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Efficiency Drifts in Euronext Stock Indexes Returns

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ABSTRACT

This paper intends to assess and test long-term memory in the Euronext stock indexes returns in the search for fractal dynamics that refute the random walk hypothesis. The Hurst exponents estimated through Rescaled-Range and Detrended Fluctuation Analysis evidence long memory in the form of persistence for all markets, with the exception of CAC 40 by the DFA. However, the Rescaled-Range Tests neither reject the absence of long dependency nor reject the existence of short dependency. On the contrary, the Fractional Differencing Test supports the presence of persistence in the PSI 20, ISE 20 and OBX indexes. This suggests that these markets are more prone to predictability, but also trends that may be unexpectedly disrupted by discontinuities, exhibiting dynamics incompatible with random walk behavior and providing evidence against the weak form of efficiency and validity of the asset pricing models.

JEL Classifications: G14, G15, C10

Keywords: long-term memory, rescaled-range analysis, detrended fluctuation analysis, fractional differencing analysis, efficient market hypothesis

I. INTRODUCTION

The prices of financial assets are usually described as a geometric Brownian motion (gBm), which is an assumption compatible with the efficient market hypothesis (EMH), wherein the stock return follow an unrelated Gaussian process through unpredictable behavior (Costa and Vasconcelos, 2003).

Although EMH is a fundamental benchmark of modern finance, efficiency drifts have been observed in several markets. The main reason has particular interest because it derives from time dependence in some stock returns series (Horta, Lagoa and Martins, 2014). This property was identified by Mandelbrot (1971) and called as "long memory or low frequency persistent temporal dependence".

The presence of long-term memory in asset prices has controversial implications on the measurement of efficiency and rationality (Maghyereh, 2007). More specifically, fractal dynamics refutes the random walk hypothesis with i.i.d. increments, which is the basis of the EMH in its weak form. Consequently, many paradigms used in the modern financial theory will be broken, namely the validity of the martingale methods of derivatives and the adequacy of the asset pricing models. This problem justifies our research.

Researchers continue to seek for a better understanding of the dynamic nature of financial time series, with most of the evidence suggesting weak base of any form of long memory (Lo, 1991; Jacobsen, 1996; Lipka and Los, 2002; Kristoufek, 2012; Braun, Jenkinson and Stoff, 2017) and another part of the evidence suggesting clear fractal structure (Fama and French, 1988; Costa and Vasconcelos, 2003; Assaf and Cavalcante, 2004; Chen and Yu, 2005; Ferreira, 2018). However, some results presented mixed findings (Sadique and Silvapulle, 2001; Christodoulou-Volos and Siokis, 2006; Eitelman and Vitanza, 2008; Núñez, Martínez and Villareal, 2017; Gomes *et al.*, 2018).

The lack of consensus on long-term memory in stock returns and its implications for EMH are the motivations for this paper. The objective of this work is to estimate and test the degree of persistence in the daily Euronext stock indexes (CAC 40, AEX, BEL 20, PSI 20, ISE 20, and OBX) returns to verify the EMH, and modelling the dynamic behaviour of the time series. The expansion and visibility of this platform justify the choice.

The econophysics approach applies theories and methods developed in statistical physics, in order to contribute for the resolution of problems in economics and finance. The widely accepted techniques to identifying long memory are those used to estimate the Hurst exponent H. Four of the most popular techniques are the rescaled-range analysis – under the classical (R/S) and modified (M-R/S) statistics –, the detrended fluctuation analysis (DFA), and the Geweke and Porter-Hudak method (GPH). The R/S analysis was first introduced by Hurst (1951), later improved by Mandelbrot and Wallis (1969a,b) and Mandelbrot and Taqqu (1979) to detect the presence of long-term memory in time series. The M-R/S analysis, proposed by Lo (1991), modifies the previous statistics to make it insensitive to short-term memory, heteroscedasticity and non-normality. The DFA analysis, proposed by Peng et al. (1994), models the series to obtain the exponent H via a single appropriate parameter, being robust to non-stationary time series. The GPH was developed by Geweke and Poter-Hudak (1993) as a semi-parametric procedure potentially more efficient than M-R/S to estimate the memory parameter in a fractionally integrated process. The GPH test has the interesting property of being robust to short-term dependence and heteroscedasticity. Furthermore, the M-R/S test and the GPH test are robust to non-normality.

The empirical study follows four research hypotheses:

H1: The time series of Euronext stock indexes returns are well described by fractional Brownian motion (fBm).
H2: The time series of Euronext stock indexes returns exhibit long-term memory.
H3: The long-term memory exhibited by Euronext stock indexes returns stems from the short-term dependency.
H4: The time series of Euronext stock indexes returns refute the EMH.

The structure of this paper is organized as follows. Section 2 provides a brief review of literature related to long-term memory in the forms of persistence and anti-persistence. Section 3 presents the data series and describes the methods employed to estimate and test long-term memory. Section 4 discusses the results of the empirical analysis. Finally, section 5 summarizes the main findings.

II. LONG-TERM MEMORY

According to Fama (1970), the random walk hypothesis with i.i.d. increments is the basis of the EMH. In a simple way, this hypothesis establishes that (1) the price variation is random, as a result of the activity of traders trying to make gains, and (2) the implementation of their strategies induces a dynamic feedback on the market randomizing the stock price (Matos et al., 2004).

The statistical analysis of financial time series has exhibited different characteristics from the random walk (Castro and Rachinger, 2020), wherein stock prices exhibit unpredictable behavior given available information (Lo, 2004; Assaf, 2006; Da Silva et al., 2007). The presence of long memory components in stock prices has controversial implications for market efficiency and is inconsistent with continuous stochastic processes employed in the martingale methods of stock valuation (Lo, 1991; Sadique and Silvapulle, 2001; Eitelman and Vitanza, 2008). A series with long-term memory is characterized by long-term dependence and by non-periodic long cycles (Mandelbrot, 1977; Cheung and Lai, 1995; Alomari et al., 2020), meaning that the market will get back to its long-term trend in the future. Mandelbrot (1971) was one of the first to recognize the possibility and implications of long-term persistent statistical dependence in the financial time series. The proper identification of the nature of persistence is crucial to decide on the type of modeling diffusion of these series. A succession of persistent or anti-persistent stock returns is characterized by an effect of long-term memory.

The most recent empirical evidence incites a renewed interest in fBm and in fractionally integrated processes (Lento, 2013; Kim, Kim and Min, 2014; Schennach, 2018), and market efficiency is one of the most frequent topics under discussion. Cajueiro, Gogas and Tabak (2009) and Wang, Liu and Gu (2009) used the Hurst exponent to examine the efficiency of the Greek and the Shenzhen stock indexes in the context of market reforms. They found that the loss of market efficiency was due to the market pressure on investors, which led to herding behavior. Onali and Goddard (2011) used the Hurst exponent to study the efficiency of 8 stock markets indexes. They confirmed the existence of strong long-range dependence on the Czech index, and weak on the Spanish and Swiss indexes, concluding that the results are generally consistent with prior expectations concerning the relative efficiency of the stock markets. Eom, et al. (2008) investigated the relationship between the Hurst exponent and the predictability of 60 different markets. They found that markets indexes with higher exponent H tend to higher levels of predictability. Horta, Lagoa and Martins (2014) analyzed how the financial crises affected the Hurst exponents in stock index returns of Belgium, France, Greece, Japan, Netherlands, Portugal, UK and US. Considering the whole sample period, they concluded that markets tend to show signs of inefficiency, with the exception of the UK and US markets that do not exhibit long memory. Núñez, Martínez and Villareal (2017) tested whether long memory in 20 stock indices depends on the model, the period or the frequency of the data. The results of the rescaled-range analysis showed long-term memory properties in all indexes. Gomes et al. (2018) analyzed 4 Euronext stock indices (CAC 40, AEX, BEL 20 and PSI 20) in search of long memory using the Hurst exponent estimated by the M-R/S analysis and the DFA. The estimated global H exponents suggest persistent long memory in the Dutch, Belgian and Portuguese markets.

There are three classifications of market prices dissemination, measured by the Hurst exponent H (the degrees of long-term dependence). For $H = 0.5$, the process corresponds to the gBm with independent innovations (Beran, 1994), following a random walk that characterizes efficient markets in the strict sense of Fama (1970). For $H \neq 0.5$, the process corresponds to fBm, wherein the increments have long-term correlation. If $0.5 < H < 1$, the increments of fBm are positively correlated and the process exhibits persistence (Embrechts and Maejima, 2002), i.e., the deviations tend to maintain the signal. If $0 < H < 0.5$, the increments are negatively correlated and the fBm exhibits anti-persistence or apparent unpredictability (Embrechts and Maejima, 2002), i.e., the deviations of a signal are usually followed by counter-signal deviations.

In a persistent market, if a change in price was up/down in the last period, then the prospect is that it will continue to be upward/downward in the following period. In this case, markets have long periods of stability interrupted by sudden and sharp discontinuities (Los and Yu, 2008). It corresponds to riskier markets and to invest in that persistence allows opportunities for abnormal gains by arbitrage. In an anti-persistent market, if a price change was upward in the last period, then the prospect is that it will be downward in the following period, and vice-versa. In this case, markets have a fast reversion to the mean and are called ultra-efficient (Kyaw, Los and Zong, 2006).

III. RESEARCH APPROACH AND METHODS

A. Sample and Data Series

Euronext was founded on September 2000 through the merger of the stock exchanges of Paris, Amsterdam and Brussels, benefiting from the harmonization of financial markets in the European Union. The group expanded its presence in Europe in 2002 with the entry of the Lisbon and Porto stock exchange, in 2018 joined the Irish stock exchange and in 2019 joined the Oslo stock exchange. The Euronext stock market reached a capitalization of EUR 4,409,881 million in 2020, making it one of the largest global markets:

Table 1
Characteristics of the Euronext stock indexes (CAC 40, AEX, BEL 20, PSI 20)

Country	Designation	Index	Date Basis	Listed Companies	Turnover (Eur millions) (Ac. Dec 2020)
France	Euronext Paris	CAC 40		40	1 276 484
Netherlands	Euronext Amsterdam	AEX	1987/12/31	25	707 149
Belgium	Euronext Brussels	BEL 20	1983/01/03	20	110 577
Portugal	Euronext Lisbon	PSI 20	1991/01/01	20	29 664
Ireland	Euronext Dublin	ISE 20	1992/12/31	20	47 149
Norway	Euronext Oslo	OBX		25	137 209

Note: The number of companies listed on the BEL 20 is equal to 20 since June 2011.
Source: Euronext, available on website: https://live.euronext.com/en/resources/statistics/nextmonth-cash

The six indexes are comparable across countries, as they were built on a consistent value-weighted basis. The start date of the study coincides with the opening of the latest index to allow time comparison of results across markets.

The original data are the series of daily closing stock indexes and cover a period longer than 22 years, since January 2, 1998 to September 18, 2020, covering several relevant events. The data used in the empirical study is the simple transformation of the stock index through the first log difference of their levels $D[log\ P_t]$. In practical terms, the returns compounded continuously X_t at time t are calculated from the consecutive daily prices P_t index:

$$X_t = \log\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

B. Methodology

In order to pursue the objectives of the empirical study, we will examine the time series of stock indexes returns to identify:

- whether exhibits long-term memory;
- whether the long-term memory is persistent or anti-persistent;
- whether the long-term memory stems from short-term dependency;
- whether empirical models of price diffusion suggest inefficiency of the markets, and thus call into question the adequacy of pricing models.

1. Rescaled-Range Analysis

(1) Classical Rescaled-Range Statistics

The range over standard deviation or rescaled-range (R/S) statistics is one approach to identify long-term dependence. The classical R/S is given by *"range of partial sums of deviations of a time series from its mean, rescaled (divided) by its standard deviation"* (Lo, 1991, p. 1287):

$$(R/S)_n = S_n^{-1}\left[\underset{1\le k\le n}{\text{Max}}\sum_{j=1}^{k}(X_j-\bar{X}_n) - \underset{1\le k\le n}{\text{Min}}\sum_{j=1}^{k}(X_j-\bar{X}_n)\right] \quad (2)$$

where X_j is the stock return in period j, $\bar{X}_n$ is the sample mean and S_n is standard deviation. The first term given in brackets is the maximum of the partial sums of the first k deviations of X_j from the sample mean and the second term given in brackets is the minimum of the same sequence. The difference between the two terms is called range (R_n) and always non-negative, as indicated by Lo (1991).

Test Procedure:

Under the null hypothesis $(V_n = 0)$ that "the series of returns is i.i.d." the R/S statistics converges asymptotically to the range of a Brownian bridge in the unit interval V. The cumulative distribution function of the range of a Brownian bridge is explicitly given in Kennedy (1976) by $F_V(v) = 1 + 2\sum_{k=1}^{\infty}(1-4k^2v^2)e^{-2(kv)^2}$, which represents the asymptotic distribution function of the normalized (i.e., divided by the square root of the sample size n) rescaled-range statistics (Lo, 1991):

$$V_n \Leftarrow \frac{1}{\sqrt{n}} \times R/S \;\sim\; V \quad (3)$$

The moments of the range V_n are calculated from the distribution function F_V, where the mean and theoretical error variance are equal to $E(V_n) = \sqrt{\pi/2}$ and $E(V_n^2) = \pi^2/6$, respectively. The test for the null hypothesis of non-existence of long-term dependence can be performed (in the absence of short-term dependency) by estimating the confidence interval for a level of significance and finding whether V_n is within or outside the desired limits, where the asymptotic *p*-values are given in Lo (1991).

(2) Modified Rescaled-Range Statistics

In order to solve the problem of sensitivity to short-term dependency, the denominator becomes the square root of a consistent estimator of the variance of partial sums until the lag q in expression (2) as follows (Lo, 1991):

$$(M\text{–}R/S)_{n,q} = S_q^{-1}\left[\underset{1\le k\le n}{\text{Max}}\sum_{j=1}^{k}(X_j-\bar{X}_n) - \underset{1\le k\le n}{\text{Min}}\sum_{j=1}^{k}(X_j-\bar{X}_n)\right] \quad (4)$$

where $S_q^2 = S_n^2 + 2\sum_{j=1}^{q}\omega_j(q)\hat{\gamma}_j$ is a heteroscedasticity and autocorrelation consistent variance estimator (Andrews, 1991), which includes the usual sample variance S_n^2 and autocovariance $\hat{\gamma}_j = (1/n)\sum_{i=1}^{n-j}(X_i-\bar{X}_n)(X_{i+j}-\bar{X}_n)$ estimators of X. The weighting function was suggested by Newey and West (1987) and given by $\omega_j(q) = 1 - j/(q+1)$, $q < n$ with the truncation lag suggested by Andrews (1991) and given by $q = \text{Int}\left[(3n/2)^{1/3} \times \left(2\hat{\rho}_1/(1-\hat{\rho}_1{}^2)\right)^{1/3}\right]$, being $\hat{\rho}_1 = \hat{\gamma}_1/\hat{\gamma}_0$ the first-order autocorrelation. The M-R/S statistics requires selection of the lag order, in relation to which exhibits high sensitivity.

Test Procedure:

The modified R/S statistics is invariant to short-term dependence, but is sensitive to long-term dependence. Under the null hypothesis $(V_n(q) = 0)$ of "short-term dependence with heteroscedasticity" (that is, absence of long-term memory), the normalized rescaled-range statistics $\left(V_n(q)\right)$ with lag q has the limit distribution (Lo, 1991):

$$V_n(q) \Leftarrow \frac{1}{\sqrt{n}} \times M\text{–}R/S \ \sim \ V \tag{5}$$

The V_n and $V_n(q)$ statistics can be used to distinguish and analyse three hypotheses: random walk, short-term memory, and long-term memory (Chow, Pan and Sakano, 1996). If both statistics are significant, the process has long-term dependence; if the statistics V_n is significant and the statistics $V_n(q)$ is insignificant, the data series exhibits short-term memory; if both statistics are insignificant, the process is independent or random walk.

(3) Rescaled-Range Analysis and Hurst Exponent

The scale where the H exponent assumes persistence or anti-persistence values reflects a trend to reinforce deviations from the mean and is characteristic of the models known as fractional Gaussian noise (Mandelbrot and Wallis, 1969b) and as fractionally integrated ARMA (Granger, 1980). In these processes, long-term dependence is shown by the slow decay of the autocorrelation function (ACF) based on the asymptotic scaling relationship (Lux, 1996):

$$(R/S)_t = \ at^H \tag{6}$$

where a is a finite positive constant independent of t and H is the Hurst exponent. The power scaling law (Weron, 2002) and the exponent H can be estimated by employing a simple linear least-squares regression (Lux, 1996) on the logarithms of both sides of the expression (6) in a sample of increasing time horizons $(s = t_1\ , t_2\ , \dots , t_n)$.
The rescaled-range analysis (sometimes called R/S analysis) involves calculating the mean of the rescaled-range for several values up to n for a given value of s. The slope of representing $\log(R/S)$ as a function of $\log(s)$ for different values of s, calculated through ordinary least-squares, provides an estimate of the Hurst exponent H (Mandelbrot and Wallis, 1969a).

2. Detrended Fluctuation Analysis

The basis of the DFA method, proposed by Peng et al. (1994), is to subtract the possible deterministic trends from the original time series and then analyse the fluctuation of detrended data.

Firstly, after subtracting the mean, one integrates the original time series $\{X_j\}$ to obtain the cumulative time series $Y(t)$ as follows (Oh, Kim and Um, 2006):

$$Y(t) = \sum_{j=1}^{t} \left(X_j - \bar{X}\right) \qquad ; \qquad t = 1, \dots , n \tag{7}$$

This accumulation process transforms the original data into a self-similar process, where $\bar{X} = \frac{1}{n}\sum_{j=1}^{n} X_j$ represents the mean.

Secondly, the series $Y(t)$, of length n, is divided by an integer equal to n/τ non-overlapping boxes, each containing τ points. Then, the local quadratic trend $z(t) = at^2 + bt + c$ in each box is defined as the standard least-squares fit of the data points. Subtracting $z(t)$ to $Y(t)$ in each box the trend is removed. This process is applied to all the boxes, and the detrended fluctuation function F is defined by the square root of the mean deviation of $Y(t)$ from the trend function $z(t)$ (Kristoufek, 2010):

$$F_k^2(\tau) = \frac{1}{\tau}\sum_{t=k\tau+1}^{(k+1)\tau} |Y(t) - z(t)|^2 \qquad ; \qquad k = 0, \dots, \frac{n}{\tau} - 1 \tag{8}$$

The calculation of the average of $F_k^2(\tau)$ over the n/τ intervals provides the definition of the fluctuation function $F(\tau)$ defined by (Matos et al., 2008):

$$F(\tau) = \sqrt{\frac{\tau}{n}\sum_{k=0}^{n/\tau-1} F_k^2(t)} \tag{9}$$

Thirdly, if the observable $X(t)$ are uncorrelated random variables, the expected behaviour should be a power-law, and the previous fluctuation function has the following scaling relation (Peng et al., 1994):

$$\langle F(\tau)\rangle \sim (const)\tau^H \tag{10}$$

Returning to run a linear least-squares regression over the relationship represented by log-log scale in the expression (10) arises a straight line, whose slope is the Hurst exponent H. Thus, from a linear (in log-log scale) regression of data corresponding to $F(\tau)$ the empirical value for exponent H can be estimated to define the degree of polynomial trend (Costa and Vasconcelos, 2003), as occurred for the R/S analysis, by $\log\langle F(\tau)\rangle = \log(const) + H\log\tau$.

3. Fractional Differencing Analysis

The fractional differential processes, developed by Granger and Joyeux (1980), may be used to model parametrically long memory dynamics. Under this approach, whether a series has long memory depends on a fractional differencing parameter. A general class of fractional processes ARFIMA(p, d, q), which are generalizations of standard ARMA models, is described by:

$$\Phi(L)(1 - L)^d X_t = \Theta(L)\varepsilon_t \tag{11}$$

where $\{x_1, \dots, x_T\}$ is a set of time series data, $\Phi(L) = 1 - \phi_1 L - \dots - \phi_p L^p$ and $\Theta(L) = 1 + \vartheta_1 L + \dots + \vartheta_q L^q$ are the AR and MA polynomials, respectively, in the lag operator L with all roots being stable, ε_t is a white noise disturbance term, and

$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)L^k}{\Gamma(-d)\Gamma(k+1)}$ is the fractional differencing operator, where $\Gamma(\cdot)$ is the standard Gamma function.

The fractional differencing parameter (or degree of fractional integration) d assumes any real values.

(1) Geweke and Porter-Hudak Method

The spectral regression method, developed by Geweke and Porter-Hudak (1983), suggests a semi-parametric procedure to estimate the memory parameter in a fractionally integrated process. The statistical procedure involves the estimation of d in expression (11), through the slope of the spectral density function around the angular frequency $\lambda_j = 0$. This process uses a simple linear regression of the log-periodogram at low Fourier harmonic frequencies $\lambda_{j\,T} = 2\pi j/T$:

$$ln\left[I\left(\lambda_{j\,T}\right)\right] = c - \hat{d}\ log\left[4\ sen^2\left(\lambda_{j\,T}/2\right)\right] + v_j \quad ; \quad j = 1, 2, \cdots, m < T \qquad (12)$$

where the disturbance v_j is asymptotically normal with variance $\pi^2/6$ under normality of the innovation ϵ_t in expression (11), m is the number of low frequency ordinates and $\propto$ is the root of the sample size, wherein $m = T^{\alpha}$ with $0 < \alpha < 1$. The test for the null hypothesis $(d = 0)$ of "short term dependence" (i.e., absence of long-term memory) can be based on the usual t-statistics.

The authors GPH show that the spectral density function of a fractional Gaussian noise with Hurst exponent is identical to that of an ARFIMA model with differencing parameter $d = H - 0.5$.

IV. DISCUSSION AND RESULTS

A. Estimation of Long-term Memory

1. Classical and Modified Rescaled-Range Analysis

In an experimentation for dependency, under fBm approach, table 2 presents the estimates of Hurst exponents, via R/S and M-R/S analysis, for Euronext stock indexes and the coefficients of determination (R^2).

Table 2

Hurst exponents H via R/S and M-R/S analysis and coefficients R^2 for daily returns of the Euronext indexes

Procedure: linear regression of $\log(R/S)_s$ and $\log(M\text{-}R/S)_s$ over log(s)

Estimates		**CAC 40**	**AEX**	**BEL 20**	**PSI 20**	**ISE 20**	**OBX**
Hurst exponent (via R/S)	H	0.548	0.552	0.570	0.585	0.555	0.565
Coefficient of determination	R^2	0.998	0.998	0.998	0.997	0.993	0.999
Hurst exponent (via M-R/S)	H	0.536	0.538	0.549	0.560	0.539	0.546
Coefficient of determination	R^2	0.997	0.999	0.998	0.998	0.993	0.999

Note: The complete series for log-returns has a length of 5925 observations, but given the need of entire divisibility in R/S procedure, we considered only the first 5920 closing prices. Specifically, the decimation for the $(R/S)_t$ ratio established the lags $s = 8, 16, 32, 40, 80, 160, 296, 592, 1184, 1480$. The $\log(R/S)_s$ and $\log(M\text{-}R/S)_s$ were calculated as the mean of a fixed number of non-overlapping intervals.

For the whole period of analysis the Hurst exponent is slightly above the benchmark $H = 0.5$, in both (R/S and M-R/S) techniques, indicating the existence of long memory in the form of persistence. The results suggest that the PSI 20 index is further and the CAC 40 index is closer to a gBm with independent innovations, with higher estimates in the R/S analysis. The excellent fit of the regressions $(R/S)_s$ and $(M\text{-}R/S)_s$ is given by R^2 close to unity.

2. Detrended Fluctuation Analysis

In another experimentation for dependence, table 3 presents the estimate of Hurst exponent via DFA for Euronext stock indexes and the coefficient of determination (R^2):

Table 3
Hurst exponent H via DFA analysis and coefficient R^2 for daily returns series of the Euronext indexes
Procedure: linear regression of log $F(\tau)$ over log(τ)

Estimates		CAC 40	AEX	BEL 20	PSI 20	ISE 20	OBX
Hurst exponent (via DFA)	H	0.488	0.504	0.511	0.545	0.526	0.529
Coefficient of determination	R^2	0.996	0.998	0.997	0.999	0.997	0.998

Note: The log $F(\tau)$ was calculated as the average of a fixed number of sliding overlapping intervals, wherein the minimum lag τ is equal to 20 days (about one month of trading).

The result of the DFA technique, for the whole period of analysis, contradicts the evidence from the R/S and M-R/S analysis in the CAC 40 index. The estimated exponent is slightly lower than 0.5, indicating the existence of long memory in the form of anti-persistence. The degree of long-term dependence indicates that the Dutch market is close to the independence of innovations in the gBm, where $H = 0.5$. Once again, the high coefficient R^2 shows the excellent fit of the regression $F(\tau)$.

B. Testing Long-term Memory

1. Classical and Modified Rescaled-Range Test

The estimates of $V_n(q)$ were calculated for the truncation parameter $q = 2, 4, 8, 16, 32$ days, in order to adjust to the possible presence of short-term autocorrelation and test the robustness of the results. Table 4 presents the statistics of R/S test and M-R/S test for Euronext stock indexes, the autocovariance component of M-R/S statistics and the influence of R/S statistics on the presence of short-term memory.

The R/S test shows that V_n statistics exceeds the mean critical value equal to 1.253 for a process without long memory in the AEX and ISE 20 indexes, indicating long-term positive dependence (i.e., persistence with $H > 0.5$). The results for these markets converge with the estimates of Hurst exponents obtained through R/S (classic and modified) analysis and DFA. The V_n statistics for BEL 20, PSI 20 (marginally) and OBX indexes suggests long-term negative dependence (i.e., anti-persistence with $H < 0.5$), diverging strongly from the estimates obtained through R/S analysis and DFA. Regarding the CAC index, the statistics V_n admits long anti-persistence convergent with the DFA estimate and divergent from the R/S estimates.

Table 4
Statistics of R/S test and M-R/S test for daily returns of the Euronext indexes, the autocovariance component of M-R/S statistics and the influence of R/S statistics on the presence of short-term memory

		$V_n(q)$									
Index	V_n	**2**	*%infl*	**4**	*%infl*	**8**	*%infl*	**16**	*%infl*	**32**	*%infl*
CAC 40	1.090	1.108	*-1.6*	1.134	*-3.9*	1.177	*-7.4*	1.205	*-9.5*	1.211	*-10.0*
Autocov. ($x10^7$)		[-0.882]		[-2.065]		[-3.806]		[-4.841]		[-5.089]	
AEX	1.261	1.259	*0.2*	1.274	*-1.0*	1.305	*-3.4*	1.293	*-2.5*	1.304	*-3.3*
Autocov. ($x10^7$)		[0.084]		[-0.533]		[-1.690]		[-1.227]		[-1.642]	
BEL 20	1.205	1.154	*4.4*	1.162	*3.7*	1.184	*1.8*	1.185	*1.7*	1.198	*0.6*
Autocov. ($x10^7$)		[1.841]		[1.542]		[0.753]		[0.704]		[0.246]	
PSI 20	1.235	1.159	*6.6*	1.140	*8.3*	1.133	*9.0*	1.118	*10.5*	1.083	*14.0*
Autocov. ($x10^7$)		[2.542]		[3.280]		[3.556]		[4.152]		[5.668]	
ISE 20	1.611	1.561	*3.2*	1.562	*3.1*	1.597	*0.9*	1.602	*0.6*	1.546	*4.2*
Autocov. ($x10^7$)		[1.713]		[1.707]		[0.471]		[0.296]		[2.270]	
OBX	1.158	1.173	*-1.3*	1.179	*-1.8*	1.194	*-3.0*	1.186	*-2.4*	1.158	*0.0*
Autocov. ($x10^7$)		[-0.722]		[-1.003]		[-1.676]		[-1.316]		[0.013]	

Note 1: The null hypothesis of an i.i.d. process, that is, non-existence of long-term memory (in the absence of short-term memory) is rejected if the V_n statistic is not contained in the confidence intervals (at 90%, 95% and 99%) defined by critical regions $[0.861\,,1.747]$, $[0.809\,,1.862]$ and $[0.721\,,2.098]$, respectively (the *p*-values were defined in Lo, 1991, Table 2, p. 1288). The moments of the range V_n are determined from their distribution function, with the mean $E(V_n) = \sqrt{\pi/2} = 1.253$ and the theoretical error variance $E(V_n^2) = \pi^2/6$.
Note 2: The null hypothesis of a short-term memory process, that is, absence of long-term memory is rejected if the $V_n(q)$ statistic is not contained in the confidence intervals (at 90%, 95% and 99%) defined by the same critical regions.
Note 3: (*), (**) and (***) indicate statistical significance (in the bilateral test) for the null hypothesis at the level of 10%, 5% and 1%, respectively.
Note 4: The %infl. is calculated using the formula $[(V_n/V_n(q)) - 1] \times 100$ and indicates the influence of classical rescaled-range statistics on the presence of short-term memory.

However, none of the V_n results is statistically significant and, therefore, one cannot reject the null hypothesis of non-existence of long-term memory (in the absence of short-term memory) in the Euronext stock markets. Furthermore, the $V_n(q)$ statistics does not reject the hypothesis of short-term memory (that is, absence of long-term memory) for any cut of lag, corroborating that conclusion.

The $V_n(q)$ statistics in the BEL 20, PSI 20 and ISE 20 indexes are lower than the V_n statistics for all q lags, suggesting that short-term dependence has influenced the value of V_n upward.

2. Geweke and Porter-Hudak Method

The GPH spectral regression procedure to estimate the parameter d and test the null hypothesis of short-term memory was subjected to different values of the root of sample size $\alpha = 0.45\,, 0.50\,, 0.55\,, 0.60, 0.65$, in order to verify the robustness of the results. Table 5 presents the estimates of fractional differencing parameter via GPH method for Euronext stock indexes, the standard errors and the t-Student statistics:

Table 5
Fractional differencing parameter d via GPH method for daily returns of the Euronext indexes, standard error deviations and t-Student statistics

		d				
Index		$m = T^{0.45}$	$m = T^{0.50}$	$m = T^{0.55}$	$m = T^{0.60}$	$m = T^{0.65}$
CAC 40		0.001	-0.072	0.018	0.045	0.010
Standard Error	*s.e.*	[0.086]	[0.060]	[0.063]	[0.051]	[0.039]
t-statistic	*t-sample*	|0.012|	|-1.200|	|0.286|	|0.882|	|0.256|
AEX		0.057	-0.032	0.016	0.047	0.050
Standard Error	*s.e.*	[0.073]	[0.058]	[0.063]	[0.047]	[0.036]
t-statistic	*t-sample*	|0.781|	|-0.552|	|0.254|	|1.000|	|1.389|
BEL 20		0.131	-0.044	0.021	-0.035	-0.009
Standard Error	*s.e.*	[0.114]	[0.081]	[0.065]	[0.049]	[0.038]
t-statistic	*t-sample*	|1.149|	|-0.543|	|0.323|	|-0.714|	|-0.237|
PSI 20		-0.035	-0.008	0.022	0.090 *	0.047
Standard Error	*s.e.*	[0.087]	[0.068]	[0.056]	[0.051]	[0.039]
t-statistic	*t-sample*	|-0.402|	|-0.118|	|0.393|	|1.765|	|1.205|
ISE 20		0.243 */**	0.109	0.144 */**	0.091 *	0.033
Standard Error	*s.e.*	[0.095]	[0.082]	[0.071]	[0.051]	[0.041]
t-statistic	*t-sample*	|2.558|	|1.329|	|2.028|	|1.784|	|0.805|
OBX		0.065	0.026	0.081	0.111 */**	0.061
Standard Error	*s.e.*	[0.088]	[0.065]	[0.056]	[0.053]	[0.040]
t-statistic	*t-sample*	|0.739|	|0.400|	|1.446|	|2.094|	|1.525|

Note: 1: The null hypothesis of a short-term memory process, that is, absence of long-term memory, is rejected if the statistic $|\text{t sample}_{\hat{d}}| > \text{t critical}$, where $\text{t sample}|_{\hat{d}} = \frac{\hat{d}-0}{\text{s.e.}_{\hat{d}}}$ and $\text{t critical} = 1.645, 1.960, 2.575$ at the significance level of 10%, 5% and 1%, respectively, for a t-Student distribution with ∞ degrees of freedom. The t statistics for the estimates of parameter d are determined from the theoretical error variance $(\pi^2/6)$.
Note 2: (*), (**) and (***) indicate statistical significance (in the bilateral test) for the null hypothesis at the level of 10%, 5% and 1%, respectively.

The estimates obtained by GPH method for ISE 20 and OBX indexes are different from zero in the stationary region $(0 < d < 0.5 \text{ or } 0.5 < H < 1)$ for all low frequency ordinates, indicating long-term memory property in the form of persistence $(d > 0 \text{ or } H > 0.5)$. Robust evidence of long-term positive dependence is found more widely in the ISE 20 index ($\alpha = 0.45, 0.55, 0.60$), and less widely in the OBX and PSI 20 indexes ($\alpha = 0.60$).

In the CAC 40, AEX and PSI 20 indexes there are marginal signs and in the BEL 20 there are moderate signs of long-term property in the form of anti-persistence $(d < 0 \text{ or } H < 0.5)$, but without statistical significance in either case.

With the relation $d = H - 0.5$, the lowest bandwidth $[0.482\ , 0.545]$ is obtained for the variation of the scaled parameter H in the French index.

The results of the semi-parametric estimator d corroborate all evidence of long-term positive dependence shown in the previous analysis for the AEX and ISE 20 indexes. The identification of long-term persistence (significant) in the PSI 20 and OBX indexes converges with the R/S (classic and modified) analysis and with the DFA, but differs from V_n and $V_n(q)$ statistics (although insignificant). In the BEL 20 index the mainly anti-persistent signal converges with both V_n statistics and in the CAC 40 index the residual positive signal converges with the parameter H obtained by both R/S analyzes.

V. CONCLUSION

In the search for evidence on the long memory property in the Euronext stock indexes, the daily returns series were modeled using a fBm formulation to obtain the Hurst exponents H through classical R/S analysis, modified M-R/S analysis and DFA with different window sizes. The regressions over the total sample data estimated slightly higher exponents H in the first method, although this is not surprising, as it tends to overestimate the parameter in small time series (Kristoufek, 2010). The excellent adjustment of the regression $(R/S)_s$ and $F(\tau)$ is consistent with the Hyp. 1, insofar as the time series of Euronext stock indexes returns seems well described by fBm. However, those methodologies provided divergent empirical results in the French market, although they do not reject the Hyp. 2. The Hurst exponents were slightly different from the benchmark $H = 0.5$, suggesting long memory in the form of persistence for all markets, with the exception of the DFA in the CAC 40 index. This means that the AEX, BEL 20, ISE 20, OBX and, even more, the PSI 20 are slightly riskier to invest and trade, while the CAC 40 has fastest reversion to the mean, and is therefore the most efficient.

In addition, statistical tests of long-term memory processes were performed for different levels of significance using the R/S test, the M-R/S test and the fractional differencing test GPH. The second and third tests consider different lag cuts and different sample size root, respectively, in order to verify the robustness of the results. Despite the persistent trend given by Hurst exponent in most Euronext stock markets, the V_n statistics evidence absence of long memory and the $V_n(q)$ statistics does not reject the hypothesis of short-term memory for any cut of lag. This analysis suggests that there may be a stochastic process of short-term memory that degenerates over long periods, supporting the Hyp. 3. Furthermore, significant positive results of the semi-parametric estimator d support the presence of long memory in the form of persistence in the PSI 20, ISE 20 and OBX indexes, contradicting the irrelevance of both V_n and $V_n(q)$ statistics. This means that the signals for Euronext stock indexes exhibit dynamics incompatible with random walk behavior and therefore refute the EMH, supporting the Hyp. 4. The shocks have a persistent impact on returns, providing evidence against the weak form of efficiency, since they imply non-linear dependence at the moments of distribution and, consequently, a potentially predictable component, but also trends that may be unexpectedly disrupted by discontinuities.

Considering that no results from the V_n statistics (in the R/S methodology) are statistically relevant, contrary to the results from the d statistics (in the GPH methodology), and that the second methodology has a potential gain in estimation efficiency (Cheung and Lai, 1995) it also seems more reliable in its evidences.

The conclusions are important for regulators and risk managers. An important issue for them is to know which stock indexes are persistent, and therefore inefficient, that can produce abnormal returns. Moreover, the research of persistence is important because (1) establishes the long-term benchmark models for pricing financial assets and derivatives, (2) provides measures for investment selection and risk management, and

(3) distinguishes the small stock markets and the global financial markets.

Further research should continue to test the degree of long-term dependence, using alternative econophysics approaches and different time sizes, and to identify the level of integration of Euronext stock indexes.

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