52)$$

The Student's table provides: P[-2.01<T<2.01]=95%. By experimentation, $t_0^* = 1.47$. Then t_0^* is obviously in the [-2.01 ; 2.01] range. Hence, with a 5% error risk, the means of the leverage ratios based on brokers on the one hand, on the Black & Scholes-Merton approach on the other hand, are equal.

4. Multiple regression to explain the growth potential of the stock price

Let g be the listed stock's growth potential, RRGD the recovery rate given default, D/EV the net debt in accounts to enterprise value and τ the maturity of the financial debt. The second table below indicates that: g = 2.08.RRGD – 0.85.D/EV + 0.05. τ – 0.47. The coefficient of determination, R^2, is around 0.8 which is high but such a regression is justified only if the 4 coefficients are significantly different from 0.

The first table below enables to test whether the 4 coefficients are simultaneously equal to 0. Under the assumption RRGD = D/EV = τ = 0, the F stat obeys a Fisher-Snedecor distribution F(k;n-k-1) with k=3 and n=28. Then $F \to F(3;24)$. The Fisher-Snedecor's table provides: P[F>3.01] = 5%. In other words, if the 4 coefficients are simultaneously equal to 0, F has a 5% probability to be higher than 3.01. By experimentation, t^*= 34.62 > 3.01. Hence, with a 5% error risk, the 4 coefficients are not simultaneously equal to 0.

Table 12
Variance analysis

	Degrees of freedom	Sum of squared	Mean of squared	F	F critical value
Régression	3	1.14	0.38	34.62	0.00
Résidual figures	24	0.26	0.01		
Total	27	1.41			

The second table enables to test whether each of the 4 coefficients is equal to 0. For each coefficient *a*, if a=0 then the T stat obeys a Student distribution with n-k-1 degrees of freedom. Here, k=3 and n=28. Then $T \rightarrow S(24)$. The Fischer-Snedecor's table enables to get P[-2.06<T<2.06] = 95%. In other words, if a coefficient is equal to 0, T has a 95% probability to be in a [-2.06 ; 2.06] range. By experimentation: $t^*(c) = -5.76 < -2.06$ where c is a constant, $t^*(RRGD) = 4.71 > 2.06$, $t^*(D/EV) = -2.44 < -2.06$, $t^*(t) = 3.51>2.06$. Hence, with a 5% error risk, none of the 4 coefficients is equal to 0.

Table 13
T-test of the four coefficients

	Coefficients	Error type	t stat = t*	Probability	Lower limit for 95% confident threshold	Lower limit for 95% confident threshold
Constant	-0.47	0.08	-5.76	0.00	-0.64	-0.30
RRGD	2.08	0.44	4.71	0.00	1.17	2.99
D/EV	-0.85	0.35	-2.44	0.02	-1.58	-0.13
τ	0.05	0.01	3.51	0.00	0.02	0.07

IV. CONCLUSION

The Discounted Cash Flow Valuation Method, DCF approach seems to undervalue the stock prices as the net debt, which is deducted from the firm value, can be found in the accounts of the firms, whereas it should be an economic value. For the CAC 40 non-financial firms, the growth potential based on a DCF including the economic value of the net debt is in average not meaningfully different. The economic value of the net debt is based on the Black & Scholes-Merton's model, which enables to take the probability of default, the maturity of the debt and the assets' volatility into account. But, in the case of CAC 40 companies, the probability of default is very low and the debt's average maturity is relatively limited. In that case, the Black-Scholes-Merton's additional value is not significant. This result is confirmed by the comparison of the leverage ratios which are not meaningfully different when based on the net debt in the accounts and on the economic value of the net debt. However, the growth potential is explained by the main parameters and an output of the Black-Scholes-Merton's model, namely the face value of debt, its maturity and the recovery rate given default. Finally, the growth potential which can be explained is not underestimated. Our analysis provides a basis for future research and can be used in other financial markets.

ENDNOTE

1. For detailed information, please contact the authors.

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